

Formalizing Arguments, Rules and Cases

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ABSTRACT

Legal argument is typically backed by two kinds of sources: cases and rules. In much AI & Law research, the formalization of arguments, rules and cases has been investigated. In this paper, the tight formal connections between the three are developed further, in an attempt to show that cases can provide the logical basis for establishing which rules and arguments hold in a domain. We use the recently proposed formalism of case models, that has been applied previously to evidential reasoning and ethical systems design. In the present paper, we discuss with respect to case-based modeling how the analogy and distinction between cases can be modeled, and how arguments can be grounded in cases. With respect to rule-based modeling, we discuss conditionality, generality and chaining. With respect to argument-based modeling, we discuss rebutting, undercutting and undermining attack. We evaluate the approach by developing a case model of the rule-based arguments and attacks in Dutch tort law. In this way, we illustrate how statutory, rule-based law from the civil law tradition can be formalized in terms of cases.

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1 INTRODUCTION

Legislation and precedents are primary sources for the backing of legal arguments, and each of these two kinds is typically associated with a specific style of reasoning: legislation with rule-based reasoning, and precedents with case-based reasoning. In rule-based reasoning, rules backed by legislation are followed when they apply in the current case, and in case-based reasoning, cases with precedential authority are adhered to when they match the current case. Both kinds of reasoning are defeasible. In rule-based reasoning, there can be an exception to an applying rule, and in case-based reasoning, adherence to a matching case can be overruled by another case that is a better match.

In Artificial Intelligence and Law, such defeasible reasoning backed by rules and cases has productively been modeled in terms of arguments for and against possible conclusions. Formal and computational models have been proposed that investigate relations between arguments, rules and cases in various ways. For instance, cases have been studied as the source of hypothetical

arguments (Aleven and Ashley 1995; Ashley 1990; Rissland and Ashley 1987), rules and cases have been studied for the construction of explanations of decisions (Branting 1991, 1993), rules and cases have been used for the construction of arguments (Prakken and Sartor 1996, 1998), and cases and the values they promote have been used to establish rules (Bench-Capon and Sartor 2003).

This and related work has shown that the formal and computational relations between arguments, rules and cases are close. The present paper aims to further develop the close formal relations between arguments, rules and cases.

For this aim, we use the recently proposed case model formalism that was previously applied to evidential reasoning and ethical system design (Verheij 2016a,b, 2017). The case model formalism was developed in an attempt to answer the semantics and normative questions for reasoning with presumptive arguments (Verheij 2016a): How are presumptive arguments grounded in interpretations; and when are they evaluated as correct? In that work, the case model formalism is shown to have equivalent qualitative and quantitative characterizations, connecting to classical logic and probability theory. Hence the formalism is simultaneously ‘with and without numbers’, and the case model formalism could be applied to evidential reasoning, involving arguments, scenarios and probabilities (Verheij 2014, 2017). In contrast with Bayesian network approaches connecting arguments, scenarios and probabilities that require the specification of a full probability distribution (Fenton et al. 2013; Hepler et al. 2007; Timmer et al. 2017; Vlek et al. 2014, 2016), the case model formalism does not require more numbers than are available. The balance between qualitative and quantitative modeling in the case model formalism was also applied to ethical systems design (Verheij 2016b). In that paper, a series of New York tort cases—earlier analyzed for the study of value-guided teleological reasoning in the law (Berman and Hafner 1995; Hafner and Berman 2002)—were formalized in the case model formalism to show the context-dependence, value-dependence and rule-dependence of ethical decision making.

The contribution of the present paper is a discussion of themes in case-based, rule-based and argument-based modeling, all using the same case model formalism. With respect to case-based modeling, we discuss the themes of analogies, distinctions and argument grounding. With respect to rule-based modeling, we discuss conditionality, generality and chaining. With respect to argument-based modeling, we discuss rebutting attack, undercutting attack and undermining attack. The proposal is evaluated by modeling Dutch tort law. That is an example domain from the rule-based, civil law tradition, and we model it in terms of the case model formalism.

By developing the formal relations between arguments, rules and cases in this way, we contribute to the explanation of the fact that comparative law research has shown that the roles of legislation and precedents as sources of arguments are closely connected in different legal systems, both in common law and in civil law

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countries with their opposing emphases on either precedents or legislation (MacCormick and Summers 1997).

Section 2 provides background material for the paper, namely a discussion of the formalism of case models and of Dutch tort law. In Sections 3, 4, and 5, we discuss characteristics connected to case-based, rule-based and argument-based reasoning in terms of the case model formalism. Section 6 provides an evaluation of the approach by developing a formalization of Dutch tort law. In Section 7, the approach is discussed in relation to previous research in AI & Law, followed by the concluding Section 8.

2 BACKGROUND

2.1 Formalism

The case model formal model used here was first presented in (Verheij 2016a), formalizing a semi-formal presentation in (Verheij 2014). The formalism was inspired by the setting of reasoning with evidence, where qualitative and quantitative reasoning methods are used. The case model formalism was applied to the combination of arguments, scenarios and probabilities as tools in evidential reasoning (Verheij 2017) and to value-guided argumentation in the context of ethical systems design (Verheij 2016b). Here we repeat core definitions, referring to the other publications for additional explanation and context. For present purposes, we focus on the qualitative characterization of case models.

The formalism uses a classical logical language L generated from a set of propositional constants in a standard way. We write \neg for negation, \wedge for conjunction, \vee for disjunction, \leftrightarrow for equivalence, \top for a tautology, and \perp for a contradiction. The associated classical, deductive, monotonic consequence relation is denoted \models . We assume a language generated by a finite set of propositional constants.

The central definition is that of case models. The cases in a case model must be logically consistent, mutually incompatible and different; and the comparison relation must be total and transitive (hence is what is called a total preorder, commonly modeling preference relations; Roberts 1985).

Definition 2.1. A *case model* is a pair (C, \geq) with finite $C \subseteq L$, such that the following hold, for all φ, ψ and $\chi \in C$:

- (1) $\not\models \neg\varphi$;
- (2) If $\not\models \varphi \leftrightarrow \psi$, then $\models \neg(\varphi \wedge \psi)$;
- (3) If $\models \varphi \leftrightarrow \psi$, then $\varphi = \psi$;
- (4) $\varphi \geq \psi$ or $\psi \geq \varphi$;
- (5) If $\varphi \geq \psi$ and $\psi \geq \chi$, then $\varphi \geq \chi$.

The strict weak order $>$ standardly associated with a total preorder \geq is defined as $\varphi > \psi$ if and only if it is not the case that $\psi \geq \varphi$ (for φ and $\psi \in C$). When $\varphi > \psi$, we say that φ is (strictly) preferred to ψ . The associated equivalence relation \sim is defined as $\varphi \sim \psi$ if and only if $\varphi \geq \psi$ and $\psi \geq \varphi$.

Example. Figure 1 shows a case model with cases $\neg p, p \wedge q$ and $p \wedge \neg q$. $\neg p$ is (strictly) preferred to $p \wedge q$, which in turn is preferred to $p \wedge \neg q$.

Although the preference relations of case models are qualitative, they correspond precisely to the relations that can be represented by real-valued functions, hence provide a formally optimal balance between a qualitative and quantitative representation. A numeric

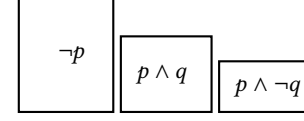


Figure 1: Example of a case model

representing function can be chosen to formally behave like a probability function.

Next we define arguments from premises $\varphi \in L$ to conclusions $\psi \in L$.

Definition 2.2. An *argument* is a pair (φ, ψ) with φ and $\psi \in L$. The sentence φ expresses the argument's premises, the sentence ψ its conclusions, and the sentence $\varphi \wedge \psi$ the *case made* by the argument. Generalizing, a sentence $\chi \in L$ is a *premise* of the argument when $\varphi \models \chi$, a *conclusion* when $\psi \models \chi$, and a *position* in the case made by the argument when $\varphi \wedge \psi \models \chi$. An argument (φ, ψ) is *properly presumptive* when $\varphi \not\models \psi$; otherwise *non-presumptive*. An argument (φ, ψ) is an *presumption* when $\models \varphi$, i.e., when its premises are logically tautologous.

Note our use of the plural for an argument's premises, conclusions and positions. This terminological convention allows us to speak of the premises p and $\neg q$ and conclusions r and $\neg s$ of the argument $(p \wedge \neg q, r \wedge \neg s)$. Also the convention fits our non-syntactic definitions, where for instance an argument with premise χ also has logically equivalent sentences such as $\neg\neg\chi$ as a premise.

A coherent argument is defined as an argument that makes a case logically implied by a case in the case model. A conclusive argument is a coherent argument, for which all cases in the case model that imply the argument's premises also imply the conclusions.

Definition 2.3. (Coherent and conclusive arguments) Let (C, \geq) be a case model. Then we define, for all φ and $\psi \in L$:

$$(C, \geq) \models (\varphi, \psi) \text{ if and only if } \exists \omega \in C: \omega \models \varphi \wedge \psi.$$

We then say that the argument from φ to ψ is *coherent* with respect to the case model. We define, for all φ and $\psi \in L$:

$$(C, \geq) \models \varphi \Rightarrow \psi \text{ if and only if } \exists \omega \in C: \omega \models \varphi \wedge \psi \text{ and } \forall \omega \in C: \text{if } \omega \models \varphi, \text{ then } \omega \models \psi.$$

We then say that the argument from φ to ψ is *conclusive* with respect to the case model.

Example (continued). In the case model of Figure 1, the arguments from \top to $\neg p$ and to p , and from p to q and to $\neg q$ are coherent and not conclusive in the sense of this definition. Denoting the case model as (C, \geq) , we have $(C, \geq) \models (\top, \neg p)$, $(C, \geq) \models (\top, p)$, $(C, \geq) \models (p, q)$ and $(C, \geq) \models (p, \neg q)$. The arguments from a case (in the case model) to itself, such as from $\neg p$ to $\neg p$, or from $p \wedge q$ to $p \wedge q$ are conclusive. The argument $(p \vee r, p)$ is also conclusive in this case model, since all $p \vee r$ -cases are p -cases. Similarly, $(p \vee r, p \vee s)$ is conclusive.

The notion of presumptive validity considered here is based on the idea that some arguments make a better case than other arguments from the same premises. More precisely, an argument is presumptively valid if there is a case in the case model implying the case made by the argument that is at least as preferred as all cases implying the premises.

Definition 2.4. (Presumptively valid arguments) Let (C, \geq) be a case model. Then we define, for all φ and $\psi \in L$:

- $(C, \geq) \models \varphi \rightsquigarrow \psi$ if and only if $\exists \omega \in C$:
- (1) $\omega \models \varphi \wedge \psi$; and
 - (2) $\forall \omega' \in C : \text{if } \omega' \models \varphi, \text{ then } \omega \geq \omega'$.

We then say that the argument from φ to ψ is *presumptively valid* with respect to the case model. A presumptively valid argument is *properly defeasible*, when it is not conclusive.

Example (continued). In the case model of Figure 1, the arguments from \top to $\neg p$, and from p to q are presumptively valid in the sense of this definition. Denoting the case model as (C, \geq) , we have formally that $(C, \geq) \models \top \rightsquigarrow \neg p$ and $(C, \geq) \models p \rightsquigarrow q$. The coherent arguments from \top to p and from p to $\neg q$ are not presumptively valid in this sense.

The three notions of validity (coherence, conclusiveness, presumptive validity) are related, as follows. Conclusive arguments are coherent, but there are case models with a coherent, yet inconclusive argument. Conclusive arguments are presumptively valid, but there are case models with a presumptively valid, yet inconclusive argument. Presumptively valid arguments are coherent, but there are case models with a coherent, yet presumptively invalid argument.

We list some of the logical properties of presumptive validity (cf. properties of nonmonotonic inference relations (Kraus et al. 1990; Makinson 1994; van Benthem 1984)). Given a case model (C, \geq) , we write $\varphi \vdash \psi$ for $(C, \geq) \models \varphi \rightsquigarrow \psi$. We write $C(\varphi)$ for the set $\{\omega \in C \mid \omega \models \varphi\}$.

For all φ, ψ and $\chi \in L$:

- (LE) If $\varphi \vdash \psi, \models \varphi \leftrightarrow \varphi'$ and $\models \psi \leftrightarrow \psi'$, then $\varphi' \vdash \psi'$.
 (Cons) $\varphi \not\vdash \perp$.
 (Ant) If $\varphi \vdash \psi$, then $\varphi \vdash \varphi \wedge \psi$.
 (RW) If $\varphi \vdash \psi \wedge \chi$, then $\varphi \vdash \psi$.
 (CCM) If $\varphi \vdash \psi \wedge \chi$, then $\varphi \wedge \psi \vdash \chi$.
 (CCT) If $\varphi \vdash \psi$ and $\varphi \wedge \psi \vdash \chi$, then $\varphi \vdash \psi \wedge \chi$.

Some key properties use a subset L^* of the language L . The set L^* consists of the logical combinations of the cases of the case model using negation, conjunction and logical equivalence (cf. the algebra underlying probability functions (Roberts 1985)). L^* is the set of *case expressions* associated with a case model.

Definition 2.5. Let (C, \geq) be a case model, $\varphi \in L$, and $\omega \in C$. Then ω expresses a *preferred case* of φ if and only if $\varphi \vdash \omega$.

Writing \vdash^* for the restriction of \vdash to L^* , we have, for all φ, ψ and $\chi \in L^*$:

- (Coh) $\varphi \vdash \varphi$ if and only if $\exists \varphi^* \in L^*$ with $\varphi^* \not\vdash \perp$ and $\varphi^* \models \varphi$;
 (Ch) If $\varphi \vdash^* \varphi$ and $\psi \vdash^* \psi$, then $\varphi \vee \psi \vdash^* \neg \varphi \wedge \psi$ or $\varphi \vee \psi \vdash^* \varphi \wedge \psi$ or $\varphi \vee \psi \vdash^* \varphi \wedge \neg \psi$;
 (OC) If $\varphi \vee \psi \vdash^* \varphi$ and $\psi \vee \chi \vdash^* \psi$, then $\varphi \vee \chi \vdash^* \varphi$.

Verheij (2016a) provides qualitative and quantitative representation results that show that the listed properties characterize presumptive validity.

2.2 Dutch tort law

In order to illustrate and evaluate our proposal, we aim to model the core rule-based arguments and attacks in Dutch tort law as grounded in a case model. In Dutch Tort law, a so-called 'onrechtmatige daad', or wrongful act, can lead to liability for the damages that follow. For instance, if you bump into another car while parking, you typically must pay for the damages incurred.

The articles 6:162 and 6:163 of the Dutch civil code (in the Netherlands referred to as Art. 6:162 and 6:163 BW, BW for 'Burgerlijk Wetboek') govern the handling of wrongful acts. Here follows the translation by Betlem (1993), also used by Verheij et al. (1997):

- Art. 6:162 BW.** 1. A person who commits an unlawful act toward another which can be imputed to him, must repair the damage which the other person suffers as a consequence thereof.
 2. Except where there is a ground of justification, the following acts are deemed to be unlawful: the violation of a right, an act or omission violating a statutory duty or a rule of unwritten law pertaining to proper social conduct.
 3. An unlawful act can be imputed to its author if it results from his fault or from a cause for which he is answerable according to law or common opinion.

Art. 6:163 BW. There is no obligation to repair damage when the violated norm does not have as its purpose the protection from damage such as that suffered by the victim.

As specified in Art. 6:162.1 BW, a duty to repair someone's damages can be established when four conditions are fulfilled:

- (1) *Someone has suffered damages by someone else's act.* For instance, the car parked into has a dent in a door panel.
- (2) *The act committed was unlawful.* In the example, the unlawfulness follows from the ownership of the damaged car.
- (3) *The act can be imputed to the person that committed the act.* In the example, it can be said that causing damages because of bumping into another car is your own fault.
- (4) *The act caused the suffered damages.* The door panel was pristine, and now has a dent.

Three kinds of unlawful acts are distinguished (Art. 6:162.2 BW):

- (1) *The act is a violation of someone's right.* In the example, the car owner's right to ownership was violated.
- (2) *The act is a violation of a statutory duty.* Examples are acts that are punishable in the sense of the Dutch criminal code or other statutes. Here we explicitly see the civil law focus on statutes. Note that also omissions ('doing nothing') can count as a wrongful act, for instance not saving someone who is in life danger.
- (3) *The act is a violation of unwritten law against proper social conduct.* In the landmark Lindenbaum-Cohen case (Supreme Court of the Netherlands, January 31, 1919, NJ 1919), this kind of wrongful act was established in a case where rights and statutory duties did not suffice for establishing liability. In the case, Lindenbaum, a printer, suffered damages because his competitor Cohen could adapt his offers by the information he got from one of Lindenbaum's servant. The servant did not violate a right or written

statutory duty, but the Supreme Court decided for liability referring to proper social conduct.

Art. 6:162.2 BW also explicates an exception to unlawfulness: the existence of grounds of justification. Examples of such grounds include force majeure, in particular a conflict of duties as they can occur in a life-endangering situation, and commands by an authority such as a police officer. In the literal text, the exception refers to the three kinds of exceptions, but Dutch doctrine often takes it that grounds of justification only occur for the first two kinds. The idea is that a violation of unwritten law against proper social conduct already implies that there is no ground of justification (e.g., Asser-Hartkamp (1998), no. 77).

In Art. 6:162.3 BW, the three forms of imputing an unlawful act to a person are specified:

- (1) *The act is imputable to someone because of the person's fault.* In the example, bad parking is your own fault.
- (2) *The act is imputable to someone because of law.* In some cases where an act is not someone's fault the legislator has decided for imputability anyway. For instance, Art 6:165.1 BW (not translated here) allows for (but does not prescribe) imputability in case of a physical or mental disability, but not for an omission to act. The legislator thought of a deaf person who does not hear the screaming of a drowning person, hence omits to save that person.
- (3) *The act is imputable to someone because of common opinion.* This statutory clause was new in the 1992 fully rewritten Dutch Civil Code and reflects a Supreme Court decision about a car accident (November 11, 1983, NJ 1984, 331). In the case, a roe deer suddenly jumped onto an 80 km/h road. A driver, trying to prevent a collision, steered his car to the left lane, crashing into a car coming from the opposite direction. The Supreme Court decided that the act was not imputable as a fault of the driver, but established liability.

Art. 6:163, the relativity constraint, is used as a general exception to the establishment of liability on the basis of Art. 6:162 BW. It provides for cases where a damage-causing act is a violation of a statutory duty, but that statute's aim is not to prevent those damages. For instance, when a dentist suffers damages because of the competition of a nearby, unlicensed dentist, these damages cannot be recovered on the basis of the lack of a license. Reason: the licensing rules are meant to protect the dentist's customers, not the dentist's competitors.

The above discussion of Dutch tort law suffices for present purposes. For further details and subtleties, the interested reader is referred to the discussion by (Verheij et al. 1997).

3 CASES AND ANALOGY

We discuss three themes of case-based reasoning: analogies, distinctions and argument grounding.

Analogy In case-based reasoning, the current situation is compared to the cases that serve as prototypes or exemplars. The result of such comparison is a specification of analogies and distinctions between a situation and the cases. A specific role for cases in case-based reasoning is that they can be used to suggest possible arguments.

We say that cases (and other situations expressed by sentences of the language) are analogous when they have a common property.

Definition 3.1. Let (C, \geq) be a case model. A sentence $\alpha \in L$ expresses an *analogy* of a case $\omega \in C$ and the situation expressed by $\sigma \in L$ if $\omega \models \alpha$ and $\sigma \models \alpha$. Then the case and situation are *analogous* by sharing the property α .

Example 3.2. Consider a case model with two cases $\neg a$ and $a \wedge b$. A situation $a \wedge c$ and the case $a \wedge b$ have analogy a . The logically most general analogy between this case and situation is $(a \wedge b) \vee (a \wedge c)$.

The case $\neg a$ is also analogous to the situation $a \wedge c$ since the case and situation share the property $\neg a \vee c$, also the most general analogy between $\neg a$ and $a \wedge c$. No elementary proposition or its negation is an analogy between the case and situation.

The example illustrates the following property of analogy.

PROPOSITION 3.3. *Let ω be a case $\in C$ and $\sigma \in L$. Then:*

- (1) \top is an analogy between the case and the situation.
- (2) $\alpha \in L$ is an analogy between the case and the situation if and only if $\omega \vee \sigma \models \alpha$.

Distinction We turn to distinctions. There are two kinds: case distinctions and situation distinctions.

Definition 3.4. Let (C, \geq) be a case model. A sentence $\delta \in L$ expresses a *case distinction* between a case $\omega \in C$ and the situation expressed by $\sigma \in L$ if $\omega \models \delta$ and $\sigma \not\models \delta$. A sentence $\delta \in L$ expresses a *situation distinction* between a case $\omega \in C$ and the situation expressed by $\sigma \in L$ if $\omega \not\models \delta$ and $\sigma \models \delta$. When δ is a case distinction or a situation distinction, the case and situation are *distinct* by not sharing the property δ .

Example 3.5. Consider again the case model with equally preferred cases $\neg a$ and $a \wedge b$, and the comparison with situation $a \wedge b$. b is a case distinction between the case $a \wedge b$ and the situation $a \wedge c$. $\neg a$ is a case distinction between the case $\neg a$ and this situation. c is a situation distinction between the case $a \wedge b$ and this situation.

Grounding A specific role for cases in case-based reasoning is that they can be used to suggest possible arguments. We say that an argument (φ, ψ) has grounding in a case ω in the case model when $\varphi \wedge \psi$ (i.e., the case made by the argument) logically follows from ω .

Definition 3.6. An argument (φ, ψ) has *grounding* in case ω if $\omega \models \varphi \wedge \psi$.

Example 3.7. An argument (a, b) making the case $a \wedge b$ has grounding in the case $a \wedge b \wedge c$. The logically most general case in which an argument has grounding is the case made by the argument.

The grounding of an argument has a straightforward formal connection to its coherence, as follows.

PROPOSITION 3.8. *Let (C, \geq) be a case model and (φ, ψ) an argument. Then the following are equivalent:*

- (1) (φ, ψ) is coherent.
- (2) There is an $\omega \in C$, such that (φ, ψ) has grounding in ω .

4 RULES AND SUPPORT

We discuss three themes related to rules and support: conditionality, generality and chaining.

Conditionality The rules used in rule-based reasoning are connected to the support of conclusions. Such support has a conditional nature, with the antecedent of a rule providing support for the consequent of the rule. The conditionality that comes with rules is already apparent in our notion of arguments (Definition 2.2). We give an example of the conditional nature of a conclusive argument and of a presumptively valid argument.

Example 4.1. Consider the case model with three cases $\neg a \wedge \neg b$, $\neg a \wedge b$ and $a \wedge b$, all equally preferred. The argument (a, b) is conclusive in this case model. The argument's condition a is needed for concluding b , since the argument (\top, b) is not conclusive. That argument is presumptively valid, though. Note also that reversing premises and conclusions gives (b, a) , an argument that is conclusive.

The example must be adapted in the case of a presumptively valid argument.

Example 4.2. Consider the case model with the same three cases $\neg a \wedge \neg b$, $\neg a \wedge b$ and $a \wedge b$, but now with a non-trivial preference ordering: $\neg a \wedge \neg b > \neg a \wedge b > a \wedge b$. Now the argument (a, b) is presumptively valid (even conclusive), but (\top, b) and (b, a) are not presumptively valid.

Generality Rules have a general character, in the sense that they can generalize over more than one case, abstracting from redundant properties of cases.

Example 4.3. Consider the case model with cases $\neg a$, $a \wedge b \wedge c$ and $a \wedge b \wedge \neg c$, all equally preferred. The arguments $(a \wedge c, b)$ and $(a \wedge \neg c, b)$ are both conclusive arguments with grounding in the case model, but they have redundant premises: in the former c is redundant, in the latter $\neg c$. The two arguments can be generalized to the conclusive argument (a, b) that has no redundant premises.

Such an argument without redundant premises is what we refer to as a rule; cf. the following definition.

Definition 4.4. A coherent/presumptively valid/conclusive argument (φ, ψ) is a coherent/presumptively valid/conclusive rule in a case model if for all coherent/presumptively valid/conclusive arguments (φ', ψ) with grounding in a case in the case model with $\varphi' \models \varphi$ it holds that $\varphi \models \varphi'$. The premises φ of a rule are also referred to as its *antecedent*, the conclusions ψ as its *consequent*.

Example 4.5. Consider the case model in the previous example. (a, b) is a conclusive rule in the case model, $(a \wedge c, b)$ is not.

This example of a rule has an elementary proposition as premises. We give an example with composite premises.

Example 4.6. Consider the case model with the five cases $\neg a \wedge \neg a'$, $\neg a \wedge a'$, $a \wedge \neg a'$, $a \wedge a' \wedge b \wedge c$ and $a \wedge a' \wedge b \wedge \neg c$, all equally preferred. The presumptively valid arguments $(a \wedge a', b)$ and $(a \wedge a' \wedge c, b)$ are both arguments with grounding in the case model. Of these, only $(a \wedge a', b)$ is a presumptively valid rule in the case model. The arguments (a, b) and (a', b) are not, showing that both conditions are needed.

Chaining Intuitively, rules (and other unstructured arguments) can be chained in the sense that when the antecedent of one rule follows from the consequent of another, they can be consecutively applied. For instance, rules (a, b) and (b, c) can be applied in a situation a , thereby supporting c .

Example 4.7. Consider the case model with four cases $a \wedge b \wedge c$, $\neg a \wedge \neg b \wedge c$, $\neg a \wedge b \wedge c$ and $\neg c$, all equally preferred. Then (a, b) and (b, c) are presumptively valid, even conclusive arguments grounded in the case model. In fact, they are rules as their premises are maximally general. In the case model, the arguments (b, a) and (c, b) with order of premises and conclusions reversed are also presumptively valid, but not conclusive. The argument $(a, b \wedge c)$ is presumptively valid and conclusive and represents the argument for the case made by chaining the two rules.

Slightly more generally, chaining unstructured arguments (in the sense of Definition 2.2) is possible when the premises of one argument logically follow from the case made by another.

The result of chaining rules is not necessarily a rule, as this example shows.

Example 4.8. Consider a case model with two cases $a \wedge b \wedge \neg c$ and $\neg a \wedge b \wedge c$, equally preferred. Then (a, b) and (b, c) are presumptively valid rules. The result of chaining these rules is the argument $(a, b \wedge c)$ making the case $a \wedge b \wedge c$. As an incoherent argument, it is not a rule.

5 ARGUMENTS AND ATTACK

The three themes that we discuss about arguments and attack are the main kinds of attack typically distinguished: rebutting attack, undercutting attack and undermining attack.

We first define attack in general.

Definition 5.1. (Successful attack) Let (C, \geq) be a case model, and (φ, ψ) a presumptively valid argument. Then circumstances χ are *defeating* or *successfully attacking* the argument when $(\varphi \wedge \chi, \psi)$ is not presumptively valid. We write $(C, \geq) \models \varphi \rightsquigarrow \psi \times \chi$. Defeating circumstances are *excluding* when $(\varphi \wedge \chi, \psi)$ is not coherent. A case $\omega \in C$ provides *grounding* for the attack if $\omega \models \varphi \wedge \chi$.

Rebutting attack Rebutting attack is a special kind of attack. Rebutting attack occurs when an argument is attacked, while supporting the opposite conclusion.

Definition 5.2. When circumstances χ successfully attack presumptively valid argument (φ, ψ) , the circumstances are *rebutting* when $(\varphi \wedge \chi, \neg\psi)$ is presumptively valid.

Example 5.3. Consider the case model with three cases $\neg a$, $a \wedge b$ and $a \wedge \neg b$, the first two preferred to the third. Then (a, b) is a presumptively valid argument. $(a \wedge \neg b, b)$ is not a presumptively valid argument. $(a \wedge \neg b, \neg b)$ is presumptively valid. Hence $\neg b$ provides rebutting circumstances for (a, b) . Since $(a \wedge \neg b, b)$ is not even coherent, $\neg b$ is also excluding. The case $a \wedge \neg b$ provides grounding for the rebutter, showing that the attack is coherent with the argument's premises.

This example uses the opposite of an argument's conclusion as a rebutting attack. Here is an example that uses an independent proposition for rebutting.

Example 5.4. Consider the case model with three cases $\neg a$, $a \wedge b \wedge \neg c$ and $a \wedge \neg b \wedge c$, the first two preferred to the third. Then (a, b) is a presumptively valid argument and $(a \wedge c, b)$ is not. $(a \wedge c, \neg b)$ is presumptively valid. Hence c is rebutting (a, b) . $(a \wedge c, b)$ is not coherent, so c excludes (a, b) .

Undercutting attack Undercutting occurs when the attacking circumstances are not rebutting.

Definition 5.5. When circumstances χ successfully attack presumptively valid argument (φ, ψ) , and are not rebutting, the circumstances are *undercutting*.

Example 5.6. Consider the case model with three cases $\neg a$, $a \wedge b \wedge \neg c$ and $a \wedge c$, the first two preferred to the third. Then (a, b) is a presumptively valid argument and $(a \wedge c, b)$ is not. Now $(a \wedge c, \neg b)$ is not presumptively valid (not even coherent), hence c is undercutting (a, b) . $(a \wedge c, b)$ is not coherent, so c excludes (a, b) . The case $a \wedge c$ provides grounding for the undercutter.

Undermining attack Undermining is the special kind of attack of a presumption, i.e., an argument that has logically tautologous premises (cf. Definition 2.2).

Definition 5.7. When circumstances χ successfully attack a presumption (\top, φ) , the circumstances are *undermining*.

Undermining can be of the rebutting and the undercutting kind, as the following two examples show.

Example 5.8. Consider the case model with two cases $a \wedge \neg b$ and $\neg a \wedge b$, the former preferred to the latter. Then (\top, a) is a presumption. b undermines the presumption since (b, a) is not presumptively valid. In fact, b rebuts the presumption (\top, a) since $(b, \neg a)$ is presumptively valid. The underminer has grounding in the case $\neg a \wedge b$.

Example 5.9. Consider the case model with two cases $a \wedge \neg b$ and b , the former preferred to the latter. Then (\top, a) is a presumption. b undermines the presumption since (b, a) is not presumptively valid. This time, b undercuts the presumption (\top, a) since $(b, \neg a)$ is not presumptively valid. The underminer has grounding in the case b .

6 APPLICATION TO TORT LAW

In this section, we apply the formalism and the treatment of cases, rules and arguments to the domain of tort law in the Netherlands (Section 2.2), as an evaluation of the approach.

We start with a list of the (unstructured) presumptively valid arguments that we expect to hold in this domain, following the discussion in Section 2.2. It is convenient to have abbreviations for the elementary propositions needed for representing the Dutch tort law domain. These are listed in Table 1.

We now consider the arguments that we expect to be presumptively valid. We use the conditional notation \rightsquigarrow as encountered in the description of the formalism in Definition 2.4, where presumptively valid arguments are defined.

Presumptively valid arguments

- $\text{dmg} \wedge \text{unl} \wedge \text{imp} \wedge \text{cau} \rightsquigarrow \text{dut}$
This argument corresponds to Art 6:162.1 BW where the

Table 1: Elementary propositions for the tort law domain

dut	There is a duty to repair someone's damages
dmg	Someone has suffered damages by someone else's act.
unl	The act committed was unlawful.
imp	The act can be imputed to the person that committed the act.
cau	The act caused the suffered damages.
VRT	The act is a violation of someone's right.
vst	The act is a violation of a statutory duty.
vun	The act is a violation of unwritten law against proper social conduct.
jus	There exist grounds of justification.
ift	The act is imputable to someone because of the person's fault.
ila	The act is imputable to someone because of law.
ico	The act is imputable to someone because of common opinion.
prp	The violated statutory duty does not have the purpose to prevent the damages.

four main conditions for establishing a duty to repair damages are specified: damages, unlawfulness, imputability and causality.

- $\text{VRT} \rightsquigarrow \text{unl}$
- $\text{vst} \rightsquigarrow \text{unl}$
- $\text{vun} \rightsquigarrow \text{unl}$

These three arguments correspond to Art 6:162.2 BW about the three kinds of unlawful acts: violation of a right, of a statutory duty or of unwritten law.

- $\text{ift} \rightsquigarrow \text{imp}$
- $\text{ila} \rightsquigarrow \text{imp}$
- $\text{ico} \rightsquigarrow \text{imp}$

These three arguments correspond to Art 6:162.3 BW about the three kinds of imputability of unlawful acts: imputability because of fault, by law and by common opinion.

Several of these arguments are not conclusive and have defeating circumstances. We provide a list, corresponding to the discussion of Dutch tort law in Section 2.2.

Defeating circumstances

- The argument $\text{dmg} \wedge \text{unl} \wedge \text{imp} \wedge \text{cau} \rightsquigarrow \text{dut}$ can be successfully attacked by defeating circumstances $\text{vst} \wedge \neg \text{prp}$. These defeating circumstances correspond to Art 6:163 BW expressing the relativity constraint. By including vst in the defeating circumstances, we represent that the relativity constraint is only used for violations of statutory duties.
- The argument $\text{VRT} \rightsquigarrow \text{unl}$ can be successfully attacked by defeating circumstances jus .
- The argument $\text{vst} \rightsquigarrow \text{unl}$ can be successfully attacked by defeating circumstances jus .
These two correspond to the exception to unlawfulness expressed in Art 6:162.2 BW. As discussed, grounds of justification are not used for the third kind of unlawfulness, viz. violations of unwritten law.

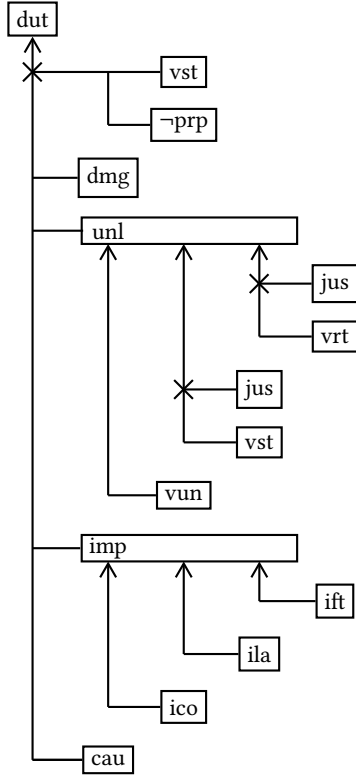


Figure 2: Arguments and their attacks in the domain of Dutch tort law

The connections between these arguments and their defeating circumstances are shown graphically in Figure 2 (following the format of Verheij 2005).

We now discuss a case model in which these arguments and their defeating circumstances hold. The case model is shown in full in Table 2. In total there are 16 cases, specified in the table's columns. For each case, the table contains the elementary propositions or their negations as they conjunctively hold in the case. For instance, case 2 is $\neg\text{dut} \wedge \text{dmg} \wedge \neg\text{unl} \wedge \neg\text{vrt} \wedge \neg\text{vst} \wedge \neg\text{vun}$. At the bottom of the table, the preference ordering of the cases is specified.

Cases and their preferences Case 1 represents the situation where there are no damages. It has highest preference, hence represents the default case, and $\top \rightsquigarrow \neg\text{dmg}$ is a presumptively valid argument in the case model. In other words, $\neg\text{dmg}$ is a presumption. Any deviation from this presumption requires additional information. Case 1 is represented as just $\neg\text{dmg}$, thereby representing that the other propositions are not relevant in the default situation. For instance, when there are no damages, it does not matter whether an act was unlawful or not (whether unl or $\neg\text{unl}$ does not matter). Moreover, the issue whether there is a duty to repair damages cannot arise when there are no damages.

Case 2 represents the situation where there are damages, but there is no unlawfulness because none of the three kinds of unlawfulness apply: there is no violation of a right, not of a statutory

duty, nor of unwritten law. The argument $\text{dmg} \wedge \neg\text{vrt} \wedge \neg\text{vst} \wedge \neg\text{vun} \rightsquigarrow \neg\text{unl} \wedge \neg\text{dut}$ is presumptively valid in the case model.

Case 3 represents the situation where there are damages and unlawfulness, but no imputability because in this case none of the three kinds of imputability ift , ila or ico hold.

Case 4 represents the lack of causality. The preference ordering $1 > 2 > 3 > 4$ models that the lower numbered case takes precedence over the higher numbered case, which has the effect that the issue of considering unl arises only when $\neg\text{dmg}$ does not hold; the issue of considering imp arises only when $\neg\text{unl}$ does not hold, etc.

The cases 5 to 13 represent the nine forms of tort that arise by the different combinations of the three kinds of unlawfulness with three kinds of imputability. We here assume that the three kinds of unlawfulness are mutually exclusive, as are the three kinds of imputability. For instance, the argument $\text{dmg} \wedge \text{vrt} \wedge \text{ift} \wedge \text{cau} \rightsquigarrow \text{unl} \wedge \text{imp} \wedge \text{dut}$ is presumptively valid in the case model. Note that also $\neg\text{jus}$ and prp follow presumptively from these same premises, modeling that by default there are no grounds of justification and the relativity constraint is fulfilled.

Cases 14 and 15 represent the general exception to unlawfulness of Art. 6:162.2 BW, when there are grounds of justification. As discussed, this exception can apply to vrt and vst (here, simplifying, assumed to be mutually exclusive), but cannot apply to vun . As these cases have lower preference than cases 5 to 10, grounds of justification must obtain before they have their exceptional effect. Formally, $\text{dmg} \wedge \text{vrt} \rightsquigarrow \text{unl}$ is presumptively valid, and $\text{dmg} \wedge \text{vrt} \wedge \text{jus} \rightsquigarrow \neg\text{unl}$ is not. In fact, $\text{dmg} \wedge \text{vrt} \wedge \text{jus} \rightsquigarrow \neg\text{unl}$ is presumptively valid, and jus rebuts the argument to unl .

Finally, case 16 is used to represent the relativity constraint of Art. 6:163 that the violated statutory duty does not have the purpose to prevent the damages. This case has lower preference than cases 8 to 10, hence $\neg\text{prp}$ provides defeating circumstances to the presumptively valid inference $\text{dmg} \wedge \text{vst} \wedge \text{imp} \wedge \text{cau} \rightsquigarrow \text{dut}$. In fact, prp rebuts the argument since $\neg\text{dut}$ follows presumptively from these same premises.

Summarizing, for the case model (C, \geq) shown in Table 2, the following hold, using the notation of Definition 5.1 to denote defeating circumstances:

- $(C, \geq) \models \text{dmg} \wedge \text{unl} \wedge \text{imp} \wedge \text{cau} \rightsquigarrow \text{dut} \times \text{vst} \wedge \neg\text{prp}$
- $(C, \geq) \models \text{vrt} \rightsquigarrow \text{unl} \times \text{jus}$
- $(C, \geq) \models \text{vst} \rightsquigarrow \text{unl} \times \text{jus}$
- $(C, \geq) \models \text{vun} \rightsquigarrow \text{unl}$
- $(C, \geq) \models \text{ift} \rightsquigarrow \text{imp}$
- $(C, \geq) \models \text{ila} \rightsquigarrow \text{imp}$
- $(C, \geq) \models \text{ico} \rightsquigarrow \text{imp}$

This list of presumptively valid arguments and their attacks correspond exactly to the list at the start of this section, as illustrated in Figure 2.

7 DISCUSSION

In this section, we discuss ideas developed in this paper in the context of earlier work on case-based, rule-based and argument-based reasoning in the field of AI & Law. These topics are closely related, as reflected in the state of the art. As we will see, much of

Table 2: Case model for the tort law domain

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	¬dut	¬dut	¬dut	dut	dut	dut	dut	dut	dut	dut	dut	dut	¬dut	¬dut	¬dut
¬dmg	dmg	dmg	dmg	dmg	dmg	dmg	dmg	dmg	dmg	dmg	dmg	dmg	dmg	dmg	dmg
	¬unl	unl	unl	unl	unl	unl	unl	unl	unl	unl	unl	unl	¬unl	¬unl	unl
		¬imp	imp	imp	imp	imp	imp	imp	imp	imp	imp	imp			imp
			¬cau	cau	cau	cau	cau	cau	cau	cau	cau	cau			cau
	¬vrt			vrt	vrt	vrt	¬vrt	¬vrt	¬vrt	¬vrt	¬vrt	¬vrt	vrt	¬vrt	
	¬vst			¬vst	¬vst	¬vst	vst	vst	vst	¬vst	¬vst	¬vst	¬vst	vst	vst
	¬vun			¬vun	¬vun	¬vun	¬vun	¬vun	¬vun	vun	vun	vun			
		¬ift		ift	¬ift	¬ift	ift	¬ift	¬ift	ift	¬ift	¬ift			
		¬ila		¬ila	ila	¬ila	¬ila	ila	¬ila	¬ila	ila	¬ila			
		¬ico		¬ico	¬ico	ico	¬ico	¬ico	ico	¬ico	¬ico	ico			
			¬jus	¬jus	¬jus	¬jus	¬jus	¬jus	¬jus	¬jus	¬jus	¬jus	jus	jus	
			prp	prp	prp										¬prp

1 > 2 > 3 > 4 > 5 ~ 6 ~ 7 ~ 8 ~ 9 ~ 10 ~ 11 ~ 12 ~ 13 > 14 ~ 15 ~ 16

this work is not limited to one of the three categories cases, rules and arguments, and discusses two or three in connection with one another.

The idea of *stare decisis* in the law, i.e., the principle that decisions in past cases provide insight—if not determine—decisions in current cases, has been extensively studied in AI & Law. A prominent line of research in this direction started with Rissland’s work on examples in mathematics, later developed in the law (Rissland 1983; Rissland and Ashley 1987, 2002). A key role in this line of research is played by HYPO, as described in (Ashley 1990), which inspired the CATO model (Aleven and Ashley 1995). An insightful recent description of this history is given by Rissland (2013). At the heart of much of this work is the idea that cases can be modeled as a set of factors, which can be regarded as a kind of reasons for and against the decision. When cases share factors they are analogous, and the analogy is stronger when more factors are shared. A key notion occurring in this work is the idea of a three-ply argument: one party cites a case to follow (ply 1), the other party (e.g.) cites another case as a counterargument, perhaps pointing to distinguishing factors (ply 2), and then the first party can in turn respond to this argument by citing a third case (ply 3). In this way, the cases and their factors serve as building blocks grounding the argumentative discussion. By combining factors with dimensions it is possible to model ranges of values and strengths of support. The models in this line of research do not focus on the mathematical formalization (as in the logic-styled formalism of this paper) since the emphasis is more on the value of the models for analyzing real cases and on implementation.

If we look at our cases (using the ones in Table 2 as representative examples), they are not modeled as sets of factors for and against decisions, but in terms of the logical properties that hold in them. Whether a property supports or attacks a decision (or another property) is not modeled as a primitive (as in HYPO and related models), but can here be derived from the case model as a whole. For instance, in the tort law case model, a violation of a right supports the decision that there is a duty to repair the damages. Formally: the argument $vrt \rightsquigarrow dut$ is presumptively valid. We have

defined notions of analogy and distinction that have counterparts in HYPO and related models, in this paper defined in a logic-styled formalization. In HYPO and related models, three-ply arguments provide a grounding of ‘hypothetical’ arguments in cases. We formally defined a notion related to such grounding, but also showed how different kinds of validity (coherence, conclusiveness, presumptive validity) can be determined by the cases in a case model. In our approach, arguments that are valid in one of these ways may or may not have the decision or its opposite as conclusion. CATO added a factor hierarchy, similar in spirit to the hierarchical structure in Figure 2, without the attacks. Branting (1991, 1993) emphasised the interplay between rules and cases as grounds of legal arguments in his computational model. Skalak and Rissland (1992) provide a taxonomy of argument patterns based on precedents and show how the patterns are supported in the CABARET system, a hybrid case-based/rule-based reasoner. Roth discussed the analogy and distinction of cases in terms of the comparison of the structured arguments they contain (Roth 2003; Roth and Verheij 2004). Following the argument structure allowed by the DefLog formalism implemented in the ArguMed argument diagramming software (Verheij 2003a,b, 2005), Roth used an entangled factor hierarchy, in which attack and support relations can be attacked. Horty and Bench-Capon (2012) discuss a reason model of precedent-based reasoning. Recently, Čyras et al. (2016) have studied case-based arguments in terms of abstract argumentation (Dung 1995).

The rule-based tradition in AI & Law is strongly connected with the development of logic programming in AI, more generally. Well-known contributions showed how the British Nationality Act and benefit legislation can be modeled as a logic program (Bench-Capon et al. 1987; Sergot et al. 1986). Important contributions were for instance provided by Loui and Norman (1995), Gordon (1995), Prakken and Sartor (1996), Hage (1997) and Prakken (1997). These works connect rules to the reasons and arguments that can be derived from them: when a rule’s condition is satisfied in the current situation, its conclusion is derived, perhaps leading to another rule having its condition satisfied, etc. We have discussed a similar structuring,

using a case model to determine which rules hold, which arguments are valid, and what the effect of chaining is. We showed by example that chaining is not always successful: a chaining of two coherent arguments may not be coherent. Loui and Norman show how arguments have arguments underlying them. In our model, we see a formal version of that idea. For instance, the argument $\text{vrt} \rightsquigarrow \text{dut}$ is presumptively valid in the tort case model, and can be ‘unpacked’ into two presumptively valid steps $\text{vrt} \rightsquigarrow \text{unl}$ and $\text{unl} \rightsquigarrow \text{dut}$. In contrast with the present work, these models use a given rule-based representation as starting point. Prakken and Sartor (1997) discussed rules and exceptions for building arguments as extracted from HYPO-styled cases.

An argument-based tradition in AI & Law developed—not strictly separable from the case-based and rule-based traditions—inspired by the works of Pollock (1987, 1995) and Dung (1995). Two of the three forms of attack, namely undercutting and undermining that we discussed have been distinguished by Pollock, and we showed how they can be given a formal treatment grounded in case models. In our approach, rebutting and undercutting are special cases of a general kind of attack. A third form of attack has been distinguished, undermining, focusing on the attack of the defeasible assumptions on which arguments can be based (Bondarenko et al. 1997). We discussed two kinds of undermining as special cases of rebutting and undercutting. Computational argumentation research also led to formal and computational representations of the argumentation schemes developed in argumentation theory (van Eemeren et al. 2014; Walton et al. 2008). Argumentation schemes have been applied to the formalization of case-based reasoning (Prakken et al. 2013) and to study the logical relations between cases and rules (Verheij 2008). In contrast with Dung (1995), our argument semantics is not abstract, graph-based, but uses case models to establish the validity of arguments and their building blocks. Dung’s abstract argumentation has for instance been successfully applied to value-based practical reasoning by Atkinson and Bench-Capon (2006, 2007); Bench-Capon (2003). The present approach using case models has been connected to this work (Verheij 2016b), modeling the development of the relevance of cases. A series of tort cases related to car accidents in New York discussed by Hafner and Berman (2002) serve as examples. Al-Abdulkarim et al. (2015) discuss how factors, facts and values are connected in legal case-based argumentation.

8 CONCLUSION

In this paper, we discussed the formalization of arguments, rules and cases using case models as a shared formal basis. Case models consist of cases and their preferences. Cases are consistent and pairwise incompatible. The preference relation on cases is a total preorder, representing the kind of qualitative ordering that can be represented in terms of a numeric, quantitative representation.

With respect to cases, we discussed analogy and distinction between cases and situations, and the grounding of arguments in cases. With respect to rules, we discussed conditionality, generality and chaining. With respect to arguments, we discussed rebutting, undercutting and undermining attack.

As an evaluation of the approach, we modeled core elements of Dutch tort law in terms of a case model, and showed that the main rule-based arguments and attacks in this domain are valid in the

case model. In this way, we showed formally that statutory, rule-based law from the civil law tradition can be given a formal, logical representation in terms of a set of cases that serve as exemplars.

By the research presented in this paper, we have aimed to contribute to the understanding of arguments, rules and cases, and how they are formally connected. The close formal connections that we discussed can be regarded as a corroboration of the findings in comparative international law that showed that the roles of cases and rules in jurisdictions in a common law and civil law tradition are less strictly distinguishable than is sometimes thought (MacCormick and Summers 1997).

Whether the ideas in this paper can provide the basis of new kinds of AI & law implementations remains to be seen, as the present paper has focused on the logical, formal foundations of how arguments, rules and cases are connected. It would be a natural step to investigate whether these foundations can be the basis for a new kind of useful algorithms.

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