How to Teach Language Models to Reason

Huiyuan Lai

huiyuanlai.l@gmail.com



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Concatenate the first letter of each word

Answer the following strategy question



?

Α



Α

?

Yes or no: Would a nail sink in water?

Concatenate the first letter of each word

Answer the following strategy question

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Q

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Nails are made of materials such as steel or iron, which are denser than water. Thus, the answer is yes.

Concatenate the first letter of each word

Answer the following strategy question

Q Artificial Intelligence



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Terry eats 2 yogurts a day. They are currently on sale at 4 yogurts for \$5.00. How much does he spend on yogurt over 30 days?



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"A faculty of the human mind that enables logical inferences to be made and rational arguments to be undertaken to understand the world and solve problems."

Michael Kent. 2007. The Oxford Dictionary of Sports Science & Medicine (3 ed.)

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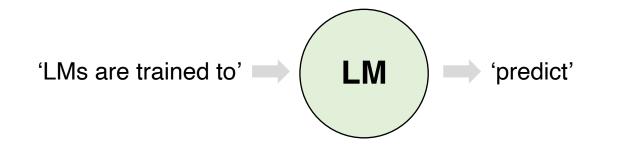
Solve the following math word problem

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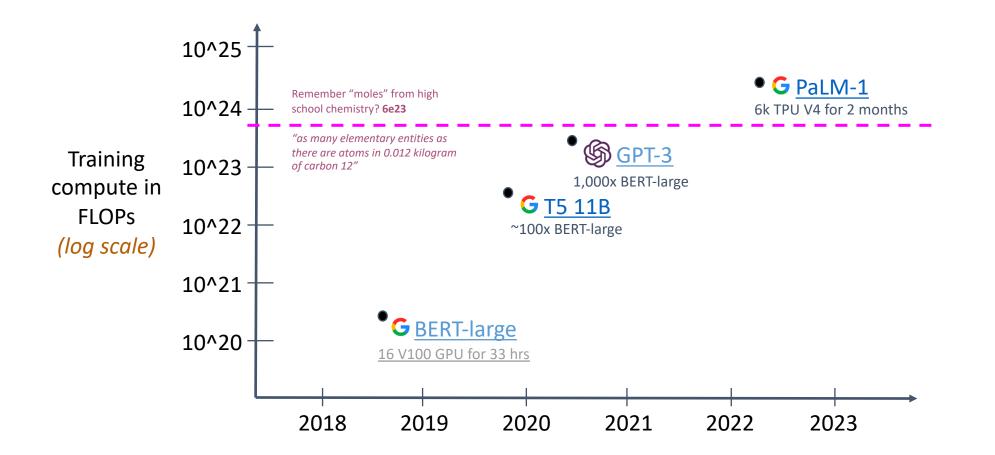
What are Language Models?

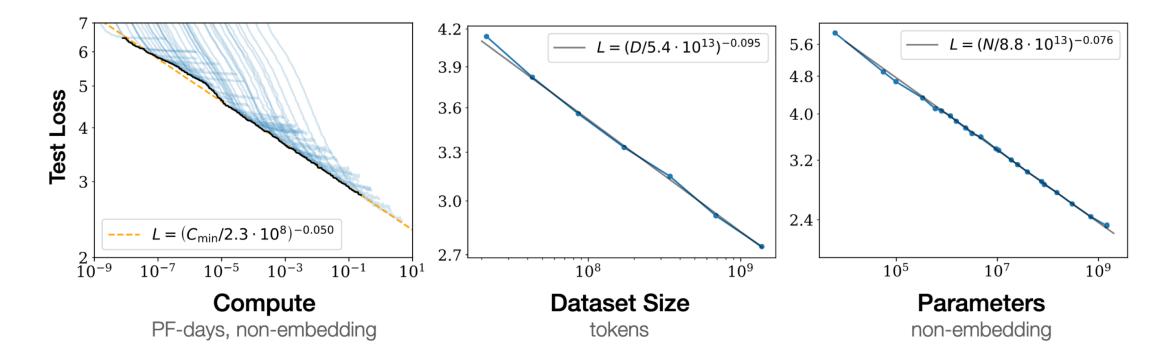
What are Language Models?

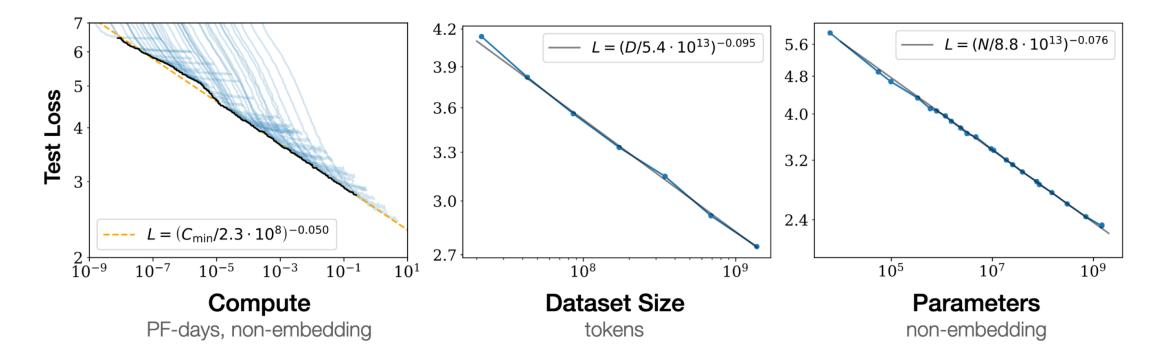
Language Models (LMs) are trained to predict the next word...



Neural Language Models

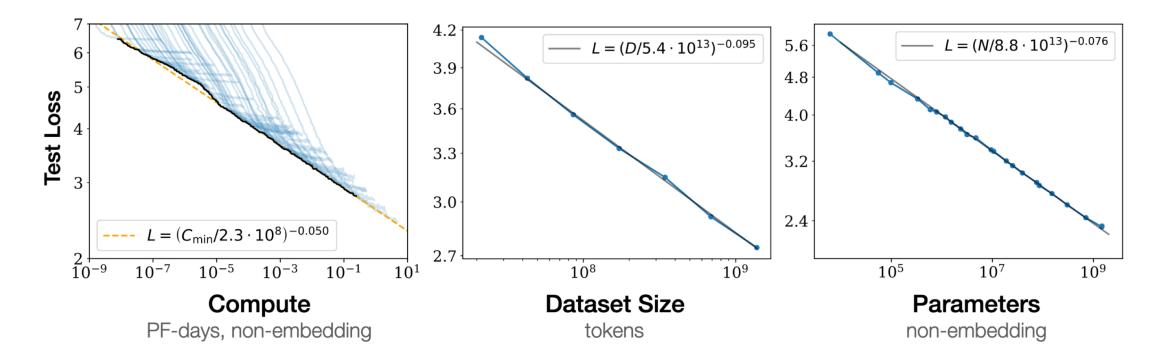






Scaling laws

Scaling laws (model size + data + compute) reliably improve model performance.

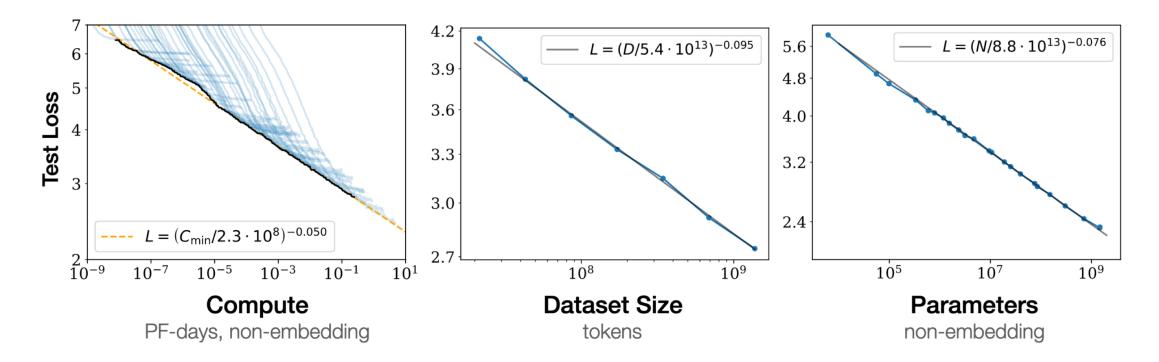


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Emergent abilities

As a result of scale, LMs gain new downstream abilities not present in small models.



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Reasoning via prompting

LMs can do multi-step reasoning, which paves the way to solving challenging problems.

How to Teach Language Models to Reason?



How to Teach Language Models to Reason?

Just like teaching kids?

How to Teach Language Models to Reason?

Give them some examples to understand the task...

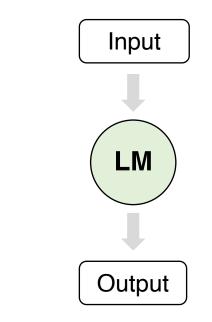
Few-shot Prompting for Toy Task



Q: Artificial Intelligence **A:** Al

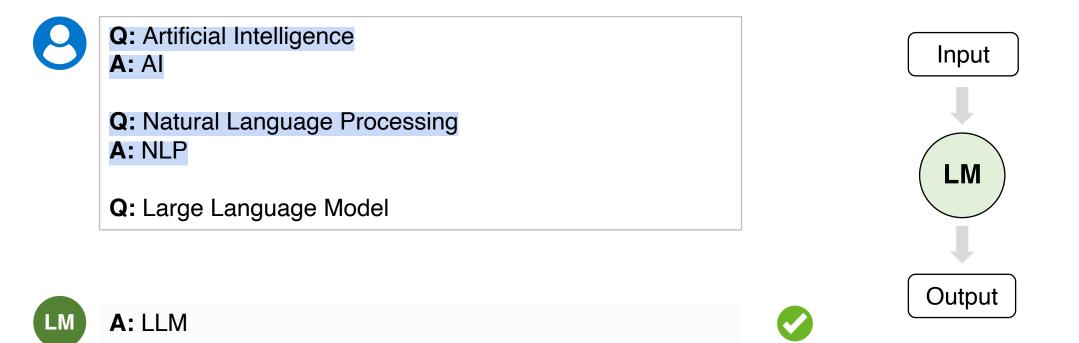
Q: Natural Language Processing **A:** NLP

Q: Large Language Model





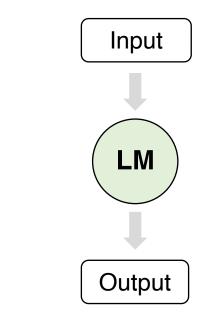
Few-shot Prompting for Toy Task



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Q: Roger has 5 tennis balls. He buys 2 more cans of tennis balls. Each can has 3 tennis balls. How many tennis balls does he have now?
A: The answer is 11.

Q: The cafeteria had 23 apples. If they used 20 to make lunch and bought 6 more, how many apples do they have?

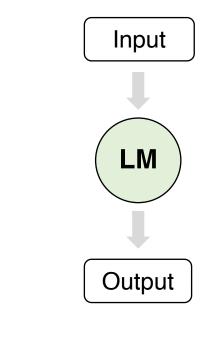


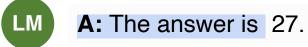


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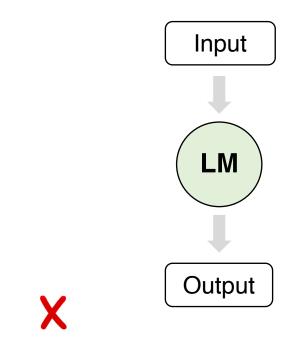


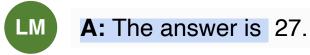


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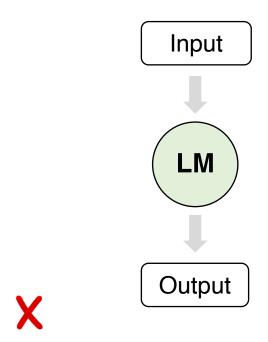


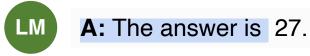
This task is hard! The model does not understand the task!

8

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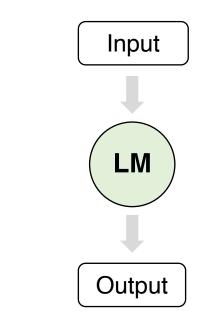


Give it more information to describe specific rationales!

8

Q: Roger has 5 tennis balls. He buys 2 more cans of tennis balls. Each can has 3 tennis balls. How many tennis balls does he have now?
A: Roger started with 5 balls. 2 cans of 3 tennis balls each is 6 tennis balls. 5 + 6 = 11. The answer is 11.

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Provide it with the example of Chain-of-Thought (CoT)!

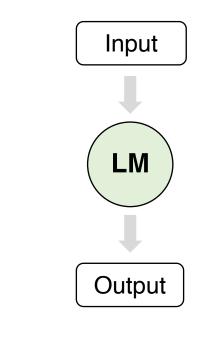
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LM

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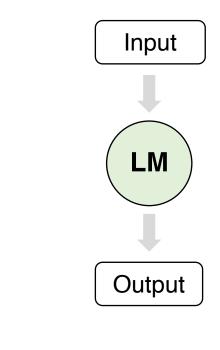
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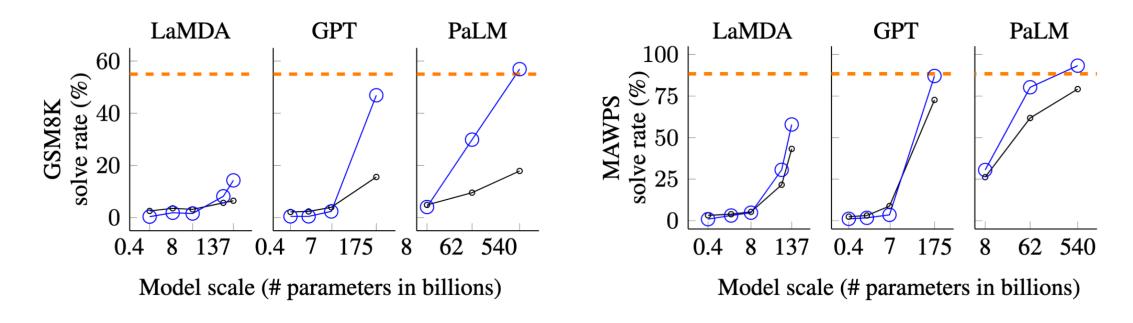
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Using natural language to describe rationales is critical for CoT!

Wei et al. 2022. Chain-of-thought prompting elicits reasoning in large language models.

- ----- Standard prompting
- ---- Chain-of-thought prompting
- --- Prior supervised best



Wei et al. 2022. Chain-of-thought prompting elicits reasoning in large language models.

CoT Prompting for Various Tasks

Math Word Problems (free response)

Q: Roger has 5 tennis balls. He buys 2 more cans of tennis balls. Each can has 3 tennis balls. How many tennis balls does he have now?

A: Roger started with 5 balls. 2 cans of 3 tennis balls each is 6 tennis balls. 5 + 6 = 11. The answer is 11.

StrategyQA

Q: Yes or no: Would a pear sink in

A: The density of a pear is about 0.6 g/cm^3, which is less than water.

Thus, a pear would float. So the

water?

answer is no.

Math Word Problems (multiple choice)

Q: How many keystrokes are needed to type the numbers from 1 to 500? Answer Choices: (a) 1156 (b) 1392 (c) 1480 (d) 1562 (e) 1788

A: There are 9 one-digit numbers from 1 to 9. There are 90 two-digit numbers from 10 to 99. There are 401 three-digit numbers from 100 to 500. 9 + 90(2) + 401(3) = 1392. The answer is (b).

Date Understanding

Q: The concert was scheduled to be on 06/01/1943, but was delayed by one day to today. What is the date 10 days ago in MM/DD/YYYY?

A: One day after 06/01/1943 is 06/02/1943, so today is 06/02/1943. 10 days before today is 05/23/1943. So the answer is 05/23/1943.

CSQA (commonsense)

Q: Sammy wanted to go to where the people were. Where might he go? Options: (a) race track (b) populated areas (c) desert (d) apartment (e) roadblock

A: The answer must be a place with a lot of people. Race tracks, desert, apartments, and roadblocks don't have a lot of people, but populated areas do. So the answer is (b).

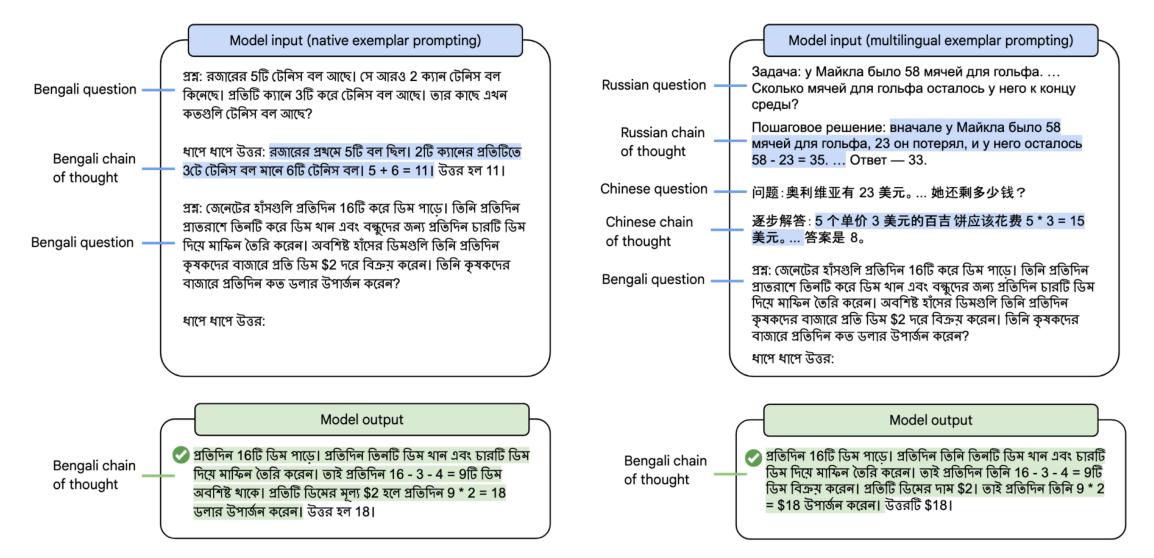
Sports Understanding

Q: Is the following sentence plausible? "Joao Moutinho caught the screen pass in the NFC championship."

A: Joao Moutinho is a soccer player. The NFC championship is part of American football, not soccer. So the answer is no.

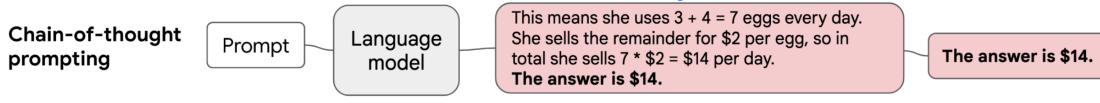
Wei et al. 2022. Chain-of-thought prompting elicits reasoning in large language models.

CoT Prompting for Multilingual Math Reasoning

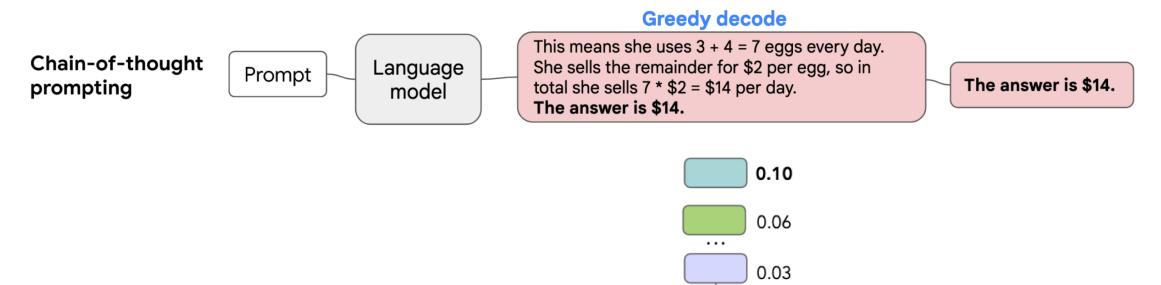


Self-consistency Improves CoT Prompting

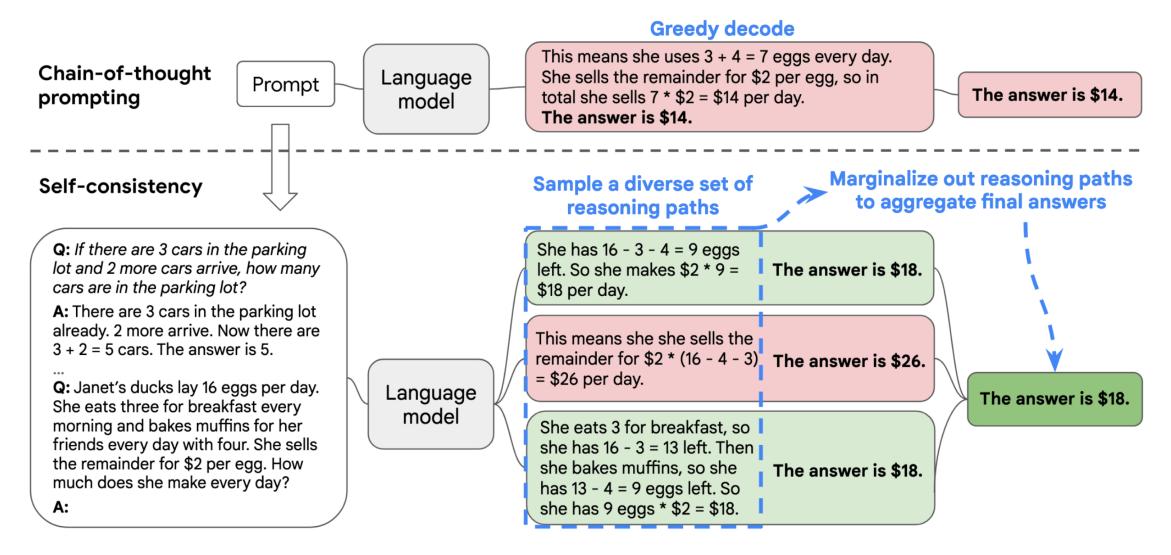
Greedy decode



Self-consistency Improves CoT Prompting



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Wang et al. 2023. Self-Consistency Improves Chain of Thought Reasoning in Language Models.

Improving CoT Prompting for Reasoning

Chen et al. 2023. **Program of thoughts** prompting: Disentangling computation from reasoning for numerical reasoning tasks. Yao et al. 2023. **Tree of Thoughts**: Deliberate Problem Solving with Large Language Models. Ding et al. 2023. **Everything of Thoughts**: Defying the Law of Penrose Triangle for Thought Generation. Zhang et al. 2023. **Automatic chain of thought** prompting in large language models. Zhou et al. 2023. **Least-to-most prompting** enables complex reasoning in large language models. Zheng et al. 2023. **Progressive-hint prompting** improves reasoning in large language models. Jiang et al. 2023. **Resprompt**: Residual connection prompting advances multi-step reasoning in large language models.

Limitations of CoT Prompting

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• Prompting engineering is not interesting

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- Cot Prompting requires scaling
- This method does not explicitly encourage LMs toward correct step-by-step reasoning
- ...

Cot Prompting requires scaling!

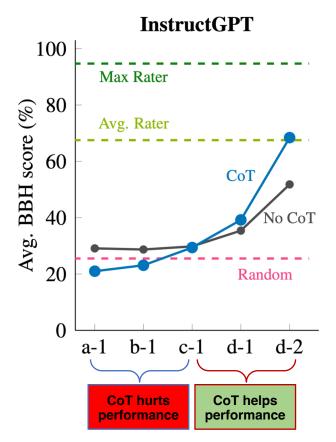
CoT Prompting

Q: Roger has 5 tennis balls. He buys 2 more cans of tennis balls. Each can has 3 tennis balls. How many tennis balls does he have now?
A: Let's think step by step. Roger started with 5 balls. 2 cans of 3 tennis balls each is 6 tennis balls. 5+6=11. The answer is 11.

Q: What is half of (3 + 7) plus one? A:



a-1: text-ada-001; b-1: text-babbage-001; c-1: text-curie-001; d-1: text-davinci-001; d-2: text-davinci-002



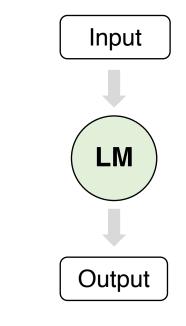
Suzgun et al. 2023. Challenging BIG-Bench Tasks and Whether Chain-of-Thought Can Solve Them.

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Q: Fred had 212 sheets of paper. He received another 307 sheets of paper from Jane and gave Charles 156 sheets of paper. How many sheets of paper does Fred have left?



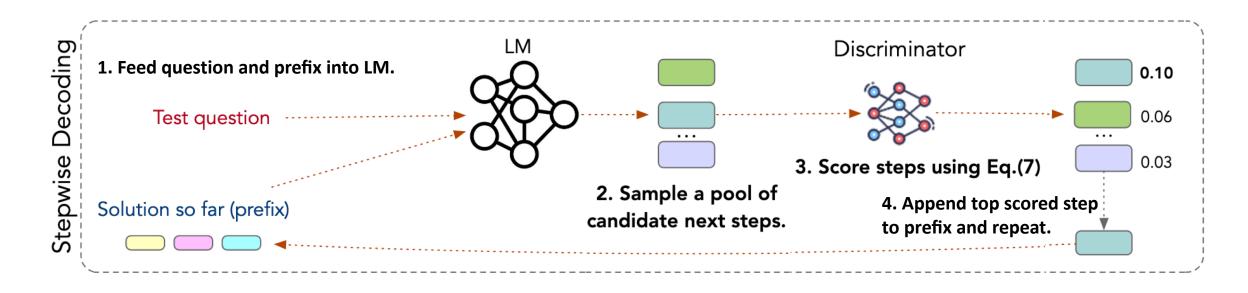


A: Fred has 212 + 156 = 368 sheets of paper. After giving Charles 156 sheets of paper, Fred has 368 - 156 = 212 sheets of paper left.

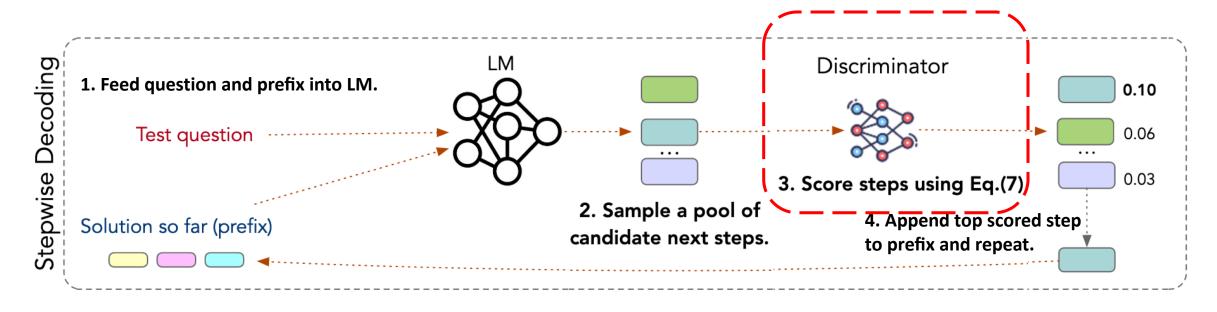
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Using a discriminator to guide the multi-step reasoning

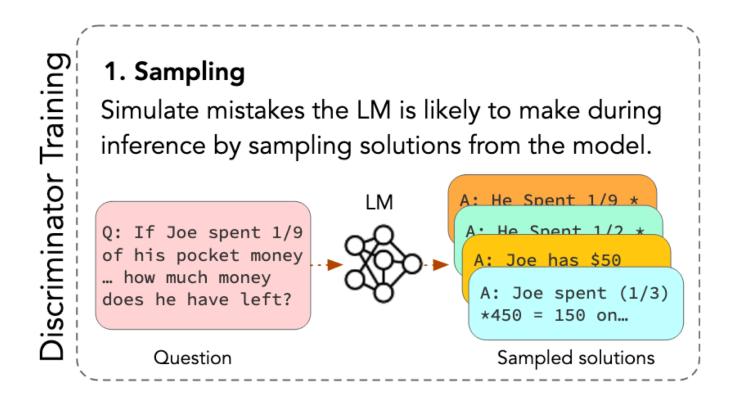
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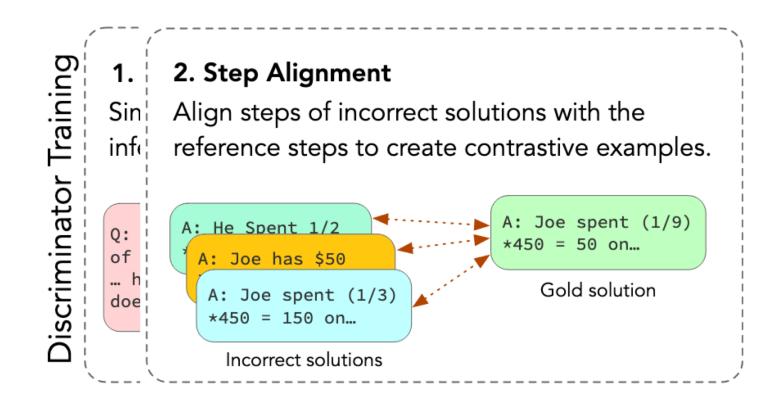
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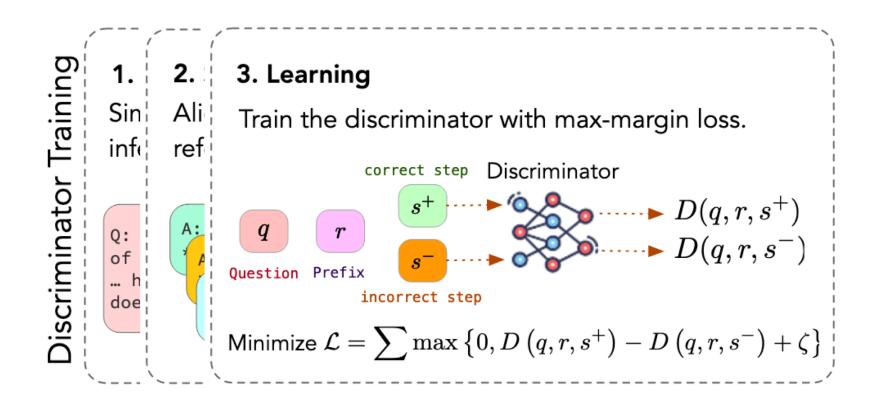
Training a discriminator to guide the multi-step reasoning



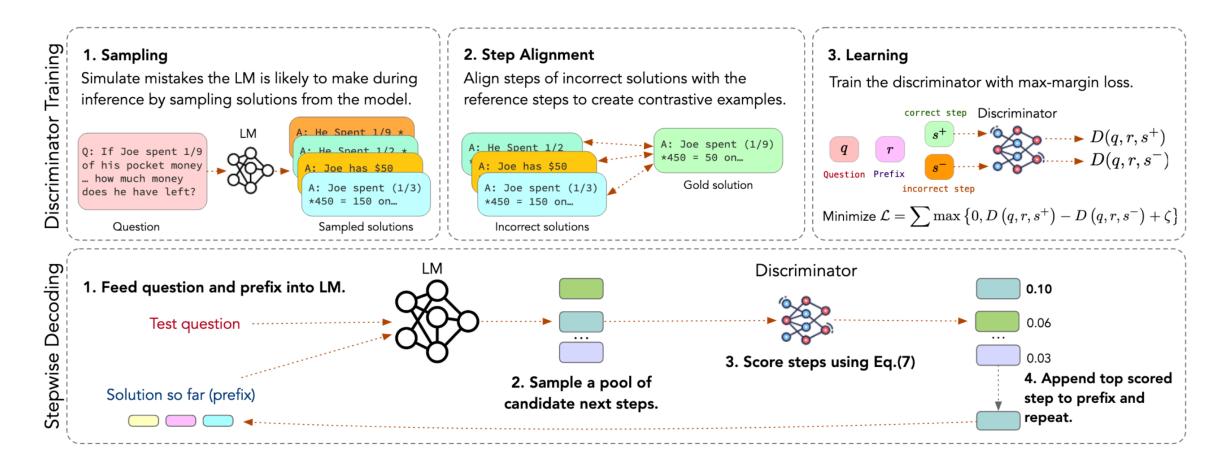
Training a discriminator to guide the multi-step reasoning



Training a discriminator to guide the multi-step reasoning



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Training a discriminator to guide the multi-step reasoning

	FLAN-T5-LARGE			LLAMA-7B		
	GSM8K	SVAMP	MathQA-Gain	GSM8K	MultiArith	SVAMP
Greedy decoding	26.9	54.5	76.5	12.9	54.0	32.8
Self-consistency	33.3	61.8	78.9	20.7	78.9	52.4
Sample-then-rank	20.5	45.9	83.7	9.60	46.4	26.1
GRACE GRACE w/ self-consistency	34.3 (+7.4) 36.3 (+3.0)	66.2 (+11.7) 68.6 (+6.80)	84.1 (+6.0) 84.4 (+0.7)	16.2 (+3.30) 30.9 (+10.2)	84.9 (+30.9) 94.6 (+15.7)	49.7 (+17.3) 55.6 (+3.20)

Cobbe et al. 2021. Training Verifiers to Solve Math Word Problems. Khalifa et al. 2023. Discriminator-Guided Multi-step Reasoning with Language Models. O'Brien et al. 2023. Contrastive Decoding Improves Reasoning in Large Language Models.

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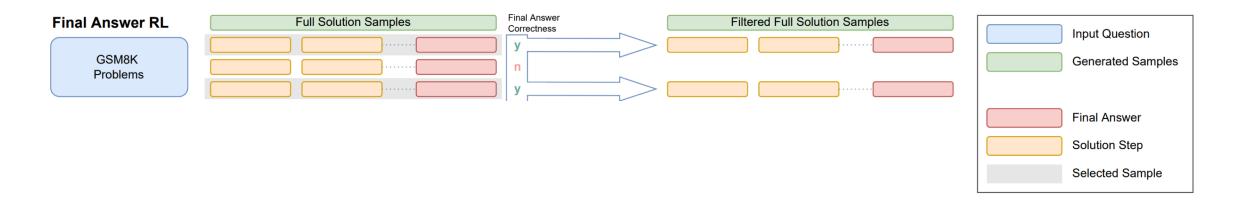
Using reinforcement learning

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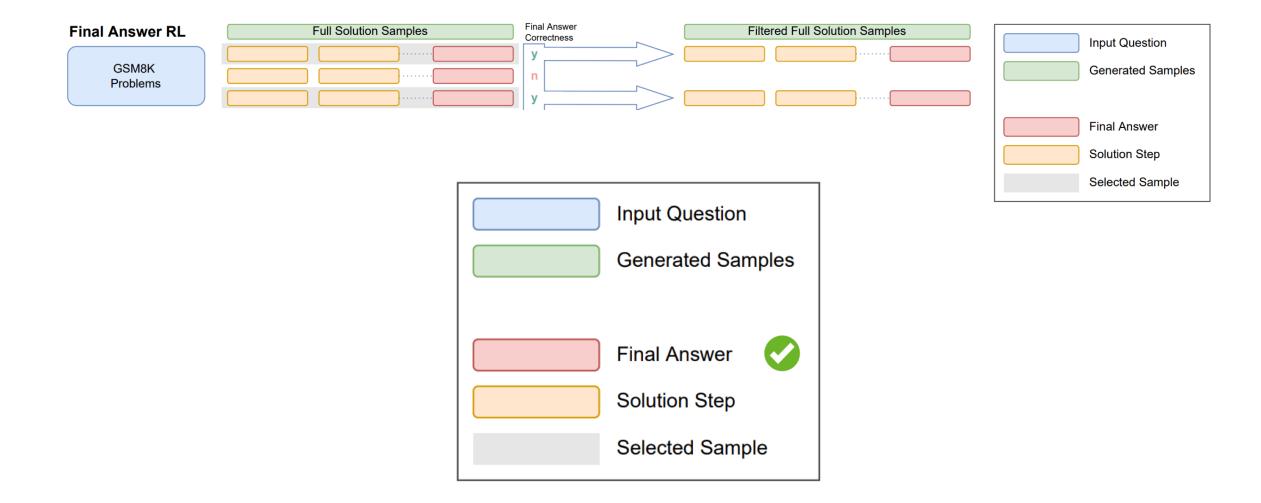
• explicitly optimize models based on feedback from metrics or humans

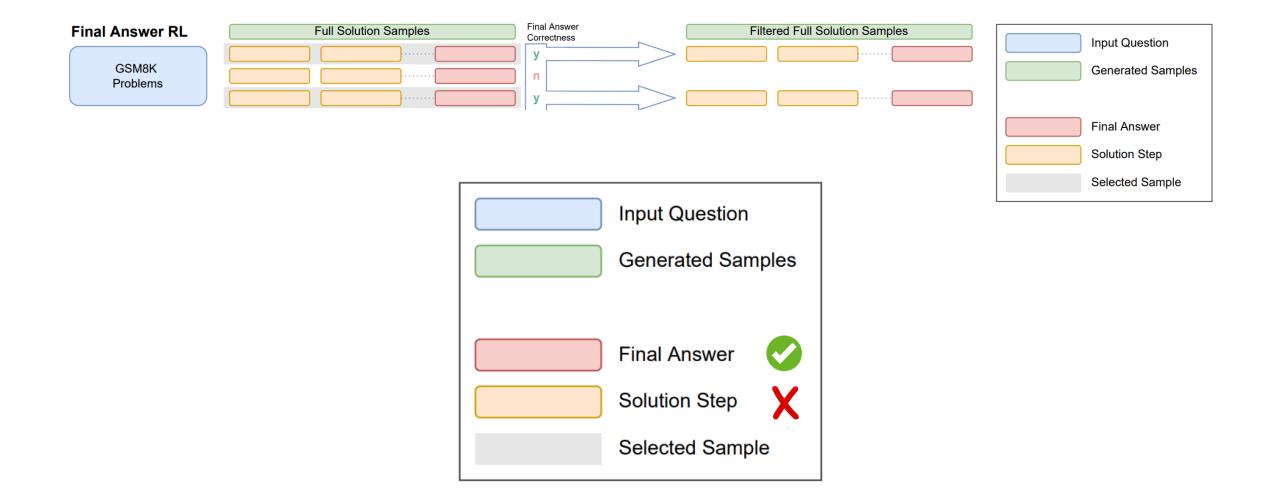
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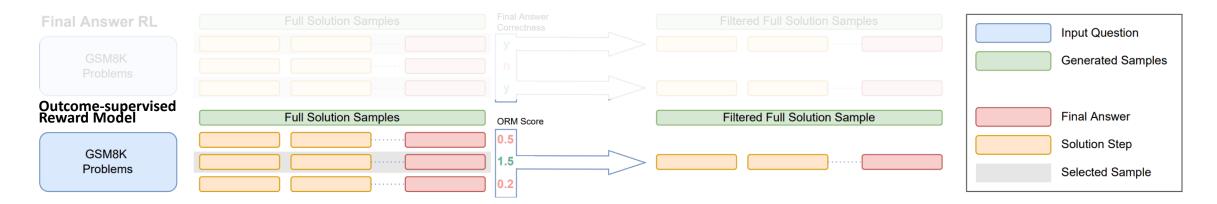
explicitly optimize models based on reasoning outcome or reasoning process



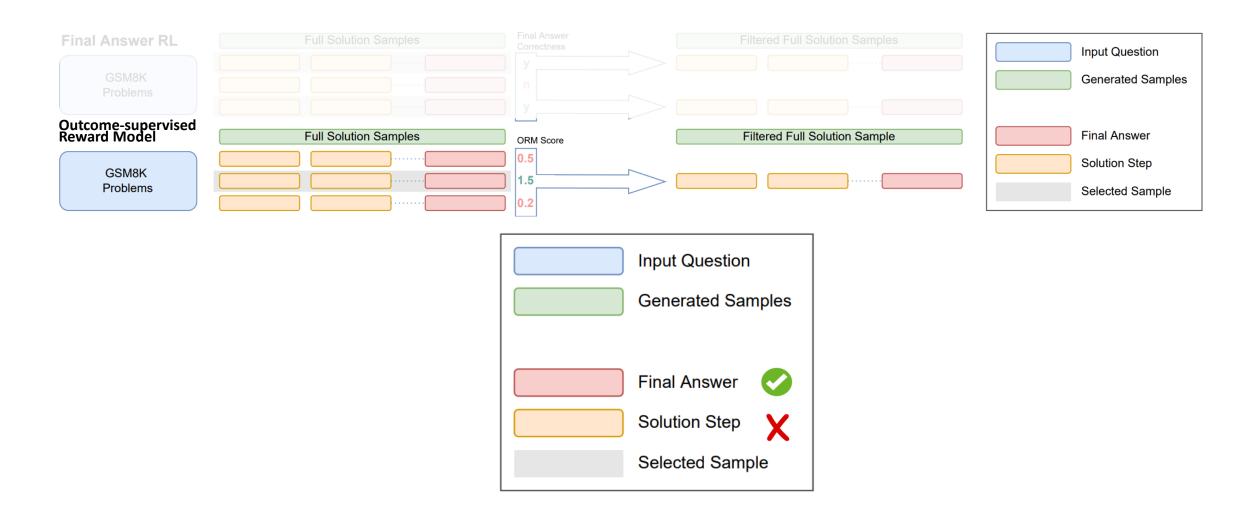
Generating *K* full potential reasoning solutions per problem and filter by final-answer correctness.







A reward model is trained to judge the correctness of solutions, with the training signal determined solely by whether or not the solution reached the correct final answer (human annotation).



Final Answer RL	Full Solution Samples			Filtered Full Solution Samples	Input Question
GSM8K Problems					Generated Samples
Outcome-supervised Reward Model	Full Solution Samples	ORM Score		Filtered Full Solution Sample	Final Answer
					Solution Step
GSM8K Problems					Selected Sample
		0.2			
Process-supervised Reward Model	Full Solution	Samples			Filtered Full Solution Sample
Reward Wodel	PRM Score	PRM Score	PRM	Score	
GSM8K		0.9	0.5		
Problems					
	0.2		0.1		

A reward model is trained to judge whether the reasoning steps so far are correct (human annotation).

		Error rate (%)		
Approach	Base model	Trace Final-and	swer	
SFT+Final-Answer RL, ORM reranking	Base-70B	3.7 (0.5-6.9)	14.2	
SFT, ORM reranking	Base-70B	4.4 (0.6-8.3)	14.8	
SFT, PRM reranking	Base-70B	3.5 (0.5-6.5)	14.1	
SFT+ORM-RL, ORM reranking	Base-70B	3.4 (0.0-6.8)	12.7	
SFT+PRM-RL, PRM reranking	Base-70B	3.8 (0.5-7.1)	12.9	

- Results-based method
 - No human annotation
 - Reasoning steps might be incorrect
- Reward-model-based method
 - Human annotation
 - Better results

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Uesato et al. 2023. Solving math word problems with process and outcome-based feedback. Zelikman et al. 2023. Star: Bootstrapping reasoning with reasoning. Luo et al. 2023. Wizardmath: Empowering mathematical reasoning for large language models via reinforced evol-instruct.

Data Augmentation Improves Reasoning

Fu et al. 2023. Specializing smaller language models towards multi-step reasoning. Hsieh et al. 2023. Distilling step-by-step! outperforming larger language models with less training data and smaller model sizes.

Magister et al. 2023. Teaching small language models to reason.

Shridhar et al. 2023. Distilling reasoning capabilities into smaller language models.

Yue et al. 2023. MAmmoTH: Building math generalist models through hybrid instruction tuning.

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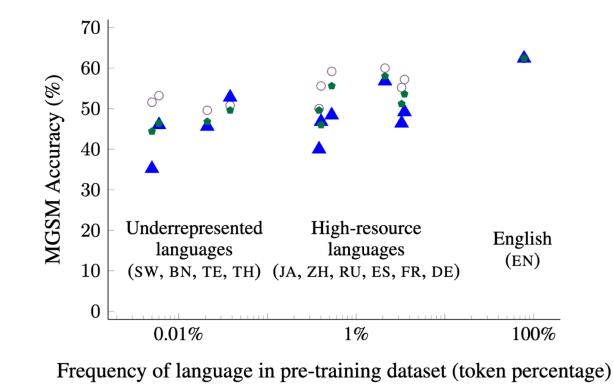
Huang et al. 2023. Large language models can self-improve.

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DATA is IMPORTANT!

Multilingual Math Reasoning

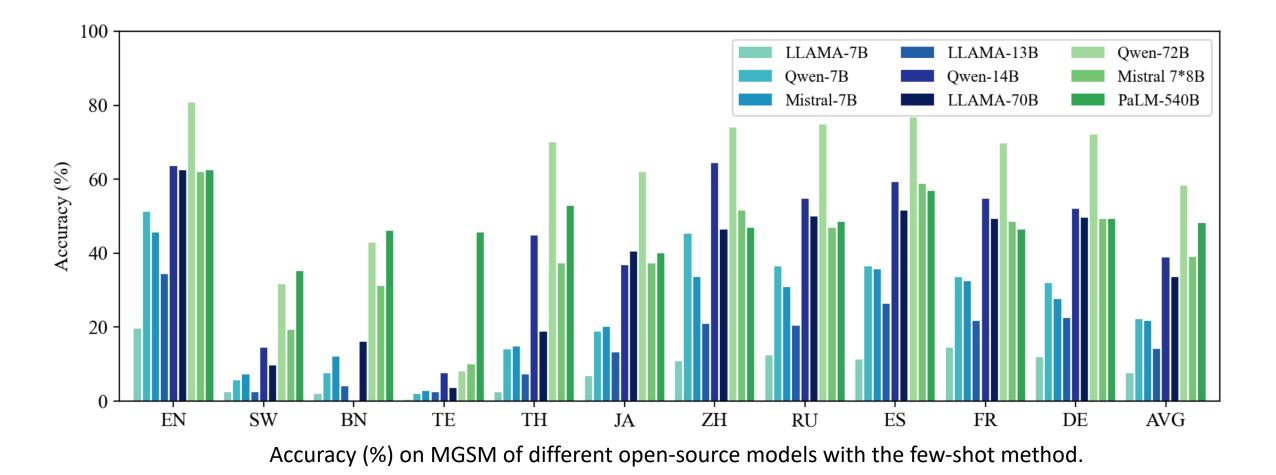
- Translate to English with Google Translate and solve with English intermediate steps
- ▲ Intermediate reasoning steps in the language of the question
- Intermediate reasoning steps in English



Correlation between language frequency and MGSM accuracy for PaLM-540B.

Shi et al. 2023. Language models are multilingual chain-of-thought reasoners.

Multilingual Math Reasoning



Multilingual Math Reasoning

Language	CoT Reasoning				
Question	Terry eats 2 yogurts a day. They are currently on sale at 4 yogurts for \$5.00. How much does he spend on yogurt over 30 days?\nAnswer:\nLet's think step by step.				
EN	He eats 2 yogurts a day so over 30 days he will need $2*30 = 60$ yogurts\n The yogurt sells as 4 units for \$5.00 and he needs 60 yogurts so that's $60/4 = 15$ packs of yogurt\n Each pack is \$5.00 and he needs 15 packs so that's $5*15 = 75.00 \n The answer is 75.				
SW	Anakunywa maziwa 2 ya mtindi kila siku hivyo zaidi ya siku 30 anakunywa $2*30 = 60$ Maziwa ya mtindi yanauzwa kwa 4 kwa \$5. na anahitaji 60 hivyo hiyo ni $60/4 = 15$ pakiti za maziwa 4 Kila pakiti ya 4 inagharimu \$5.00 na anahitaji pakiti 15 kwa hivyo hiyo n $5*15 = 75.00 Jibu ni 75.				
BN	তনিদিনিং 2টি ইঠ োগার্ট খান এবং সগেুলপি্রত 4িটি 5.00 ডলারং বকি্রহিয় যাতং 2/4 = \$2.50 তনিপি্রতদিনি ইঠ োগার্টরে জন্য \$2.50 খরচ করনে তাই 30 দনিরে বশেসিময় তনি2.5*30 = \$75.00 খরচ করনে উত্তর হল 75।				
TE	అతను రోజుకు 2 యోగర్ట్ లు తంటాడు మరియు అమ్ మకంలోఉన్ న సేల్ లో4 ఉన్ నాయి కాబట్ టి అతను రోజుకు 2/4 = 0.5 యోగర్ట్ లను కొనుగోలు చేయాలి, అపి ఒక్ కొక్ కట \$5.00 మరియు అతను రోజుకు 0.5 కొనుగోలు చేయాలి కాబట్ టి అతను రోజుకు 5*0.5 = \$2.50 ఖర్ చుచేస్ తాడు. 30 రోజుల పాటు యోగర్ట్ లు అతసిక రోజుకు 2.5*30 = \$75.00 ఖర్ చమతాయి సమాధానం 75.				
TH	เทอร _์ รี่∣กินโยเกิร์ต 2 ถัวยต่อวัน ดังนั้นใน 30 วัน เขาจะต้องการโยเกิร์ต 2 x 30 = 60 ถ้วย โยเกิร์ต 4 แพ็กลดราคาอยู่ที่ 5.00 ดอลลาร์สหรัฐฯ ดังนั้นโยเกิร์ตแต่ละถ้วยจึงมีราคา 5.00 ดอลลาร์สหรัฐฯ / 4 = 1.25 ดอลลาร์ ดังนั้น เทอร์รี่∣่จะใช้จ่าย 60 x 1.25 ดอลลาร์ = 75.00 ดอลลาร์ สำหรับโยเกิร์ตใน 30 วัน คำตอบคือ 75				
JA	彼は1日に2つのヨーグルトを食べるので、30日間で2*30=60個のヨーグルトを食べることになります。ヨーグルトは4個で5ドル で販売されており、彼は60個のヨーグルトを購入する必要があるため、60/4=15個の4パックになります。各パックの価格は5ドルで 彼は15個のパックを購入する必要があるため、5*15=75ドル相当のヨーグルトになります。答えは75です。				
ZH	他每天吃 2 份酸奶,所以 30 天他将吃 2*30 = 60 份酸奶 酸奶以 4 份的价格出售,他将购买 60 份酸奶,因此 60/4 = 15 包酸奶 每包售价 5.00 美元,他将购买 15 包,因此 5*15 = \$75.00 答案是 75。				
RU	Терри съедает 2 йогурта в день, значит, за 30 дней ему понадобится 2 x 30 = 60 йогуртов. Йогурт продается по цене 5 долларов за 4 йогурта, поэтому Терри нужно будет купить 60/4 = 15 упаковок йогурта. Каждая пачка стоит 5 долларов, поэтому Терри потратит 15 x 5 долларов = 75 долларов на йогурт за 30 дней. Ответ: 75.				
ES	Gasta 2*5=\$10 por d\u00eda en yogur. Gasta 10*30=\$300 en 30 d\u00edas. La respuesta es 300.				
FR	Il mange 2 yaourts par jour donc sur 30 jours il mange $2*30 = 60$ yaourts. Les yaourts sont vendus 4 pour 5,00 \$ et il a besoin de 60 yaourts, ce qui fait $60/4 = 15$ paquets de 4 yaourts. Chaque paquet de 4 co\u00fbte 5,00 \$ et il a besoin de 15 paquets de 4 donc il d\u00e9pense 5*15 = 75,00 \$ en yaourt. La r\u00e9ponse est 75,00.				
DE	Er isst 2 Joghurts pro Tag, also ben\u00f6tigt er \u00fcber 30 Tage hinweg $2*30 = 60$ Joghurts. Der Joghurt wird f\u00fcr 5,00 \$ pro 4er-Packung verkauft und er braucht 60 Joghurts, das sind also $60/4 = 15$ Packungen Joghurts. Jede Packung kostet 5,00 \$ und er braucht 15 Packungen, das sind also $5*15 = 75,00$ \$. Die Antwort ist 75.				

Outlook

- Scaling models
- Better prompting/verifier/optimization method
- Multilingual scenario (low-resourced languages)
- High-quality Data
- Smaller task-tailored models

What is Reasoning?

Concatenate the first letter of each word

Answer the following strategy question

Q Artificial Intelligence



Yes or no: Would a nail sink in water?

A AI



Nails are made of materials such as steel or iron, which are denser than water. Thus, the answer is yes.

Solve the following math word problem



Terry eats 2 yogurts a day. They are currently on sale at 4 yogurts for \$5.00. How much does he spend on yogurt over 30 days?

A He eats 2 yogurts a day so over 30 days he will need 2*30 = 60 yogurts. The yogurt sells as 4 units for \$5.00 and he needs 60 yogurts so that's 60/4 = 15 packs of yogurt. Each pack is \$5.00 and he needs 15 packs so that's 5*15 = \$75.00. The answer is 75.

"A faculty of the human mind that enables logical inferences to be made and rational arguments to be undertaken to understand the world and solve problems."

Michael Kent. 2007. The Oxford Dictionary of Sports Science & Medicine (3 ed.)

Do Language Models Really Perform Reasoning?

Concatenate the first letter of each word

Answer the following strategy question



Artificial Intelligence



Yes or no: Would a nail sink in water?



LM

Nails are made of materials such as steel or iron, which are denser than water. Thus, the answer is yes.

Solve the following math word problem



Terry eats 2 yogurts a day. They are currently on sale at 4 yogurts for \$5.00. How much does he spend on yogurt over 30 days?



He eats 2 yogurts a day so over 30 days he will need 2*30 = 60 yogurts. The yogurt sells as 4 units for \$5.00 and he needs 60 yogurts so that's 60/4 = 15 packs of yogurt. Each pack is \$5.00 and he needs 15 packs so that's 5*15 = \$75.00. The answer is 75.



Gasta 2*5=\$10 por d\u00eda en yogur. Gasta 10*30=\$300 en 30 d\u00edas. La respuesta es 300.



X

Thank You!