Precedent Comparison in the Precedent Model Formalism: Theory and Application to Legal Cases*

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Abstract. Comparison between precedents and case facts is a core issue in case-based reasoning, which has been discussed in a lot of research. In this paper, we use a recently developed precedent model formalism to discuss precedent comparison in case-based reasoning. With this formalism and a case study in a real legal domain, we show a new generalization and a new refinement of precedent comparison with respect to case-based reasoning approaches based on factors, such as HYPO and CATO. 1) Generalization: precedents and case facts can now be compared with general propositional formulas, and not only with factors. 2) Refinement: a distinction can be made between current analogies and distinctions in precedent models, and so-called relevances, i.e., unshared formulas between two precedents that are relevant for possible additional analogies and distinctions that can arise in a discussion. With these contributions the role of factors in case-based reasoning can be refined and compound formulas based on factors can be taken into account in case-based reasoning.

Keywords: case-based reasoning· precedents· precedent comparison

1 Introduction

Case-based reasoning, one of the main legal reasoning types, has been discussed in the Artificial Intelligence and Law community for years. It allows for a form of analogical reasoning [1], and the core issue is how to make decisions for a current case by comparing precedents, namely the doctrine of *stare decisis*. As concluded by [1], there are three kinds of approaches for modeling case-based reasoning: prototype and deformation [2]; dimension and legal factor [3–7]; and exemplar-based explanation [8]. This paper follows the direction of factor-based approaches.

Case-based reasoning with factors has been formalized using many different approaches. For instance, abductive logic programming[9, 10], formal dialogue

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games [11], context-related frameworks [12], dialectical arguments [13], ontologies in OWL [14], the ASPIC+ framework [15], reason models [16], abstract argumentation [17], abstract dialectical frameworks [18] and case-based argumentation frameworks [19]. These works often discuss precedent comparison in terms of factors, following ideas developed in HYPO [3, 4].

In [20], a new formalism on modeling case-based reasoning has been discussed in a formal logical language. It can be used for evaluating the validity of arguments in legal reasoning. These models are based on the case model formalism [21], which has been implemented in a Prolog program [22]. The precedent models we present in that paper represent precedents as conjunctions of factors and outcomes. In our approach, factors are meant to represent generalized case facts relevant to the outcome of the case decision. However, unlike in CATO [6], our use of factors does not assume that factors favor a side of the decision, either pro-plaintiff or pro-defendant, as such an assumption is not needed for our logical definitions of precedent comparison. Unlike HYPO [4], our factors do not come with a dimension that can express a magnitude.

In [20], we show that precedents can be compared through the preference relation in precedent models, however, that paper only briefly mentions precedent comparison in the form of case-based reasoning, which is the focus of the present paper. In Section 2, we show the technical aspect of comparing precedents with our formalism which can compare precedents not only in terms of shared factors, but also in terms of other, more general shared propositional formulas, as also presented in [23]. The present paper is an extension of [23], we continue our discussion by applying our approach to HYPO-style comparison in terms of factors (Section 3.1), and discussing the approach in the context of a real legal domain (Section 3.2). We generalize case-based reasoning by comparing precedents with general formulas and refine case-based reasoning by introducing the new notion of relevances (Section 2.2). In this way, we show that comparing precedents with respect to general properties, represented by general propositional formulas, offers a novel angle on case-based reasoning.

2 The precedent model formalism

In this section, we present the precedent model formalism and apply it to precedent comparison in case-based reasoning (also shown in [23]). The precedent models defined in Section 2.1 are based on the case models formalism addressed by Verheij [21]. Precedent comparison is based on the analogies and distinctions defined in the case models [21].

2.1 Precedents and precedent models

The formalism introduced in this paper uses a propositional logic language L generated from a set of propositional constants. We fix language L. We write $\neg$ for negation, $\wedge$ for conjunction, $\vee$ for disjunction, $\leftrightarrow$ for equivalence. The associated classical, deductive, monotonic consequence relation is denoted $\vDash$.

Precedents consist of factors and outcomes. As explained in the introduction, our use of factors is related to but differs from other uses of factors in the literature, and we do not assume that factors favor a side since that is not necessary for our focus of logical precedent comparison. We consider both *factors* and *outcomes* are literals. A literal is either a propositional constant or its negation. We use $F \subseteq L$ to represent a set of factors, $O \subseteq L$ to represent a set of outcomes. The sets F and O are disjoint and consist only of literals. If a propositional constant p is in F (or O), then $\neg p$ is also in F (respectively in O). A factor represents an element of a case, namely a factual circumstance. Its negation describes the opposite fact. For instance, if a factor φ is "A kills B", then its negation $\neg\varphi$ is "A does not kill B". An outcome always favors a side in the precedent, its negation favors the opposite side. For instance, an outcome ω is "A is guilty", its negation $\neg\omega$ is "A is not guilty".

Following existing work in case-based reasoning, a *precedent* is a logical consistent conjunction of factors and outcomes. If a precedent contains an outcome, then we say it is a *proper precedent.* If a precedent doesn't have any outcome, then it is a *situation* that describes a current case. The outcomes of these situations need to be decided upon.

Definition 1. (Precedents) *A* precedent *is a logically consistent conjunction of distinct factors and outcomes* $\pi = \varphi_0 \wedge \varphi_1 \wedge \ldots \wedge \varphi_m \wedge \omega_0 \wedge \omega_1 \wedge \ldots \wedge \omega_{n-1}$*, where* m *and* n *are non-negative integers. We say that* $\varphi_0, \varphi_1, ..., \varphi_m$ *are the* factors *of* π*,* $\omega_0, \omega_1, ..., \omega_{n-1}$ *are the* outcomes *of* π*. If* $n = 0$*, then we say that* π *is a* situation *with no outcomes, otherwise* π *is a* proper precedent*.*

Notice that both m and n can be equal to 0. When $m = 0$, there is one single factor. When $n = 0$, the precedent has no outcome and the empty conjunction $\omega_0 \wedge \ldots \wedge \omega_{n-1}$ is equivalent to $\top$. We do not assume precedents are complete descriptions. That is, factors may exist which do not occur in the precedent. Furthermore, we do not assume that the negation of a factor holds when the factor does not occur in the precedent.

Example 1. Assume sentences $\pi_0, \pi_1 \in L$ are two precedents. $\pi_0 = f_1 \wedge f_2 \wedge f_3 \wedge o$, $\pi_1 = f_1 \wedge \neg f_2 \wedge \neg o$. f_1, f_2, $\neg f_2$ and f_3 are factors, o and $\neg o$ are outcomes.

Precedents can be compared through the preference relation between precedents in precedent models, which has been discussed in [20]. A *precedent model* is a set of logically incompatible precedents forming a total preorder representing a preference relation among the precedents.

Definition 2. (Precedent models) *A* precedent model *is a pair* $(P, \geq)$ *such that for all* $\pi, \pi' \in P$ *with* $\pi \neq \pi'$*,* $\pi \wedge \pi' \vDash \bot$*, and* $\geq$ *is a total preorder over* P*.*

As customary, the asymmetric part of $\geq$ is denoted $>$. The symmetric part of $\geq$ is denoted $\sim$. Let π, π' be two precedents, $\pi \geq \pi'$ means π is *at least as preferred as* π'; $\pi > \pi'$ means π is *more preferred than* π'; and $\pi \sim \pi'$ means π is *as preferred as* π'.

Example 2. Following Example 1, we assume π_0 is more preferred than π_1. In a precedent model with only precedents π_0 and π_1, the preference relation of this model is $\pi_0 > \pi_1$.

2.2 Comparing precedents in the formalism

Notions of comparing precedents in case-based reasoning include analogies, distinctions and relevances, they are related to general formulas, not only the factors or outcomes. *Analogies* between two precedents are the formulas that follow logically from both two precedents. *Distinctions* are the unshared formulas between two precedents, that only follow logically from one of the precedents and its negation is logically implied by the other precedent. *Relevances* are the unshared formulas between two precedents, that are relevant to the analogies and distinctions between them. These formulas only follow from one of the precedents, but both themselves and their negation are not logically implied by the other one.

Definition 3. (Analogies, distinctions and relevances) *Let $\pi, \pi' \in L$ be two precedents, we define:*
1. *a sentence $\alpha \in L$ is an* analogy between π and π' *if and only if $\pi \models \alpha$ and $\pi' \models \alpha$. A* most specific analogy between π and π' *is an analogy that logically implies all analogies between π and π'.*
2. *a sentence $\delta \in L$ is a* distinction in π with respect to π' (π-π' distinction) *if and only if $\pi \models \delta$ and $\pi' \models \neg\delta$. A* most specific π-π' distinction *is a distinction that logically implies all π-π' distinctions.*
3. *a sentence $\rho \in L$ is a* relevance in π with respect to π' (π-π' relevance) *if and only if $\pi \models \rho$, $\pi' \not\models \rho$ and $\pi' \not\models \neg\rho$. ρ is a* proper π-π' relevance *if and only if ρ is a π-π' relevance that logically implies the most specific analogy between π and π'. A most specific π-π' relevance is a relevance that logically implies all π-π' relevances.*

Both π-π' distinctions and π'-π distinctions are called the *distinctions between π and π'*. Both π-π' relevances and π'-π relevances are called the *relevances between π and π'*. When a most specific analogy/distinction/relevance exists it is by definition unique, and we can refer to it as the most specific analogy/distinction/relevance.

Example 3. Following Example 1, we have:
- Analogies between π_0 and π_1: *e.g.*, f_1, $f_1 \vee f_2$, $f_1 \vee f_3$;
- π_0-π_1 distinctions: *e.g.*, f_2, $f_2 \wedge o$, $f_2 \vee \neg f_1$;
- π_0-π_1 relevances: *e.g.*, f_3, $f_2 \vee f_3$;
- Proper π_0-π_1 relevances: *e.g.*, $f_1 \wedge f_3 \wedge (\neg o \vee f_2) \wedge (\neg f_2 \vee o)$.

Now we further discuss notions in Definition 3. Figure 1 illustrates analogies, distinctions and relevances using Venn diagrams representing sets of worlds in which sentences are true. As shown in Figure 1, for any analogy α between precedents π and π', the sets of π and π' worlds are subsets of the set of α

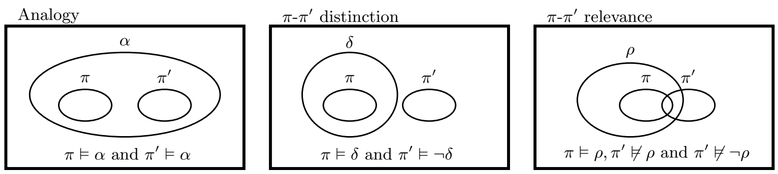

Fig. 1. Precedent comparison illustrated by worlds in which sentences are true

worlds; for any π-π' distinction δ, the π worlds are a subset of the δ worlds, while the π' worlds and the δ worlds are disjoint; for any π-π' relevance ρ, the π worlds are a subset of the ρ worlds, while the π' worlds and the ρ worlds are not subsets of each other and the intersection of the π' worlds and the ρ worlds are always not empty. The following proposition shows the properties of analogies, distinctions and relevances between precedents.

Proposition 1. *Let $\pi, \pi' \in L$ be precedents. Then the following holds:*

1. *The most specific analogy between π and π' always exists and is logically equivalent to $\pi \vee \pi'$;*
2. *There exists a distinction between π and π' if and only if $\pi \wedge \pi' \vDash \bot$; If a π-π' distinction exists, then the most specific π-π' distinction exists and is logically equivalent to π;*
3. *The most specific π-π' relevance does not always exist;*
4. *If the most specific π-π' distinction exists, then the most specific π-π' distinction logically implies each proper π-π' relevance. Each proper π-π' relevance logically implies the most specific analogy between π and π'.*

Proof. Let $\pi, \pi' \in L$ be precedents. For Property 1, by Definition 3, for any analogy α, $\pi \vDash \alpha$ and $\pi' \vDash \alpha$. By propositional logic it follows that any analogy α is logically implied by $\pi \vee \pi'$. By Definition 3, $\pi \vee \pi'$ is therefore a most specific analogy.

For Property 2, assume a π-π' distinction δ exists. By Definition 3, $\pi \vDash \delta$ and $\pi' \vDash \neg\delta$. It follows by propositional logic that $\pi \wedge \pi' \vDash \bot$. If $\pi \wedge \pi' \vDash \bot$, then by propositional logic $\pi' \vDash \neg\pi$. By Definition 3 and propositional logic, π is therefore the most specific π-π' distinction.

For Property 3, assume language L is generated from a set of propositional constants $\{f_1, f_2\}$. If $\pi = f_1$, $\pi' = \neg f_1$, the most specific π-π' relevance does not exist. π-π' relevances like $f_1 \vee f_2$, $f_1 \vee \neg f_2$ cannot be logically implied by a unique π-π' relevance.

For Property 4, by Property 2 if the most specific π-π' distinction exists, then it is logically equivalent to π. By Definition 3, π logically implies all π-π' relevances, including proper ones, and proper π-π' relevances always logically imply the most specific analogy between π and π'. □

As shown in Proposition 1, $\pi \vee \pi'$ is always the most specific analogy between π and π'. In legal case-based reasoning, the most specific analogy between two precedents we define here may seem to be counterintuitive. However, as the definition of precedent comparison is based on propositional logic, $\pi \vee \pi'$ is the only sentence that can logically imply all the analogies between two precedents.

The most specific π-π' distinction is logically equivalent to π if it exists. From the perspective of case-based reasoning with factors, such as HYPO, it is odd that the precedent itself is a distinction, but notice that here we represent precedents as conjunctions of factors and outcomes, hence precedents themselves can be considered as a formula. Since precedent comparison we define in this section aims at comparing all formulas in precedents, in this sense, we can say the precedent itself, as a propositional formula, is the most specific distinction.

Property 4 in Proposition 1 shows why we have singled out proper relevances: in the formally precise sense of the proposition, they are logically 'in between' the most specific distinction (if it exists) and the most specific analogy.

The following corollary shows a property of precedents in precedent models.

Corollary 1. *Let $(P, \geq)$ be a precedent model, for all π and $\pi' \in P$, the most specific π-π' distinction always exists.*

Proof. The corollary follows from Definition 2 and Property 2 of Proposition 1. □

Two precedents can be compared with a third precedent using the analogy relation defined below, which is similar to what is called on-pointness in HYPO [4]. The analogy relation is based on the shared formulas between precedents. When comparing precedents π and π' in terms of precedent π'', if the most specific analogy between π and π'' logically implies the most specific analogy between π' and π'', then we say that π is at least as analogous as π' with respect to π''. Formally we define the analogy relation as follows.

Definition 4. (Analogy relation between precedents) *Let π, π' and $\pi'' \in L$ be precedents. We define:*

$$\pi \succeq_{\pi''} \pi' \text{ if and only if } \pi \vee \pi'' \vDash \pi' \vee \pi''.$$

Then we say π is at least as analogous as *π' with respect to π''.*

As customary, the asymmetric part of the relation is denoted as $\pi \succ_{\pi''} \pi'$, which means π is *more analogous* than π' with respect to π''. The symmetric part of the relation is denoted as $\pi \sim_{\pi''} \pi'$, which means π is *as analogous as* π' with respect to π''. If it is not the case that $\pi \succeq_{\pi''} \pi'$ and $\pi' \succeq_{\pi''} \pi$, then we say π and π' are *analogously incomparable with respect to* π''.

Example 4. Comparing π_0 and π_1 in Example 1 in terms of precedent $\pi_2 = f_1 \wedge f_2$, we have $\pi_0 \succ_{\pi_2} \pi_1$; If $\pi_2 = f_1 \wedge \neg f_2$, then we have $\pi_1 \succ_{\pi_2} \pi_0$; If $\pi_2 = f_1$, then we have $\pi_0 \sim_{\pi_2} \pi_1$; If $\pi_2 = \neg f_3$, then π_0 and π_1 are analogously incomparable with respect to π_2.

Proposition 2. *Let π, π' and $\pi'' \in L$ be precedents. Then the following holds:*
1. *The analogy relation is reflexive and transitive, hence a preorder;*
2. *$\pi \succeq_{\pi''} \pi'$ if and only if $\pi \vDash \pi' \vee \pi''$;*
3. *If $\pi \succeq_{\pi''} \pi'$, then $\pi \succeq_{\pi'} \pi''$ and vice versa;*

4. *For any $\alpha \in L$, if $\pi \succeq_{\pi''} \pi'$, and α is an analogy between π' and π'', then α is also an analogy between π and π''.*

Proof. For property 1, the analogy relation is reflexive, since $\pi \vee \pi'' \vDash \pi \vee \pi''$. The relation is also transitive because of the transitivity of entailment in propositional logic. Assume $\pi = f_1 \wedge f_2$, $\pi' = f_1 \wedge f_3$ and $\pi'' = f_1 \wedge f_2 \wedge f_3$, π and π' are analogously incomparable with respect to π'', hence the relation is not in general total.

For Property 2, from left to right, by Definition 4 we obtain $\pi \vee \pi'' \vDash \pi' \vee \pi''$, and by propositional logic $\pi \vDash \pi' \vee \pi''$. From right to left, from $\pi \vDash \pi' \vee \pi''$ and propositional logic, we obtain $\pi \vee \pi'' \vDash \pi \vee \pi''$, and by Definition 4 $\pi \succeq_{\pi''} \pi'$.

Property 3 then follows directly from Property 2 by the commutativity of $\vee$.

Property 4 follows directly from Definition 3 and 4. □

Notice that if $\pi \succeq_{\pi''} \pi'$, then it is still possible that $\pi \nvDash \pi'$ and $\pi \nvDash \pi''$. For instance, if $\pi = f_1$, $\pi' = f_1 \wedge f_2$, $\pi'' = f_1 \wedge \neg f_2$, then we have $\pi \succeq_{\pi''} \pi'$, but both π' and π'' are not logically implied by π. Also notice that if $\pi \succeq_{\pi''} \pi'$, it cannot be concluded that $\pi \vDash \pi'$. For instance, $\pi = f_1 \wedge f_2$, $\pi' = f_3$ and $\pi'' = f_1$. In this example, $\pi \succeq_{\pi''} \pi'$ but $f_1 \wedge f_2 \nvDash f_3$.

3 Application: HYPO in the precedent model formalism

In this section, we formalize notions of comparison in HYPO with the precedent model formalism, and discuss them by a case study in a real legal domain. Factors in our approach are different from similar notions in HYPO (as dimensions) and CATO (as binary factors), which always favor a specific outcome.

3.1 Formalizing notions of comparison in HYPO

We assume that all factors in F favor a specific outcome in O, and $O = \{\omega, \neg\omega\}$, where ω stands for an outcome that one of the parties in the court wins the claim, $\neg\omega$ stands for the other party winning the claim. $F = F_\omega \cup F_{\neg\omega}$, such that for all $\varphi \in F$, if $\varphi \in F_\omega$, then $\neg\varphi \in F_{\neg\omega}$ and vice versa. F_ω stands for the factors that support outcome ω, $F_{\neg\omega}$ stands for the factors against outcome ω, namely supporting outcome $\neg\omega$.

In HYPO, shared factors between two cases are called *relevant similarity*, while the unshared factors are called *relevant difference*. Unshared factors can be used for pointing out the two cases should be decided differently. When comparing two precedents in terms of a current situation, HYPO always makes sure that the precedents are *on point* to the situation, namely they share at least one factor. If a precedent shares more factors with the situation than a second one, then this precedent is *more on point* than second with respect to the situation.

Let π, π' and π'' be precedents, $\omega \in O$ be an outcome of π'. Notions related to HYPO-style comparison can be formalized by the precedent model formalism as follows:

1. *Relevant similarity* $\psi_s \in L$ is the relevant similarity between π and π' if and only if ψ_s is the conjunction of all the factors that are analogies between π and π'.
2. *Relevant difference against* ω *in* π' $\psi_d \in L$ is the relevant difference against ω in π' between π and π' if and only if ψ_d is the conjunction of all the factors φ that are distinctions or relevances between π and π', such that:
   (a) if φ is a π-π' distinction or π-π' relevance, then $\varphi \in F_{\neg\omega}$; and
   (b) if φ is a π'-π distinction or π'-π relevance, then $\varphi \in F_{\omega}$.
3. *On pointness* Precedents π is *on point to* π' if and only if the relevant similarity between π and π' is not an empty conjunction. Assume $\psi_s \in L$ is the relevant similarity between precedents π and π'', $\psi'_s \in L$ is the relevant similarity between precedents π' and π'', π is *more on point* than π' with respect to π'' if and only if both π and π' are on point to π'' and $\psi_s \vDash \psi'_s$, $\psi'_s \nvDash \psi_s$.

Comparing with the notions defined in Definition 3, the relevant similarity between precedents is always an analogy between them, since the conjunction of factors as the relevant similarity can be logically implied by both of them. However, the relevant difference can not be simply equal to distinctions or relevances between precedents.

Notice that in on pointness, only factors are compared and there is no outcome involved in the comparison, however, in the analogy relation, both factors and outcomes are taken into account. Therefore, if precedent π is more on point than precedent π' with respect to precedent π'', then it is not always that $\pi \succeq_{\pi''} \pi'$. For instance, if $\pi = f_1 \wedge f_2 \wedge o$, $\pi' = f_1 \wedge \neg o$ and $\pi'' = f_1 \wedge f_2 \wedge f_3$, then it is obviously that π is more on point than π' with respect to π''. However, since $\pi \nvDash \pi' \vee \pi''$, π is not more analogous than π' with respect to π'', π and π' are analogously incomparable with respect to π''.

3.2 Case study

In this section, we use an example from a real legal domain to discuss precedent comparison introduced in Section 2.2 with notions of comparison in HYPO and comparison with preference relation in precedent models. This example has also been discussed in [1, 20].

Let $(P, P \times P)$ be a precedent model containing two precedents (the *Yokana* case[3] and the *American Precision* case[4]). Notice that $P \times P$ denotes the trivial preference relation where all precedents are as preferred as each other. The current situation is adapted from the *Mason* case[5]. We suppose outcome $\omega \in O$ is logically equivalent to Pla. As shown in Figure 2, *Yokana* favors defendant ($\neg$Pla) and *American Precision* favors plaintiff (Pla).

When comparing *Mason* with *Yokana* in $(P, P \times P)$ according to the notions in HYPO, we have:

[3] Midland-Ross Corp. v. Yokana, 293 F.2d 411 (3rd Cir.1961)
[4] American Precision Vibrator Co. v. National Air Vibrator Co., 764 S.W.2d 274 (Tex.App.-Houston [1st Dist.] 1988)
[5] Mason v. Jack Daniel Distillery, 518 So.2d 130 (Ala.Civ.App.1987)

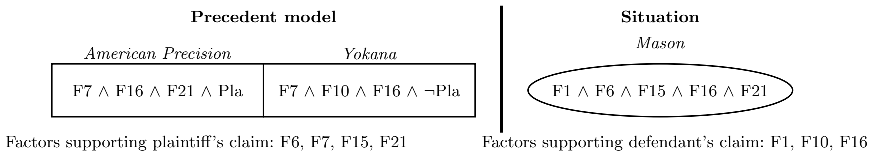

Fig. 2. Precedent model for the *Mason* case

- The relevant similarity between *Mason* and *Yokana*: F16;
- The relevant difference between *Mason* and *Yokana* against $\neg$Pla in *Yokana*: F6 $\wedge$ F15 $\wedge$ F21 $\wedge$ F10.

When comparing *Mason* with *Yokana* through the notions in Section 2.2:

- Analogies between *Yokana* and *Mason*: *e.g.*, F16, F16 $\vee$ F21;
- The most specific analogy between *Yokana* and *Mason*: (F7 $\wedge$ F10 $\wedge$ F16 $\wedge$ $\neg$Pla) $\vee$ (F1 $\wedge$ F6 $\wedge$ F15 $\wedge$ F16 $\wedge$ F21).
- *Mason*-*Yokana* relevances: *e.g.*, F6 $\wedge$ F15 $\wedge$ F21, F1 $\wedge$ F21;
- *Yokana*-*Mason* relevances: *e.g.*, F10, F16 $\wedge$ $\neg$Pla;
- There is no distinction between *Mason* and *Yokana*, as they are not incompatible.

When comparing *American Precision* and *Yokana* in $(P, P \times P)$ in terms of *Mason*, *American Precision* is more on point than *Yokana* with respect to *Mason*, however, *American Precision* and *Yokana* are analogously incomparable with respect to *Mason*.

According to different comparison relations (preference relation/analogy relation/on pointness), the selection of better precedent can be different. Notions in HYPO can be well-defined with the precedent model formalism. The analysis also shows that HYPO-style precedent comparison is different from the comparison in Section 2.2. We will further discuss these points in Section 4.

4 Discussion and conclusion

In this paper, we discuss precedent comparison in case-based reasoning with the precedent model formalism, which is described in a formal propositional logic language. Unlike other case-based reasoning models, in which precedents are represented as dimensions [4], sets of rules [11], sets of factors [15], combinations of rules, facts and outcome [16] and hierarchies [6, 18]. The formalism we present here represents precedents and current situations with conjunctions of factors and outcomes. Comparing with the case model formalism in [21], we give a more concrete account of precedents. The representation we use here allows us to discuss case-based reasoning from a perspective that is closer to logic, thereby allowing the comparison of precedents in terms of general formulas. In this way, we are able to present a new generalization and a new refinement of precedent comparison in case-based reasoning with the precedent model formalism.

The new generalization of precedent comparison in case-based reasoning Case-based reasoning models following HYPO often discuss comparison between precedents in terms of factors. In the formalism we present here, we generalize the comparison approach in case-based reasoning, namely comparing precedents not only with factors, but also with more general propositional formulas.

Section 3.1 shows a key difference between our comparison approach and formalizations that follow HYPO [4, 6, 16]. Factors in HYPO-style comparison typically favor a side in the court case, showing which factors can strengthen or weaken the arguments given by the parties involved, hence constraining possible argument moves. However, the more general formulas used in our comparison approach may not favor a specific side in the court (as shown in Section 2). The formulas we discuss are more general logical expressions than factors.

As shown in Section 3.2, when comparing *American Precision* and *Yokana* in terms of the analogy relation defined in Definition 4, the result of this comparison is different from other comparison relations, namely the preference relation in precedent models and the on pointness in HYPO. According to the preference relation in $(P, P \times P)$, *American Precision* and *Yokana* are as preferred as each other; according to the on pointness, *American Precision* is more on point with respect to *Mason*; and according to the analogy relation, these two precedents are analogously incomparable with respect to *Mason*. This is because the analogy relation discusses comparison in terms of the most specific analogy between precedents, while other comparison relations are in terms of other notions.

The new comparison approach allows us to discuss general formulas beyond factors in case-based reasoning, such as conjunctions or disjunctions of factors, which can bring new discussion on case-based reasoning with the precedent model formalism. For instance, for future research we can discuss hierarchical factors shown in CATO [6], as higher level factors can be represented with compound formulas based on base-level factors. Therefore, it seems possible to compare abstract factors between precedents directly, and discuss argument moves like downplaying a distinction in the formalism.

The new refinement of precedent comparison in case-based reasoning In [20], we don't distinguish the distinctions and relevances between precedent, nor do HYPO [4] and other case-based reasoning models [6, 15]. In the formalism we present here, relevances between precedents are distinguished from analogies and from distinctions. While analogies between two precedents refer to formulas that hold in both precedents, and distinctions to formulas that hold in one precedent and are negated in the other, relevances are formulas that are not yet determined in a precedent and hence have the potential to turn out as an analogy or distinction once determined. Although both distinctions and relevances are related to the unshared factors, these formulas cannot be considered as distinctions directly, since if such relevant formulas in a precedent can be found in a situation, they will be considered as analogies rather than distinctions between the precedent and the situation.

As shown in Section 3.1, the relevant similarity and the relevant difference between cases in HYPO can be formalized by analogies, distinctions and relevances defined in Definition 3. Although the relevant similarity is an analogy between precedents, factors in the relevant difference are not always distinctions between precedents, but also can be relevances. In this sense, our approach compares precedents in a more specific way than HYPO. However, as we haven't defined dimension of factors in the formalism, it is unable to discuss magnitude of factors in relevant differences, which means we cannot compare precedents in terms of dimensions, such as finding a contrary case which has some factors with extreme magnitude. This needs further discussion in the future.

This refinement also points to case-based reasoning in a dynamic scenario, in which the situation can change accordingly when new facts are found. For instance, in the example shown in Figure 2, when comparing *Yokana* with *Mason*, F1 $\wedge$ F21 is a relevance between them. If these two factors can be found in *Yokana*, then it will be more on point than *American Precision* with respect to *Mason*. However, if the $\neg$F21 is implied by *Yokana*, then F21 and $\neg$F21 will become the distinctions between these two cases.

Continuing from the preliminary report [20] and the technical note [23], in this paper, we applied the approach of precedent comparison to a real legal domain, and discuss it with the case comparison in HYPO [4]. With the precedent model formalism, we provide a way that both generalizes and refines case-based reasoning with factors. We discuss not only the shared factors between precedents, but also other compound formulas based on factors, which allows us to compare precedents from a more logical perspective and discuss other features among precedents. In this way, we show a new generalization of case-based reasoning with factors. We further distinguish the unshared formulas as distinctions and as relevances, i.e., unshared formulas between two precedents that are relevant for the analogies and distinctions that can arise in a discussion. In this way, we show a new refinement of case-based reasoning with factors. These ideas show that the precedent model formalism has the potential to help analyze argument moves in case-based reasoning and support the selection of good precedents to cite in a court discussion, these still need further research in the future. It would also be interesting to investigate computational mechanisms to discover good arguments move in a legal discussion using precedent models.

References

1. K. D. Ashley. *Artificial Intelligence and Legal Analytics: New Tools for Law Practice in the Digital Age.* Cambridge University Press, Cambridge, 2017.
2. L. Thorne McCarty. An implementation of Eisner v. Macomber. In *Proceedings of the Fifth International Conference on Articial Intelligence and Law*, ICAIL 1995, pages 276–286. ACM, New York, 1995.
3. E. L. Rissland and K. D. Ashley. A Case-Based System for Trade Secrets Law. In *Proceedings of the 1st International Conference on Artificial Intelligence and Law*, ICAIL 1987, pages 60–66. ACM, New York, 1987.
4. K. D. Ashley. *Modeling Legal Arguments: Reasoning with Cases and Hypotheticals.* MIT Press, Cambridge, 1990.
5. D. B. Skalak and E. L. Rissland. Arguments and Cases: An Inevitable Intertwining. *Artificial Intelligence and Law*, 1(1):3–44, 1992.
6. Vincent Aleven. *Teaching case-based argumentation through a model and examples.* PhD thesis, University of Pittsburgh, 1997.

7. K. D. Ashley and S. Brüninghaus. Automatically classifying case texts and predicting outcomes. *Artificial Intelligence and Law*, 17(2):125–165, 2009.
8. L. K. Branting. Building explanations from rules and structured cases. *International Journal of Man-Machine Studies*, 34(6):797–837, 1991.
9. K. Satoh. Translating Case-Based Reasoning into Abductive Logic Programming. In W. Wahlster, editor, *Proceedings of the 12th European Conference on Artificial Intelligence, ECAI 1996*, pages 142–146. Wiley, Chichester, 1996.
10. K. Satoh. Using Two Level Abduction to Decide Similarity of Cases. In Prade H., editor, *Proceedings of the 13th European Conference on Artificial Intelligence, ECAI 1998*, pages 398–402. Wiley, Chichester, 1998.
11. H. Prakken and G. Sartor. Modelling Reasoning with Precedents in a Formal Dialogue Game. *Artificial Intelligence and Law*, 6:231–287, 1998.
12. C. L. Hafner and D. H. Berman. The Role of Context in Case-Based Legal Reasoning: Teleological, Temporal, and Procedural. *Artificial Intelligence and Law*, 10(1–3):19–64, 2002.
13. B. Roth and B. Verheij. Dialectical Arguments and Case Comparison. In T.F. Gordon, editor, *Legal Knowledge and Information Systems. JURIX 2004: The Seventeenth Annual Conference*, pages 99–108. IOS Press, Amsterdam, 2004.
14. A. Wyner. An ontology in OWL for legal case-based reasoning. *Artificial Intelligence and Law*, 16(4):361, 2008.
15. H. Prakken, A. Wyner, T. Bench-Capon, and K. Atkinson. A formalization of argumentation schemes for legal case-based reasoning in ASPIC+. *Journal of Logic and Computation*, 25(5):1141–1166, 05 2013.
16. J. Horty and T. Bench-Capon. A factor-based definition of precedential constraint. *Artificial Intelligence and Law*, 20(2):181–214, 2012.
17. K. Cyras, K. Satoh, and F. Toni. Abstract Argumentation for Case-Based Reasoning. In *Proceedings of the Fifteenth International Conference on Principles of Knowledge Representation and Reasoning (KR 2016)*, pages 549–552. AAAI Press, 2016.
18. L. Al-Abdulkarim, K. Atkinson, and T. Bench-Capon. A methodology for designing systems to reason with legal cases using Abstract Dialectical Frameworks. *Artificial Intelligence and Law*, 24(1):1–49, 2016.
19. H. Prakken. Comparing Alternative Factor- and Precedent-based Accounts of Precedential Constraint. In M. Araszkiewicz and V. Rodriguez-Doncel, editors, *Legal Knowledge and Information Systems. JURIX 2019: The Thirty-Second Annual Conference*, pages 73–82. IOS Press, Amsterdam, 2019.
20. H. Zheng, D. Grossi, and B. Verheij. Case-Based Reasoning with Precedent Models: Preliminary Report. In H. Prakken, S. Bistarelli, F. Santini, and C. Taticchi, editors, *Computational Models of Argument. Proceedings of COMMA 2020*, volume 326, pages 443–450. IOS Press, Amsterdam, 2020.
21. B. Verheij. Formalizing Arguments, Rules and Cases. In *Proceedings of the Sixteenth International Conference on Articial Intelligence and Law*, ICAIL 2017, pages 199–208. ACM, New York, 2017.
22. H. Zheng, M. Xiong, and B. Verheij. Checking the Validity of Rule-Based Arguments Grounded in Cases: A Computational Approach. In M. Palmirani, editor, *Legal Knowledge and Information Systems. JURIX 2018: The Thirty-first Annual Conference*, volume 313, pages 220 – 224. IOS Press, Amsterdam, 2018.
23. H. Zheng, D. Grossi, and B. Verheij. Precedent Comparison in the Precedent Model Formalism: A Technical Note (to appear). In *Legal Knowledge and Information Systems. JURIX 2020: The Thirty-third Annual Conference*. IOS Press, Amsterdam, 2020.