Two Approaches to Dialectical Argumentation: Admissible Sets and Argumentation Stages

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Abstract

Currently there is a revival of the study of dialectical argumentation in the artificial intelligence community. There are good reasons why: First, the notions of argument and counterargument shed new light on nonmonotonic reasoning. Second, the process character of dialectical argumentation inspires new computational techniques.

In a recent important paper, Dung [1] has studied the relations of (unstructured) arguments and their counterarguments in terms of admissible sets. He has investigated the relations between several types of extensions of argumentation theories.

In this paper, we propose a model of the stages of argumentation, related to that of Verheij [2, 3]. Each stage is characterized by the arguments that have been taken into account and by the status of these arguments, either undefeated or defeated. This stage approach provides better understanding of the process of argumentation, because sequences of stages can be interpreted as lines of argumentation. The stage approach also gives naturally rise to two new types of extensions. Their definitions formalize the idea that as many arguments are taken into account as possible.

We show the connections with Dung's work and give a number of examples. It turns out that the argumentation stage approach generalizes the admissible set approach. The main conclusion of the paper is that the argumentation stage approach can give more insight in the procedural nature of dialectical argumentation than the admissible set approach.

1 Introduction

Dialectical argumentation has two main characteristics (cf. for instance Rescher [4]):

1. The arguments used to support a conclusion can be challenged by *counterarguments*.
2. Whether an argument justifies a conclusion depends on the stage of the argumentation *process*.

These characteristics have recently led to renewed attention of the artificial intelligence community (cf. Bench-Capon [5]; Loui [6]), for two reasons: First, the notion of counterargument sheds new light on nonmonotonic reasoning, and second, the process character of argumentation directly inspires new computational techniques.

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In a recent paper, Dung [1] has thoroughly investigated the relations of (unstructured[1]) arguments and counterarguments in terms of admissible sets. In this paper, a model of the stages of the argumentation process is discussed, related to the model of Verheij [2, 3], and compared to Dung's approach.

We show that there are close connections between the two approaches, but that the stage approach gives insight into the process character of argumentation, while the admissible set approach does not. It will turn out that sequences of argumentation stages can be interpreted as lines of argumentation. Especially interesting are the so-called argumentation diagrams, that include all stages and lines of argumentation of a given argumentation theory. Argumentation diagrams are important since specific argumentation strategies and protocols (that are currently the subject of active research) can be considered as constraints on these diagrams.

In section 2, the main definitions of the two approaches are discussed. In section 3, we discuss their (close) formal connections. In section 4, we give some examples by means of argumentation diagrams. Section 5 summarizes the conclusions of the paper.

2 Definition of admissible sets and argumentation stages

In this section, we give the main definitions of the admissible set approach, taken from Dung [1], and the argumentation stage approach, adapted from Verheij [2, 3].

Argumentation depends on the arguments that can be taken into account, and on which arguments challenge other arguments. In this paper, arguments are considered as abstract unstructured objects. Defeaters represent which arguments challenge other arguments. This leads to the following definition of an argumentation theory.[2]

Definition 1.

An *argumentation theory* is a pair (*Arguments*, *Defeaters*), where *Arguments* is any set, and *Defeaters* is a subset of *Arguments* × *Arguments*. The elements of *Arguments* are the arguments of the theory, the elements of *Defeaters* the defeaters. In a defeater (*Arg*, *Arg*'), the argument *Arg* is the *challenging* argument, and *Arg*' the *challenged* argument.

The following definitions and results depend on a not explicitly mentioned argumentation theory (*Arguments*, *Defeaters*), unless specified otherwise.

Definition 2 summarizes some of Dung's definitions. For an extended discussion, we refer to the original paper [1]. Central in his definitions is the notion of an *acceptable* argument. An argument *Arg* is acceptable with respect to some set of arguments *Args* if all arguments that challenge the argument *Arg* are themselves challenged by an argument in the set *Args*.

Definition 2. (Dung, [1])

(1) A set of arguments *Args* is *conflict-free* if there is no defeater (*Arg*, *Arg*'), such that *Arg* and *Arg*' both are elements of *Args*.

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1 For some recent discussions of structured arguments, the reader is referred to e.g. the work of Pollock [7], Vreeswijk [8], or Verheij [2, 3].

2 Our argumentation theories correspond to Dung's argumentation frameworks. Our defeaters are his attacks.

(2) An argument *Arg* is *acceptable* with respect to a set of arguments *Args* if for all arguments *Arg*' of the theory the following holds:

    If (*Arg*', *Arg*) is a defeater, then there is an argument *Arg*'' in *Args*, such that (*Arg*'', *Arg*') is a defeater.

(3) A set of arguments *Args* is *admissible* if it is conflict-free and all arguments in *Args* are acceptable with respect to *Args*.

(4) A *preferred extension* of an argumentation theory is an admissible set of arguments, that is maximal with respect to set inclusion.

(5) A conflict-free set of arguments *Args* is a *stable extension* of an argumentation theory if for any argument of the theory *Arg* that is not in *Args*, there is an argument *Arg*' in *Args*, such that (*Arg*', *Arg*) is a defeater.[3]

An argumentation theory has at least one preferred extension, since the empty set ∅ is admissible, and unions of increasing sequences of admissible sets are admissible, as Dung [1] shows. A theory does not always have a stable extension. For instance, the argumentation theory ({α}, {(α, α)}) has the empty set ∅ as unique preferred extension, which is not stable. A theory can have more than one preferred extension. For instance, the argumentation theory ({α, β}, {(α, β), (β, α)}) has the preferred (and stable) extensions {α} and {β} (see section 4.2).

In the following definition, the argumentation stage approach is summarized. It is a restricted version of the definitions by Verheij [2, 3].[4] The adaptation was made to make the relations with Dung's admissible set approach clearly visible.

Intuitively, an argumentation stage is characterized by the arguments that have been taken into account, and by the statuses of these arguments. Each argument has one of two statuses: either undefeated or defeated. Formally, an argumentation stage is a status assignment, that satisfies a constraint: Any argument challenged by an undefeated argument must be defeated, and any defeated argument must be challenged by an undefeated argument. A stage extension is now an argumentation stage in which a maximal number of arguments is taken into account, i.e., a stage that has maximal range.

Definition 3.

(1) A *defeat status assignment* is a pair of disjoint sets of arguments. In a defeat status assignment (*UndefeatedArgs*, *DefeatedArgs*), the arguments in *UndefeatedArgs* are *undefeated*, those in *DefeatedArgs* are *defeated*. The union of the sets *UndefeatedArgs* and *DefeatedArgs* is the *range* of the defeat status assignment.

(2) An *argumentation stage* (or *stage*, for short) is a defeat status assignment (*UndefeatedArgs*, *DefeatedArgs*), such that for each argument *Arg* in its range the following holds:

    Arg is an element of *DefeatedArgs* if and only if there is an argument *Arg*' in *UndefeatedArgs*, such that (*Arg*', *Arg*) is a defeater.

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[3] Dung's definitions of complete and grounded extensions are left out.

[4] In this paper the definitions of Verheij [2, 3] are restricted in two ways. First, there the influence of the structure of arguments on argumentation is considered, in particular in cases of accrual of reasons and sequential weakening. Second, Verheij [3] argues that defeat can be compound, meaning that the status of arguments depends on relations of *groups* of arguments. In this paper, and in Dung's [1], only single arguments can challenge other single arguments.

(3) A *stage extension* is an argumentation stage (*UndefeatedArgs*, *DefeatedArgs*), such that there is no argumentation stage with larger range. A stage extension is *complete* if all arguments of the argumentation theory are in its range.[5]

An argumentation theory does not always have a stage extension. For instance, the argumentation theory ($\{\alpha_i \mid i = 0, 1, 2, ...\}$, $\{(\alpha_i, \alpha_j) \mid i > j\}$) has no stage extension. It has several sensible stages, though, such as ($\{\alpha_i\}$, $\{\alpha_j \mid i > j\}$) for any i = 0, 1, 2, ..., while its preferred extension ∅ is its only admissible set of arguments (see section 4.5). A theory can have more than one stage extension. For instance, the argumentation theory ({α, β}, {(α, β), (β, α)}) has the (complete) stage extensions ({α}, {β}) and ({β}, {α}).

Each argument in an admissible set must be defended against *all* challenging arguments. In an argumentation stage, each undefeated argument must only be defended against the challenging arguments that have been *taken into account*, i.e., against the arguments in the range of the stage. This is intuitively the main difference between the two approaches.

3 Connections between the two approaches

In this section, we investigate the connections between the admissible set and the argumentation stage approach. The following notation is used.

Notation.

Challenging(*Args*) = {*Arg* | There is a defeater (*Arg*, *Arg*') with *Arg*' in *Args*}
Challenged(*Args*) = {*Arg* | There is a defeater (*Arg*', *Arg*) with *Arg*' in *Args*}

The following lemma reformulates some of the definitions in terms of these sets.

Lemma.

(1) An argument *Arg* is acceptable with respect to a set of arguments *Args* if and only if Challenging({*Arg*}) is a subset of Challenged(*Args*).
(2) For any set of arguments *Args* the following are equivalent:
    (i) *Args* is conflict-free.
    (ii) *Args* contains no element of Challenged(*Args*).
    (iii) *Args* contains no element of Challenging(*Args*).
(3) A set of arguments *Args* is admissible if and only if it is conflict-free and Challenging(*Args*) is a subset of Challenged(*Args*).
(4) A defeat status assignment (*UndefeatedArgs*, *DefeatedArgs*) is an argumentation stage if and only if *UndefeatedArgs* contains no elements of Challenged(*UndefeatedArgs*) and *DefeatedArgs* is a subset of Challenged(*UndefeatedArgs*).

Admissible sets of arguments are closely related to the sets of undefeated arguments of a stage. However, not all such sets are admissible. For instance, ({α}, ∅) is a stage of the argumentation theory ({α, β}, {(β, α)}), while {α} is not admissible. The following result characterizes when the undefeated arguments of a stage form an admissible set and which stages have the same admissible set as set of undefeated arguments.

[5] Our complete stage extensions have no relation with Dung's complete extensions (cf. note 3).

Theorem 1.

(1) For any argumentation stage (*UndefeatedArgs*, *DefeatedArgs*) the following holds:

UndefeatedArgs is admissible if and only if Challenging(*UndefeatedArgs*) is a subset of *DefeatedArgs*.

(2) For any admissible set *AdmissibleArgs* the following holds:

(*AdmissibleArgs*, *DefeatedArgs*) is an argumentation stage if and only if *DefeatedArgs* is a subset of Challenged(*AdmissibleArgs*).

PROOF: (1) First notice that the set of undefeated arguments of an argumentation stage is conflict-free. Then the result follows from the lemma. (2) Follows from the lemma.

A consequence of the second part of the theorem is that any admissible set occurs as the set of undefeated arguments of some argumentation stage. In particular, if *AdmissibleArgs* is an admissible set of arguments, then (*AdmissibleArgs*, $\emptyset$) and (*AdmissibleArgs*, Challenged(*AdmissibleArgs*)) are argumentation stages.

The following theorem characterizes admissible sets of arguments in terms of stages.

Theorem 2.

A set of arguments *Args* is admissible if and only if (*Args*, Challenging(*Args*) $\cup$ Challenged(*Args*)) is an argumentation stage.

PROOF: First notice that from the second part of the lemma it follows that a set of arguments *Args* is conflict-free if and only if (*Args*, Challenging(*Args*) $\cup$ Challenged(*Args*)) is a defeat status assignment. The 'only if'-part follows from the third part of the lemma and the second part of theorem 1. The 'if'-part follows from the first part of theorem 1.

A stage with an admissible set of undefeated arguments is an *admissible stage*. Stages of the form (*Args*, Challenged(*Args*)) are *canonical stages*, since they have maximal range among the stages with a particular set of undefeated arguments. So, if *Args* is admissible, (*Args*, Challenging(*Args*) $\cup$ Challenged(*Args*)) is a canonical stage. An admissible stage with maximal range (which is always canonical) is an *admissible-stage extension*.

Admissible-stage extensions do not correspond to stage extensions, since stage extensions are not necessarily admissible stages. For instance, the theory ($\{\alpha_1, \alpha_2, \alpha_3\}$, $\{(\alpha_1, \alpha_2), (\alpha_2, \alpha_3), (\alpha_3, \alpha_1)\}$) has the stage ($\emptyset$, $\emptyset$) as unique admissible-stage extension, and the non-admissible-stage extensions ($\{\alpha_1\}$, $\{\alpha_2\}$), ($\{\alpha_2\}$, $\{\alpha_3\}$) and ($\{\alpha_3\}$, $\{\alpha_1\}$) as stage extensions (see section 4.3). This example shows that neither all admissible-stage extensions are stage extensions, nor vice versa. In a sense, however, admissible-stage extensions are 'smaller' than stage extensions, since the range of any admissible-stage extension is smaller than (or equal to) the range of any stage extension.

One might expect that admissible-stage extensions correspond to Dung's preferred extensions. This is however not true, since there can be canonical stages with a preferred extension as set of undefeated arguments that are not admissible-stage extensions. For instance, the theory ($\{\alpha, \beta, \gamma_1, \gamma_2, \gamma_3\}$, $\{(\alpha, \beta), (\beta, \alpha), (\gamma_1, \gamma_2), (\gamma_2, \gamma_3), (\gamma_3, \gamma_1), (\alpha, \gamma_1)\}$) has the (canonical) admissible stages ($\{\beta\}$, $\{\alpha\}$) and ($\{\alpha, \gamma_2\}$, $\{\beta, \gamma_1, \gamma_3\}$). The range of the first is a proper subset of the second, while both $\{\beta\}$ and $\{\alpha, \gamma_2\}$ are preferred extensions (see section 4.4).

The following corollary characterizes Dung's preferred extensions in terms of stages.

Corollary 1.

A set of arguments *Args* is a preferred extension if and only if (*Args*, Challenging(*Args*) ∪ Challenged(*Args*)) is an admissible stage with maximal set of undefeated arguments.

PROOF: Follows immediately from theorem 2 and the observation that Challenging and Challenged are monotonic operators.

Canonical stages with a preferred extension as set of undefeated arguments, as in corollary 1, are *preferred stages*. Admissible-stage extensions are preferred stages, but not vice versa, as we have seen.

Dung's stable extensions and our complete stage extensions coincide, however, as the following corollary shows.

Corollary 2.

A set of arguments *Args* is a stable extension if and only if (*Args*, Challenging(*Args*) ∪ Challenged(*Args*)) is a complete stage extension.

The relations between the discussed types of stages are summarized in Figure 1. The continuous arrows indicate conceptual inclusion. We have given counterexamples for all missing arrows. Stage extensions and admissible-stage extensions have no counterpart in Dung's paper [1]. The dotted arrows indicate that any stage extension has a range larger than (or equal to) the range of any admissible-stage extension or preferred stage.

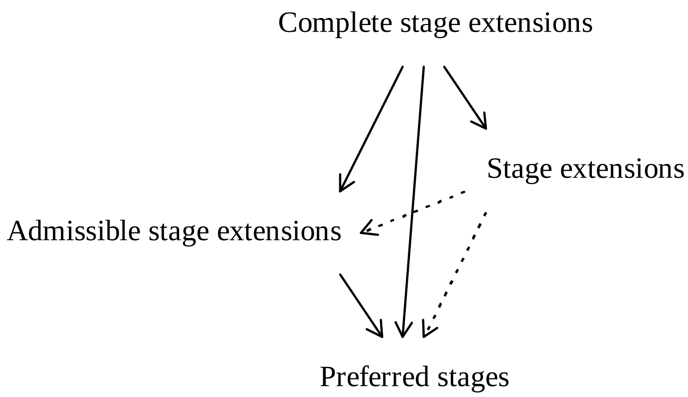

Figure 1: Relations between types of argumentation stages

The previous results have shown that the stages of an argumentation theory generalize the admissible sets of that theory. In these results, the argumentation theory was fixed. The section ends with a comparison result that says that the stages of a theory correspond exactly to the admissible sets of certain *other* theories. The following notation is used.

Notation.

For any argumentation theory (*Arguments*, *Defeaters*) and set of arguments *Range*, the *restriction* of the theory to *Range*, denoted $(Arguments, Defeaters)|_{Range}$ = $(Arguments|_{Range}, Defeaters|_{Range})$, is defined as follows:

$Arguments|_{Range} = Range$

$Defeaters|_{Range}$ = {(*Arg*, *Arg*') ∈ *Defeaters* | *Arg* and *Arg*' are elements of *Range*}

Theorem 3 is based on the observation that for an argumentation stage a defeater is only relevant if its challenging and challenged argument are both taken into account, i.e. are elements of the range of the stage.[6] The theorem follows straightforwardly from the definitions of admissible sets and argumentation stages. It shows that the argumentation stage approach is the 'local' version of the 'global' admissible set approach.

Theorem 3.

(*UndefeatedArgs*, *DefeatedArgs*) is an argumentation stage with range *Range* of the argumentation theory (*Arguments*, *Defeaters*) if and only if *UndefeatedArgs* is admissible with respect to the theory $(\mathit{Arguments}, \mathit{Defeaters})|_{\mathit{Range}}$.

Clearly, the set of undefeated arguments of an argumentation stage is in fact a stable extension of the restricted theory.

4 Examples and argumentation diagrams

In this section, some example argumentation theories are discussed. They are used to compare the admissible set and the argumentation stage approach. We make use of diagrams, which show all admissible sets and argumentation stages of a theory. It turns out that the diagrams of the stages of a theory can be interpreted as diagrams of the process of argumentation. As a result, they are called *argumentation diagrams*.[7]

4.1 Counterattack and reinstatement

The argumentation theory $(\{\alpha, \beta, \gamma\}, \{(\beta, \alpha), (\gamma, \beta)\})$ is an example of a counterattack: one argument, α, is challenged by another, β, which is itself challenged by a third argument, γ.

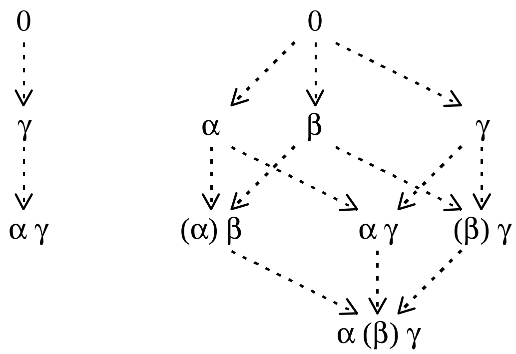

Figure 2: Counterattack and reinstatement

Figure 2 shows on the left side a diagram of the admissible sets of the theory, and on the right side a diagram of the argumentation stages. Each node corresponds to an admissible set or an argumentation stage. The arrows represent inclusion (for admissible sets) or

[6] Verheij [2, 3] defines which defeaters are *relevant* for a stage with range *Range*. The relevant defeaters are exactly the defeaters in $\mathit{Defeaters}|_{\mathit{Range}}$.

[7] Verheij [2, 3] gives similar diagrams.

inclusion of range (for stages). An admissible set is denoted by listing its elements; a stage is denoted by listing the arguments in its range and putting its defeated arguments in brackets. (The 0 indicates the empty list of arguments.)

The canonical stages of the admissible sets ∅, {γ} and {α, γ} are (∅, ∅), ({γ}, {β}) and ({α, γ}, {β}), respectively. For this theory, the unique preferred stage ({α, γ}, {β}) coincides with the (complete) stage extension.

Whereas the diagram on the left has no clear intuitive interpretation (except for set theoretical inclusion of admissible sets), the diagram of the argumentation stages on the right can be interpreted as a diagram of the process of argumentation, in the sense that each path in the diagram corresponds to a line of argumentation, in which the arguments of the theory are taken into account in a particular order.

In that interpretation, each arrow indicates that a new argument is taken into account. Arrows with the same direction correspond to the same argument that is taken into account. For instance, the arrows between the stages (∅, ∅) and ({β}, ∅) and between ({α, γ}, ∅) and ({α, γ}, {β}) both indicate that the argument β is taken into account. Different paths through the diagram from the initial stage (∅, ∅) to the final stage ({α, γ}, {β}) correspond to different orders in which the arguments are taken into account.

Of course the status of arguments can change. This theory has a line of argumentation in which α is *reinstated*: In the line of argumentation ({α}, ∅), ({β}, {α}), ({α, γ}, {β}), in which first α, then β, and finally γ is taken into account, the argument α is first undefeated, then defeated, and finally undefeated again.

4.2 Mutual attack and multiple extensions

The argumentation theory ({α, β}, {(β, α), (α, β)}) is an example of mutual attack: the arguments α and β challenge each other. As a result, the theory has multiple extensions. The stage extensions of the theory are ({α}, {β}) and ({β}, {α}). Figure 3 shows the corresponding diagrams of admissible sets and argumentation stages. Multiple extensions with equal range are separated by a comma.

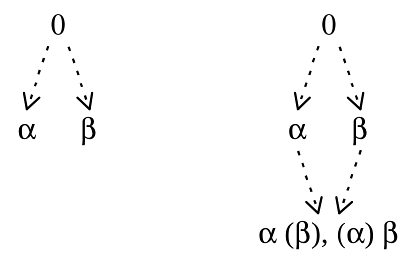

Figure 3: Mutual attack and multiple extensions

The argumentation theory ({α, β, γ}, {(β, α), (α, β), (γ, α)}) has an additional argument γ, that challenges the argument α. As a result, the theory has multiple stages, ({α}, {β}) and ({β}, {α}). The theory shows that a theory with multiple stages does not always have multiple stage extensions. Its unique stage extension is ({β, γ}, {α}). As a result, the notion of multiple stages is a proper generalization of the notion of multiple extensions. Figure 4 shows the corresponding diagrams of admissible sets and argumentation stages of the argumentation theory.

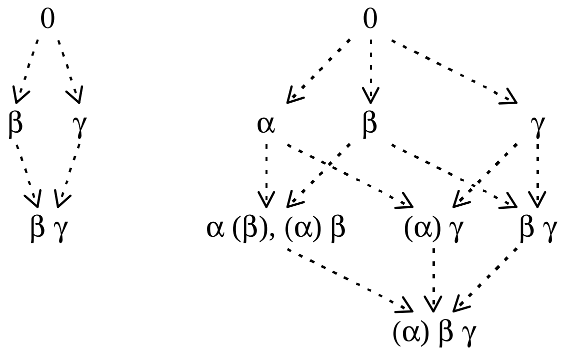

Figure 4: Multiple stages, but no multiple extensions

Clearly, multiple non-extension stages have no counterpart in the admissible set approach.

4.3 Loop of attacks and non-admissible-stage extensions

The argumentation theory ($\{\alpha_1, \alpha_2, \alpha_3\}$, $\{(\alpha_1, \alpha_2), (\alpha_2, \alpha_3), (\alpha_3, \alpha_1)\}$) contains a loop of attacks: the argument α_1 challenges the argument α_2, which challenges α_3, which on its turn challenges α_1. This theory gives an example of admissible-stage extensions (and preferred stages) that are not stage extensions. The unique admissible-stage extension of this theory is ($\varnothing$, $\varnothing$). The stage extensions are ($\{\alpha_1\}$, $\{\alpha_2\}$), ($\{\alpha_2\}$, $\{\alpha_3\}$) and ($\{\alpha_3\}$, $\{\alpha_1\}$). The theory has no complete stage extension (or, equivalently, stable extension). In Figure 5, this is indicated by the three question marks ???.

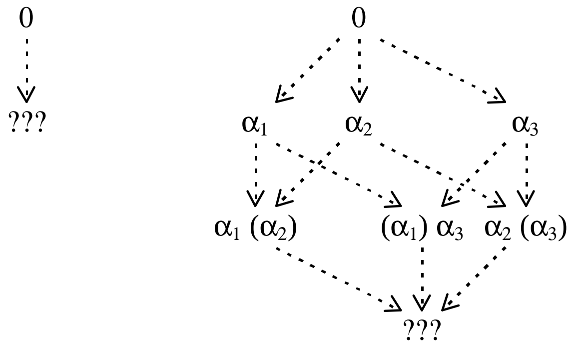

Figure 5: Loop of attacks and non-admissible-stage extensions

The diagram of the argumentation stages shows that any two of the arguments of the theory can be taken into account, but that it is impossible to take all three of them into account.

The example generalizes to loops of attacks of any odd number of arguments. It is essential that the number of arguments in the loop is odd. An even number results in two extensions of equal range.

4.4 Mutual attack skewly breaking a loop of attacks

The ornate title of this section refers to the argumentation theory ($\{\alpha, \beta, \gamma_1, \gamma_2, \gamma_3\}$, $\{(\alpha, \beta), (\beta, \alpha), (\gamma_1, \gamma_2), (\gamma_2, \gamma_3), (\gamma_3, \gamma_1), (\alpha, \gamma_1)\}$). It consists of the mutually attacking arguments α and β, one of which, α, breaks the loop of attacks of the arguments γ_1, γ_2 and γ_3. This theory shows that preferred stages are not always admissible-stage extensions. It has two preferred stages: $(\{\beta\}, \{\alpha\})$ and $(\{\alpha, \gamma_2\}, \{\beta, \gamma_1, \gamma_3\})$. The range of the first is smaller than that of the second. Only the second is an admissible-stage extension, that is even complete. Figure 6 shows on the left all admissible sets of the theory, and on the right only the canonical admissible stages (and their *twins*, i.e., stages with equal range).

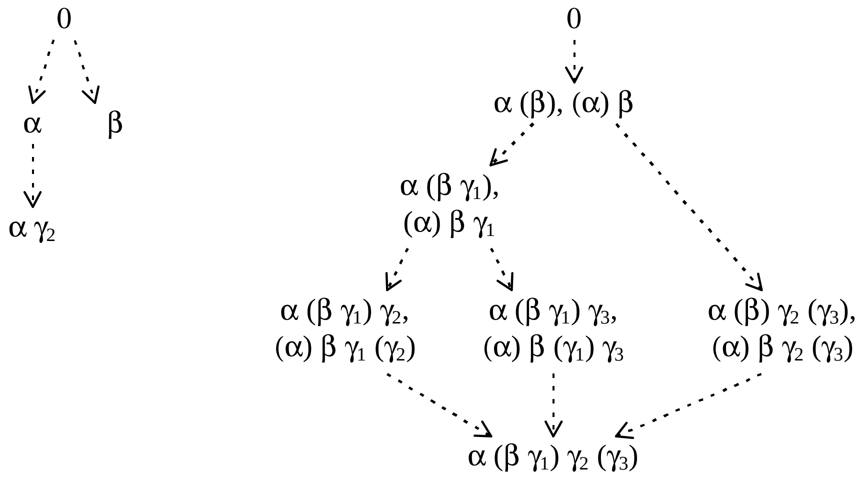

Figure 6: Mutual attack skewly breaking a loop of attacks[8]

In a sense, an admissible-stage extension is better than a preferred stage with smaller range, since more arguments are taken into account. It is therefore 'better informed'.

4.5 No exhausting sequence of compatible stages

The argumentation theory $(\{\alpha_i \mid i = 0, 1, 2, ...\}, \{(\alpha_i, \alpha_j) \mid i > j\})$ has no stage extension. The stages $(\{\alpha_i\}, \{\alpha_j \mid i > j\})$, for i = 0, 1, 2, ..., are the only canonical stages of the theory. Their ranges exhaust the arguments of the theory, but the stages are mutually not compatible. Compatibility is defined as follows: Stages $Stage_1$ = ($UndefeatedArgs_1$, $DefeatedArgs_1$) and $Stage_2$ = ($UndefeatedArgs_2$, $DefeatedArgs_2$) are *compatible* if $UndefeatedArgs_1$ contains no elements of Challenged($UndefeatedArgs_2$) and $UndefeatedArgs_2$ contains no elements of Challenged($UndefeatedArgs_1$).[9] It can be shown that a theory has a stage extension if and only if there is a sequence of compatible stages that exhaust all arguments of the theory.

The theory has one admissible set, its preferred extension $\emptyset$. Figure 7 shows the admissible sets and the canonical stages.

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[8] On the right, only the canonical admissible stages (and their twins) are shown. The directions of the arrows do not correspond to taking a particular argument into account, as in the previous diagrams.

[9] As a result, stages ($UndefeatedArgs_1$, $DefeatedArgs_1$) and ($UndefeatedArgs_2$, $DefeatedArgs_2$) are compatible if and only if ($UndefeatedArgs_1 \cup UndefeatedArgs_2$, $DefeatedArgs_1 \cup DefeatedArgs_2$) is a stage.

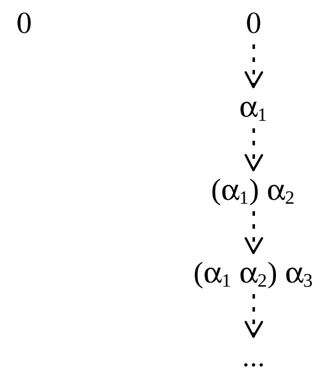

Figure 7: No exhausting sequence of compatible stages

Even though the theory has no stage extension, its canonical stages provide more information than its admissible sets.

5 Conclusions

In this paper, two approaches to dialectical argumentation have been compared: Dung's admissible sets of arguments and Verheij's argumentation stages. The main difference is that the arguments in an admissible set must be defended against all challenging arguments, while the undefeated arguments of an argumentation stage only need to be defended against the challenging arguments taken into account.

Theorem 3 showed that both approaches are strongly related: All argumentation stages of an argumentation theory correspond to the admissible sets of the restrictions of that theory. As a result, they are equivalent as far as the relations of arguments and counterarguments is concerned.

However, for a fixed theory, argumentation stages generalize admissible sets. We have shown that Dung's preferred and stable extensions correspond to preferred stages and complete stage extensions, respectively. In the stage approach, there are two natural new types of extensions: admissible-stage extensions and stage extensions. Their definitions are based on the idea that one wants to take as many arguments into account as possible. These types of extensions have no counterpart in the admissible set approach. Figure 1 summarizes the relations between the types of extensions.

We have shown that the argumentation stages give in a natural way rise to argumentation diagrams, in which paths can be interpreted as lines of argumentation. Therefore, the stages approach gives insight in the process character of dialectical argumentation.

This is especially important since recently the importance of the fundamentally procedural nature of argumentation-as-justification has been re-emphasized in the artificial intelligence community (see, e.g., Gordon [9]; Hage *et al.* [10]; Lodder [11]; Loui [12]; Vreeswijk [13]). Argumentation diagrams are useful for the understanding of the process of argumentation. For instance, specific argumentation strategies and protocols can be regarded as constraints on lines of argumentation, and therefore correspond to partial argumentation diagrams.

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