Strategies: A logic - automata study
Lecture 5: Dynamics of large games

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Many issues

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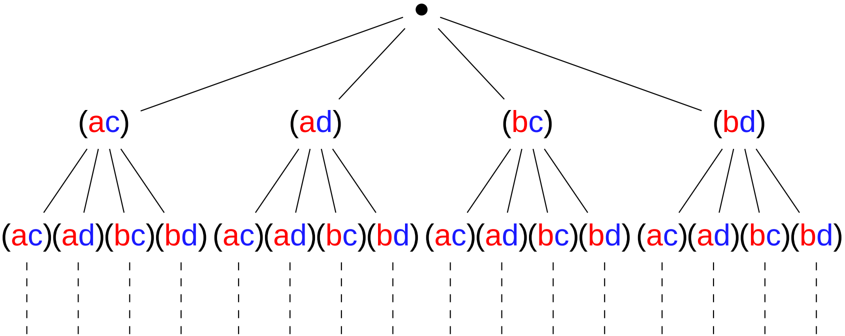
- Dynamic game forms.
- Algebraic theory of strategy composition.

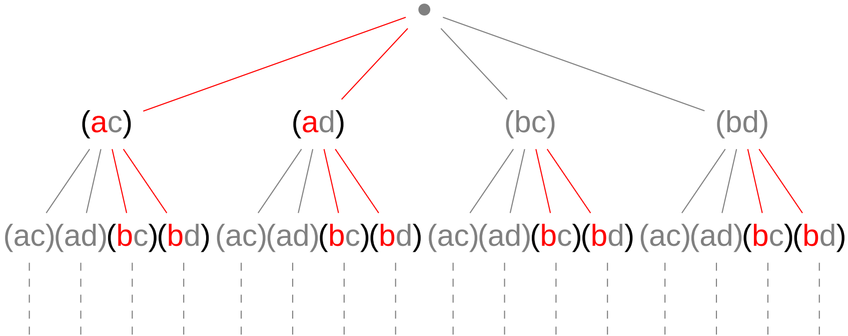
Strategy switching and stability

Concurrent game structures



$$A = \{a, b\}, \ B = \{c, d\}$$

Strategies - Subtrees



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- They compose/switch to devise new strategies.

A switch operator

$$\Omega_i ::= \sigma \in \Sigma_i \mid St_1 \cup St_2 \mid St_1 \cap St_2 \mid St_1 {}^\frown St_2 \mid (St_1 + St_2) \mid \psi ? St'$$

where ψ is a past time formula of a simple tense logic over an atomic set of observables and Σ_i is the set of all atomic (partial) strategies of player *i*.

Strategy Specifications[2]

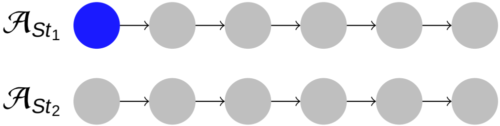
Intuitively

$St_1 \cup St_2$	chooseSt_1 or St_2 and play accordingly.
$St_1 \cap St_2$	play according to St_1 if defined else play according to St_2 if defined.
$St_1 {}^\frown St_2$	play according to St_1 and after some point switch to playing according to St_2.
$(St_1 + St_2)$	at every point play either according to St_1 or St_2.
$\psi?St'$	test if ψ holds at a point. If yes, play according to St'.

Each specification defines a set of total strategy trees. The semantics is given in terms of the strategy trees.

Transducers for operators

$St_1 \cup St_2$



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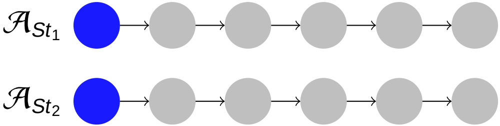

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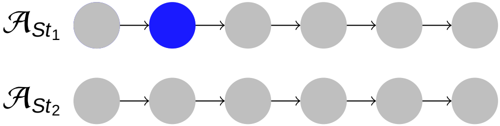

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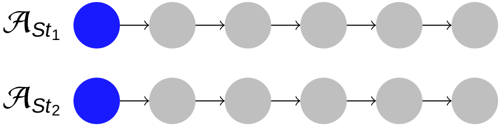

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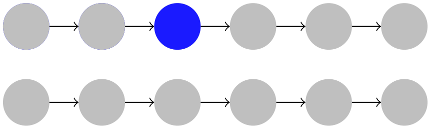

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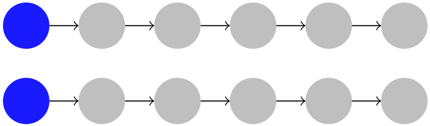

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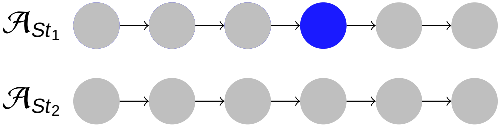

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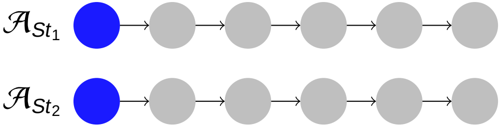

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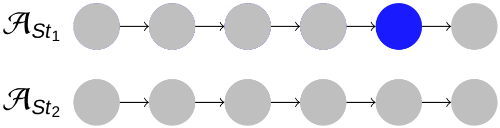

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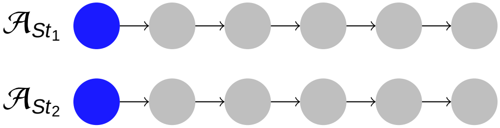

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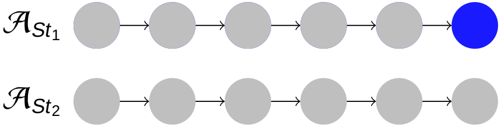

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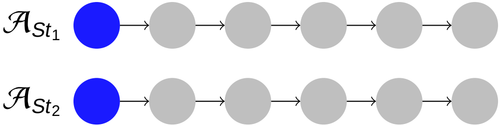

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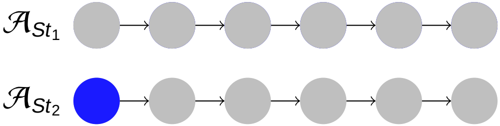

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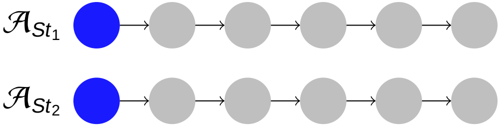

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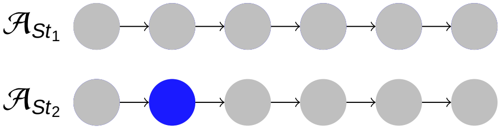

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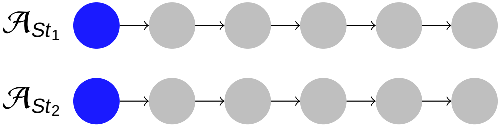

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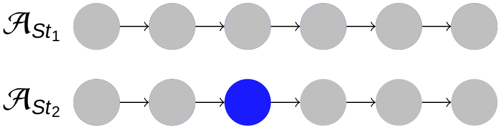

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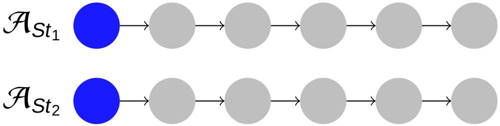

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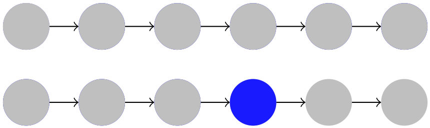

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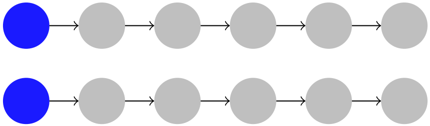

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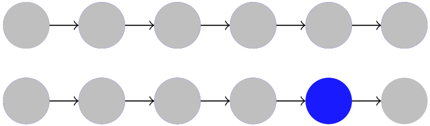

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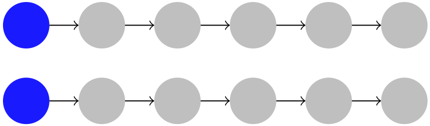

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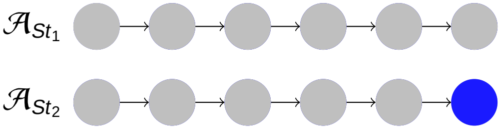

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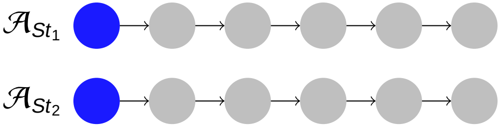

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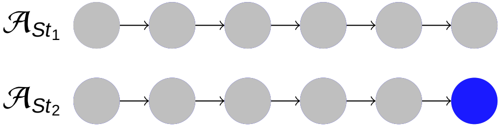

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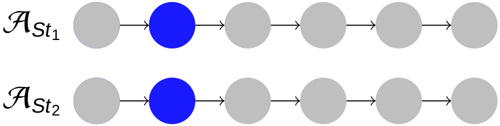

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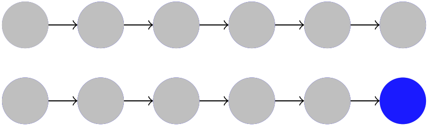

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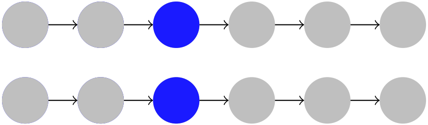

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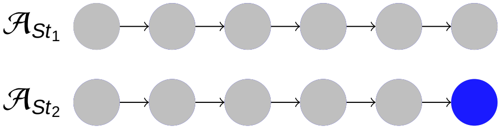

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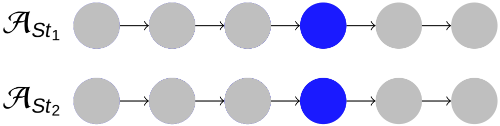

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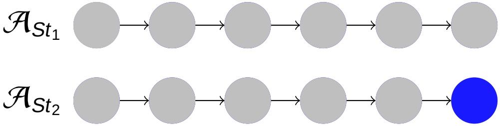

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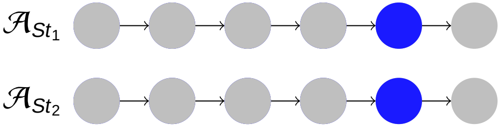

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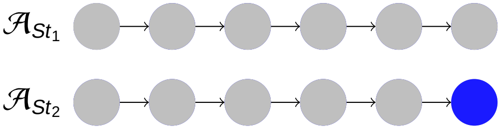

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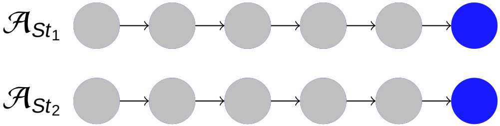

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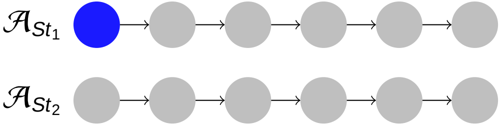

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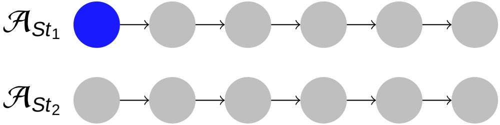

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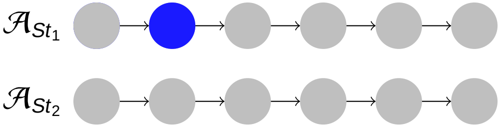

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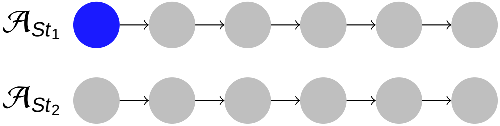

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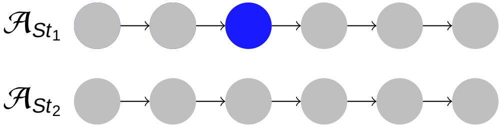

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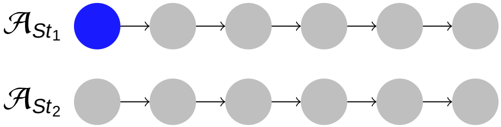

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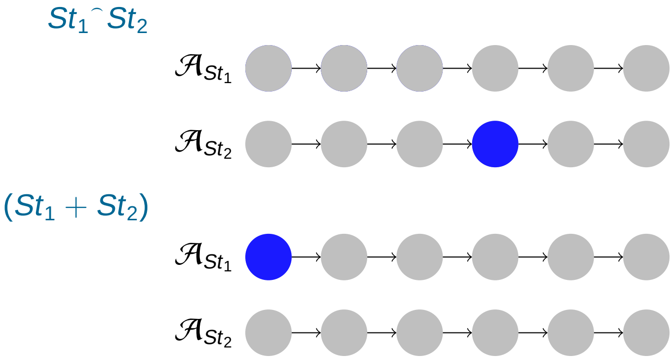

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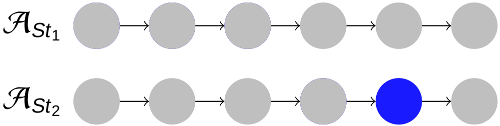

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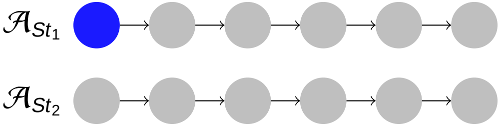

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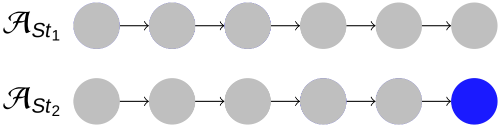

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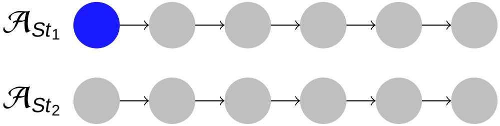

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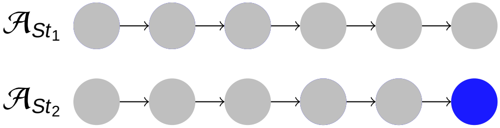

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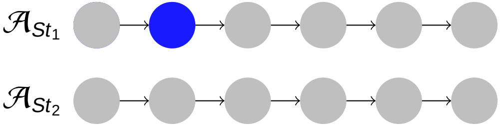

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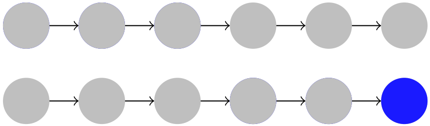

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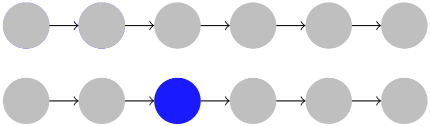

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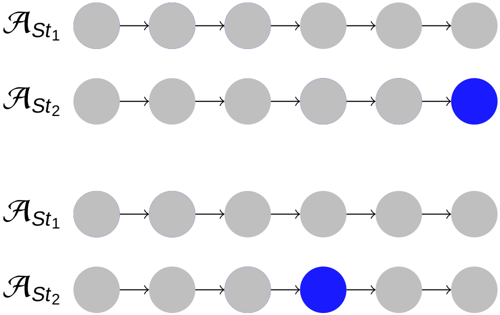

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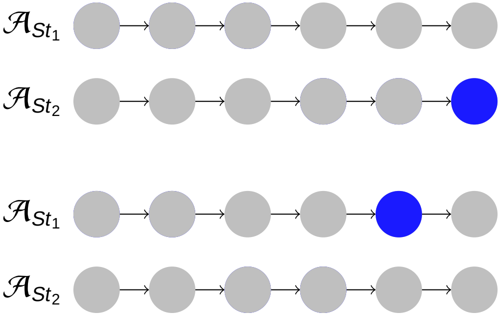

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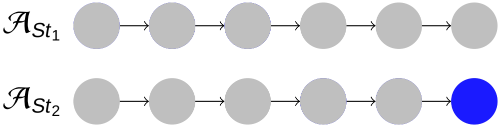

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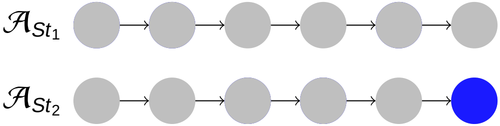

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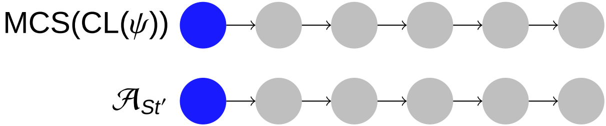

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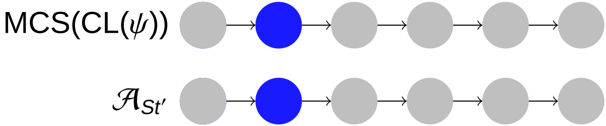

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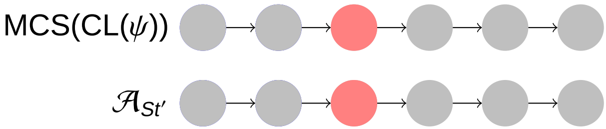

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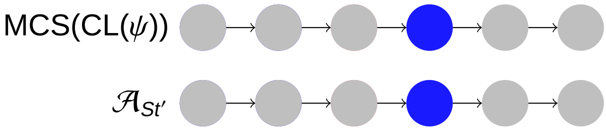

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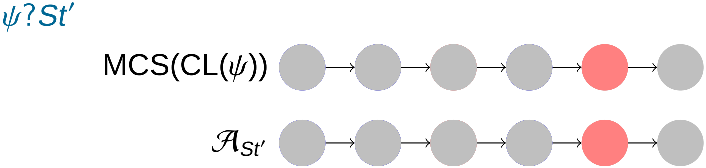
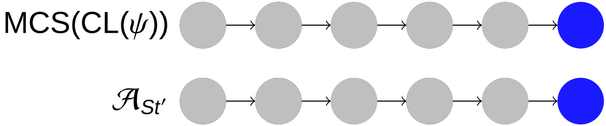


Transducers for operators

$\psi ? St'$



Questions

St is switch-free if it does not have any of the $\frown$ or the $+$ constructs.

Questions:

- Does the game eventually settle down to some subarena? (the equilibrium subset)
- In the equilibrium subset, can a player keep the game in that subset with a strategy that does not involve switching?

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Results

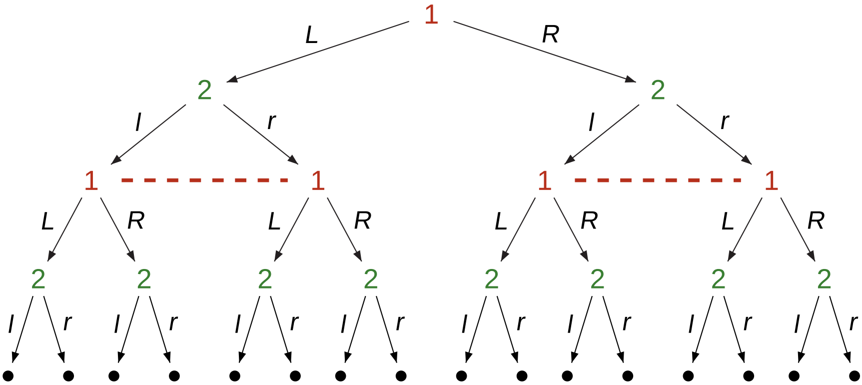
Theorem

Given a game arena $\mathcal{G}$, a valuation Val of the atomic observables on the arena, a subarena $\mathcal{G}'$ and strategy specifications $St_1, \ldots, St_n$ for the players, the following questions are decidable:

- *Do all plays conforming to these specification eventually settle down to the subarena?*
- *If all plays conforming to these specifications converge to the subarena, does the strategy of a player become eventually stable with respect to switching?*

Imperfect information and distributed games

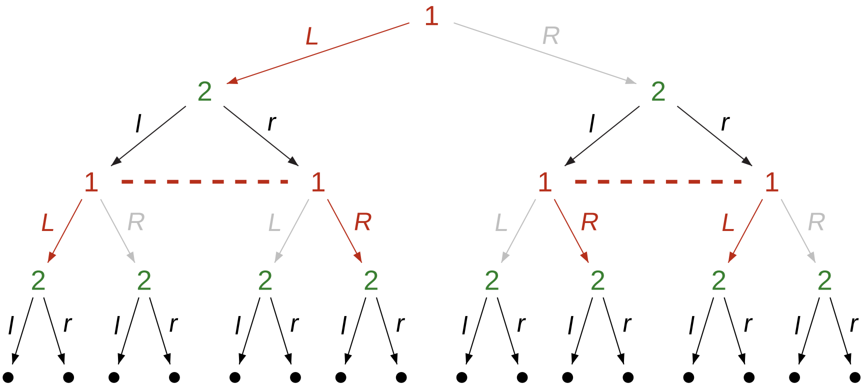
Imperfect information



Game tree + uncertainty relation: $T = (T_{PI}, (\sim_i)_i)$.

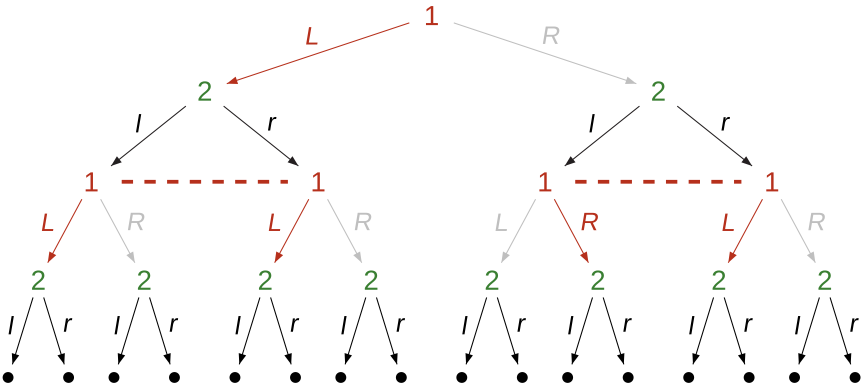
The uncertainty relation defines players' information sets.

Strategy with imperfect information



Strategy needs to respect the uncertainty relation.

Strategy with imperfect information



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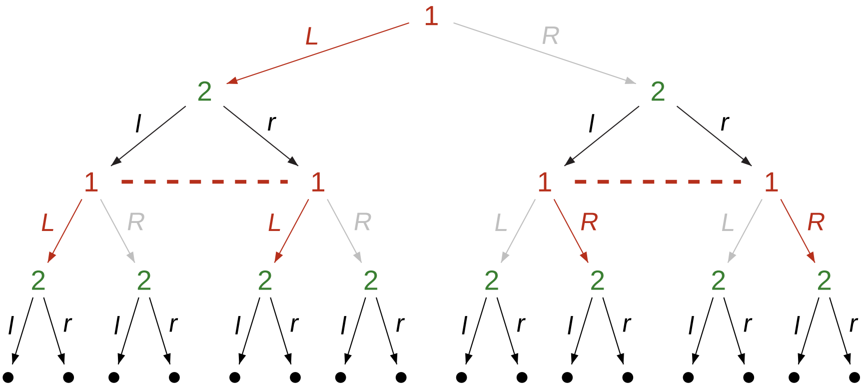

Strategy needs to respect the uncertainty relation.

Information sets

- Imperfect information:
  strategy: information sets $\rightarrow$ actions.
- Perfect information:
  all informations sets are singletons.

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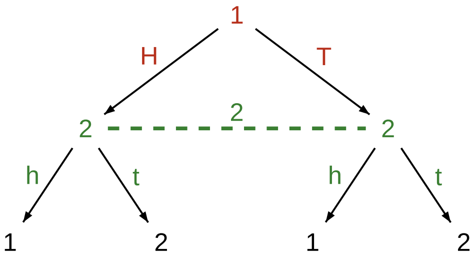
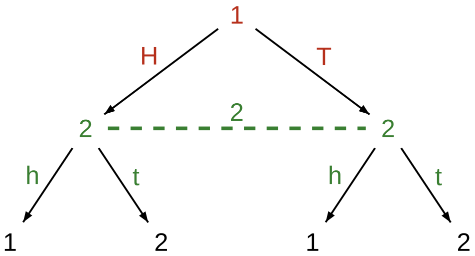
Answer: No (matching pennies).

Matching pennies



- If the outcomes match then player 1 wins else player 2 wins.

Matching pennies



- If the outcomes match then player 1 wins else player 2 wins.

Neither player has a winning strategy.

Three players

Players 1 and 2 have common winning condition Φ.

Player 3 is indifferent.

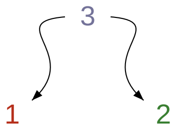
Strategy question: Are there strategies (σ_1, σ_2) such that for all σ_3 the outcome lies in Φ.

Undecidability

It is possible to reduce the halting problem to the strategy problem.

The game:

- Players 1 and 2 cannot "talk" to each other.
- Player 3 has perfect information about the game.
- Player 3 can create an information fork.



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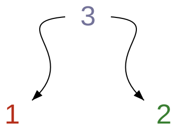


The knowledge hierarchy can become unbounded.

Undecidability

Is the strategy question decidable?

Undecidability

Is the strategy question decidable?

Answer: No, it is undecidable.

- Reachability objectives [Bernet and Janin 05].
- Safety objectives [Bernet 06, Berwanger and Kaiser 10].
- Decidable subclasses [Gastin, Sznajder and Zeitoun 09].

An alternative model

Arbitrary uncertainty relations can code up information forks.

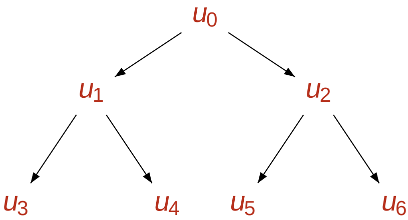
- We propose a subclass where the uncertainty relation reflects the game structure.
- Communication: Information sets are generated in terms of players' ability to communicate.

What is a model for such games ?

A possible solution

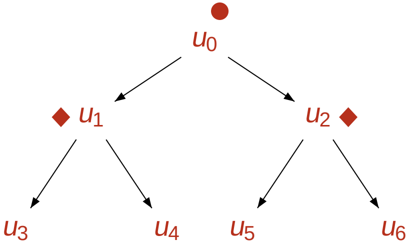
- Local game arena for each player.

Local arena



Player 1’s local arena

Local arena



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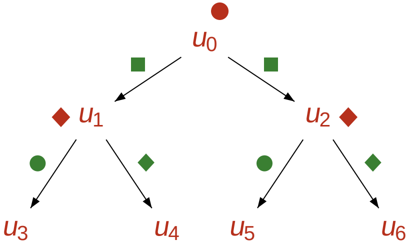

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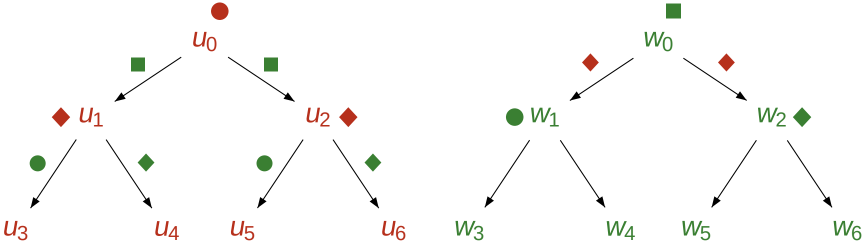

Player 1’s local arena

Distributed games

MiMaze: Multicast Internet Maze

- Each player sees only a part of the maze.
- Of course she encounters others while navigating.
- They can leave trails, put up coloured flares and so on.

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WWW: The internet itself can be seen as a distributed game.

Local arena

A finite set of announcement symbols Γ^i.

- $\Gamma^1 = \{\blacksquare, \bullet, \blacklozenge\}$, $\Gamma^2 = \{\blacksquare, \bullet, \blacklozenge\}$.

Local arena: $\mathcal{G}^i = (W^i, \rightarrow_i, w_0^i, \chi^i)$ where

- W^i - set of local game positions.
- w_0^i - initial game position.
- $\chi^i : W^i \rightarrow \Gamma^i$. $\chi^1(u_1) = \blacklozenge$.
- $\rightarrow_i : W^i \times \widehat{\mathbf{\Gamma}} \rightarrow 2^{W^i}$.

Global arena

How do players know when to move ?

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Enabling of moves depend on:

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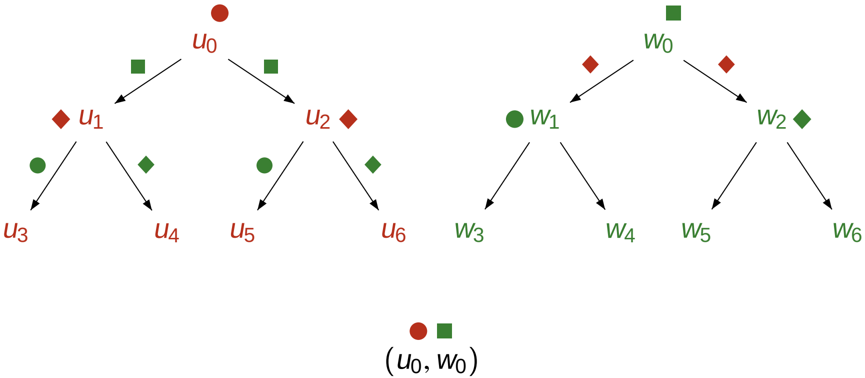
Global arena:

- Product of the local arenas.
- Only enabled moves appear.

Global game arena



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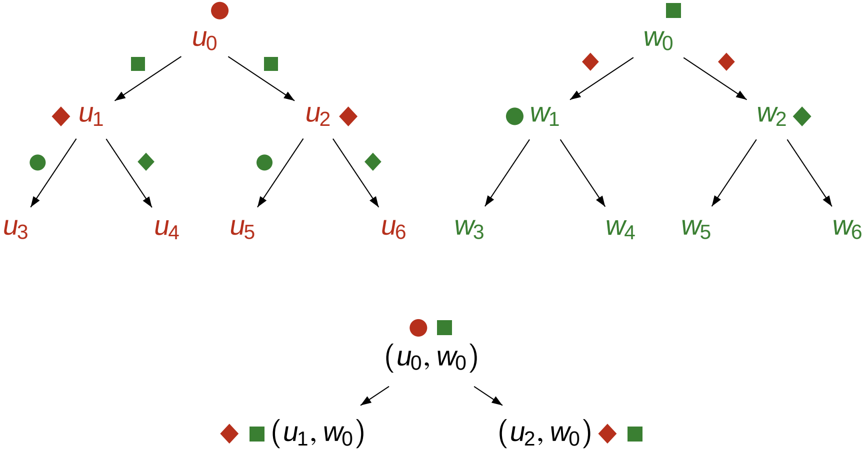

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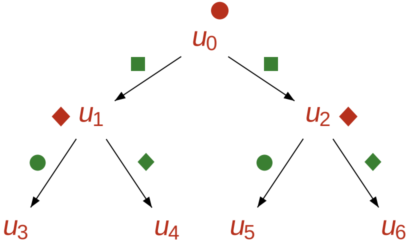

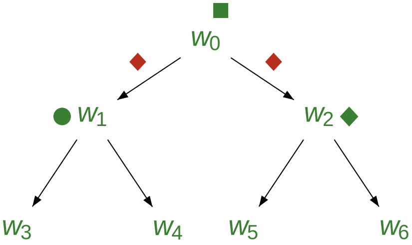

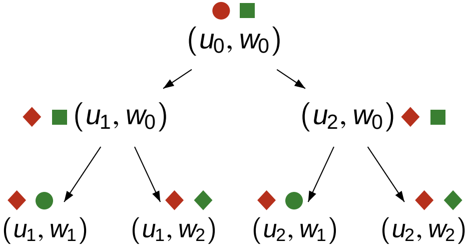

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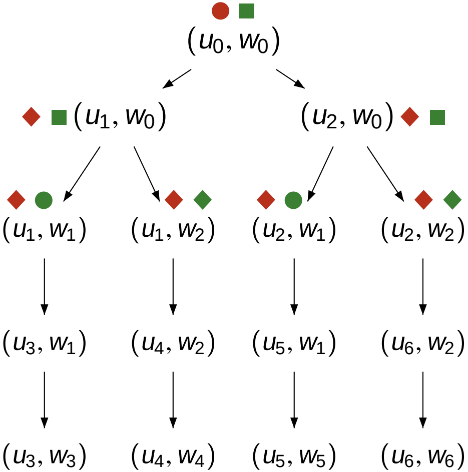


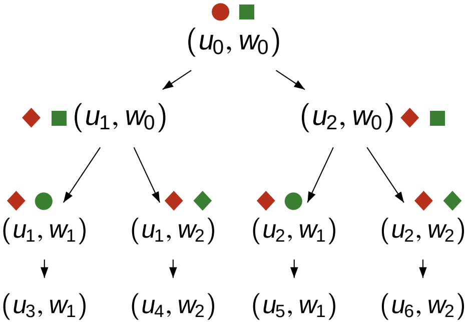
Information sets

Assumptions:

- Communication by means of public announcement.
- Existence of unique initial state.

Question: Are non-trivial information sets generated?

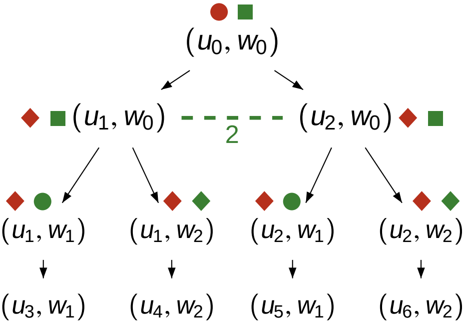
Global game arena



View of a player: local state + announcements received.

- $view^2((u_0, w_0)) = (w_0, \bullet)$.

Global game arena



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Properties

- Captures imperfect information games.
- Moves can be concurrent.
- Communication can force turn based games.

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Consequence: For all players i, j, k,

- player i's uncertainty about j = players k's uncertainty about j.

Corollary: Knowledge hierarchy collapses.

Objectives

Strategy question: Decidable.

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Non-zero sum objectives: With each player i we have:

- a reachability set, $\mathcal{R}^i \subseteq W^i$.
    - $\mathcal{R}^i$ constitutes sink nodes.
    - players announce the entire local state.
- preference ordering: $\preceq^i \subseteq \mathcal{R}^i \times \mathcal{R}^i$.

Observable behaviour

Task: We need to define best response in terms of observable behaviour of players.

Announcement plan: An announcement plan for player i, $\xi_i : W^* \to \Gamma^i$ which respects the information partition.

- Specifies an announcement instead of a local state.

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Bounded memory announcement plan - representable by a finite state transducer.

Solution concept

Local best response: A strategy σ_i is a local best response for an announcement plan profile ξ_{-i} if

- for all σ'_i and for all $\sigma_{-i} \in [\![\xi_{-i}]\!]$, $\rho_{(\sigma'_i,\sigma_{-i})} \preceq^i \rho_{(\sigma_i,\sigma_{-i})}$.

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Perfect information games

- Local best response = Best response.
- Locally consistent equilibrium = Nash equilibrium.

A new notion

In general LCE (Locally consistent equilibrium) is distinct from NE (Nash equilibrium) in distributed games.

- We have games for which NE exists but not LCE, and vice versa.
- We have games for which both exist and coincide, as well as games for which both exist and are different.

Algorithmic questions

Questions of interest:

- Verification: Is it decidable to check if a local best response strategy exists for ξ_{-i} ?
- Synthesis: Synthesize a local best response strategy for ξ_{-i} (when it exists).

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Local best response computation

Problem: Given a game $G = (\{\mathcal{G}^i\}_{i \in N}, \{\preceq^i\}_{i \in N})$ and an announcement profile ξ_{-1} synthesize the local best response for player 1.

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Proposition: Player 1 has a local best response in $\mathcal{G}$ iff she has a dominant strategy in $\mathcal{G}^{\mathsf{K}}$.

Equilibrium computation

Question: Given a game $G = (\{\mathcal{G}^i\}_{i \in N}, \{\preceq^i\}_{i \in N})$,

- check whether G has a locally consistent equilibrium profile.
- synthesize an equilibrium profile (when it exists).

Task: Construct a finite structure which preserves the equilibrium behaviour of players.

Core issue: Identify the knowledge set (or memory) that players need to keep track of in the game.

Bounded memory: We show that bounded memory strategies suffice and construct automata for this.

Private communication

Communication by means of private channels

Extend the communication alphabet: $\Gamma^i \quad \rightsquigarrow \quad \Gamma^i_j$ for each player j.

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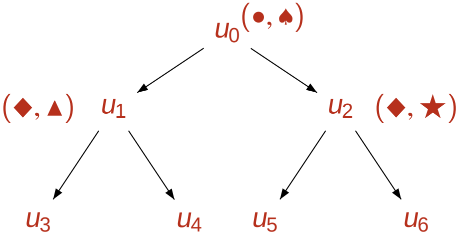

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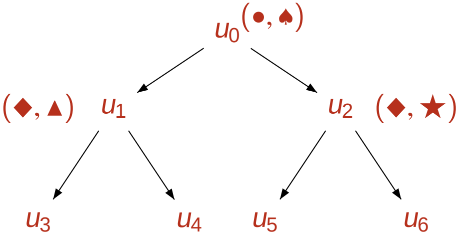

Three player games: Strategy problem is undecidable.

Remarks

Folklore wisdom: Multiplayer imperfect information games are intractable.

Main message: Does not imply that imperfect information games are always uninteresting.

- More careful analysis of how imperfect information arises in games.
- Uncertainty due to structural reasons - solvable by communication.

Remarks

- Communication is a strategic concern.
- Public and private communication constitute extremes in the model.
  - Card games.
  - Bounded revelation games.
- Asynchronous communication.
- Complexity of locally consistent equilibria.

Large games

Infrastructure questions

Recently Singapore decided to make the entire city Wi-Fi enabled. How is it decided that a facility be provided as infrastructure ?

- Typically such analysis involves determining when usage crosses a threshold.
- But then understanding why usage of one facility increases vastly, despite the presence of several alternatives, is tricky.

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The invisible traffic

Similar situations occur in the management of the Internet.

- Policies for bandwidth allocation are not static.
- They are dynamic, based on studying both volumes of traffic and type of traffic.
- The popularity of an application like YouTube dramatically changes such usage, calling for changes in Internet policies.
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Lost in the arena

These are generic examples of games with structure and involving a large population of players.

- A player does not even know the number of players in the game, let alone the identity of every other player.
- Payoffs are not determined by strategy profiles but by *how many* players play a particular strategy.
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Type distributions

In large games, payoffs are associated not with strategy profiles, but with type distributions.

- Suppose there are k strategies used in the population. Then the outcome is specified as a map $\mu : \Pi_k(n) \rightarrow P^k$.
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- Can one carry out all the analysis using only the t types and then lift the results to the entire game ?

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Rationale for reductions

Why should such an analysis be possible ?

- We confine our attention to finite memory players.
- For n players, the strategy space is the n-fold product of these memory states.
- What we wish to do is to map this space into a t-fold product, whereby we wish to identify two players of the same type.
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Equivalence

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- We show: $\mathcal{A} \otimes \mathcal{A} \equiv \mathcal{A}$.
- We show: α is stable in $\mathcal{G} \otimes \mathcal{A}$ iff α is stable in $\mathcal{G} \otimes (\mathcal{A} \otimes \mathcal{A})$.
- α is stable in $\mathcal{G} \otimes \mathcal{A}_1 \otimes \mathcal{A}_2$ iff α is stable in $\mathcal{G} \otimes \mathcal{A}_2 \otimes \mathcal{A}_1$.

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A population of 1000 players with only two types needs to be represented only by pairs of states and not 1000-tuples.

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A tradeoff

When is the determinising procedure worthwhile?

- Suppose we have n players, t types, and p the maximum size of the state space of any nondeterministic type FST.
- We can show that the construction is worthwhile when

$$n > 0.693 \cdot t \cdot \pi(p)$$

  where $\pi(p)$ is the number of primes less than or equal to p.
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Quantitative outcomes

When outcomes are distribution determined, we can consider quantitative objectives as well.

- Every player i has a function $f_i : V \to \mathbb{Q}$ which can be seen as the payoff of i for a particular distribution.
- Given an infinite play $\rho = v_0 \xrightarrow{a_1} v_1 \xrightarrow{a_2} \ldots$, we study the discounted-payoff Player i gets:

$$p_i(\rho) = \lim_{n\to\infty} \inf \frac{1}{n} \sum_{j=1}^{n} f_i(v_j).$$

- We add atomic formulas of the form $p_i < d$ in the syntax.
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Neighbourhood structures

In large games, it is convenient to think of players arranged in neighbourhoods.

- A player strategizes locally, observing behaviour and outcomes within her neighbourhood, but may switch to an adjacent neighbourhood.
- Example of a vegetable seller in India: small player in a large game.
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Dynamic game forms

Meta-strategies

Social situations often involve strategies that are generic, (almost) game-independent.
They have some (limited) efficacy in many interaction situations.

- Threat and punishment.
- Go with the winner / Follow the leader.
- Try to take the lead, and if you can't, follow a leader.
- Imitate someone you think well of.
- ...

But when a significant proportion of players use such heuristics, it may affect game dynamics significantly.

Dynamic game changes

Examples

- Tailors in India.
- Tollbooth equipped with RFID.

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The scenario

General situation

- Individual has to make choices; making choices has a cost.
- Society provides choices; incurs cost.
- Society revises choices and costs from time to time based on the history and prediction of the future.
- This effects individual strategies who switch between the available choices.
- The game arena is not static but changes dynamically.

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Questions

Questions to ask - eventual patterns dictated by the dynamics:

- Does the play finally settle down to some subset of the game?
- Can a player ensure certain objectives using a strategy that doesn't involve switching?
- Given a subarena, is a particular strategy live?
- Does an action profile eventually become part of the social infrastucture?
- Do the rules of the society and the behaviour of other players drive a particular player out of the game?

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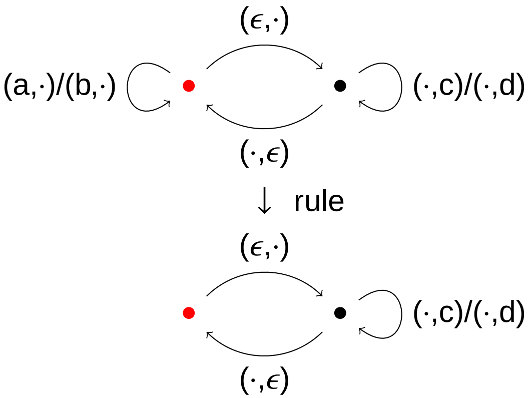
Dynamic Game Restriction[2]

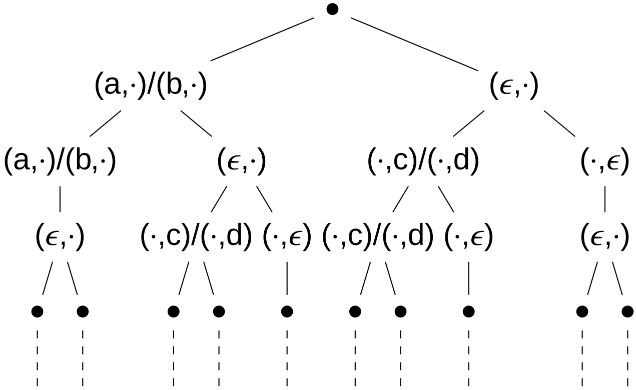
- Of the form rule = precondition $\rightarrow$ new arena $\mathcal{G}'$.
- Rule r is enabled at (old arena $\mathcal{G}$, partial play t) if
  - t conforms to the precondition of r.
  - $\mathcal{G}'$ is a subarena of the old arena.
  - $last(t) \in \mathcal{G}'$.

Dynamic Game Restriction[3]



Dynamic Game Restriction[4]

Induced Game Tree



Logical Specifications

- Homomorphisms
  - $h : A \cup \{\epsilon\} \to A \cup \{\epsilon\}$ such that $h(a) = a/\epsilon,\ h(\epsilon) = \epsilon$
  - $\mathbf{a} = (a_1, \ldots, a_n)$ implies $h(\mathbf{a}) = (h(a_1), \ldots, h(a_n))$
- A restriction specification is of the form $\varphi \supset h$ where φ is a restriction precondition and h is a homomorphism specification. Restriction precondition:

$$\varphi ::= p \in \mathcal{P} \mid \neg\varphi \mid \varphi_1 \vee \varphi_2 \mid \langle \mathbf{a} \rangle^{-} \varphi \mid \langle \mathbf{a} \rangle^{+} \varphi \mid \boxminus \varphi$$

Evaluated on the tree unfolding $\mathcal{T}_{\mathcal{G}}$ of the arena.

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Results

Theorem

Given an arena $\mathcal{G}$, an initial vertex v_0 in $\mathcal{G}$, a finite set of restriction rules $\mathbb{R}$, finite sets of strategy specifications $\{St_i\}$ for each player i and a formula α, the following question is decidable:

- *Is α stable in $(\mathcal{G}, \mathbb{R}, \{St_i\})$?*

Results

Corollary

Given a game arena $\mathcal{G}$ and specifications $\mathbb{R}$ and $\{St_i\}$, the following questions are decidable:

1. *Does player i eventually get removed by the dynamics of the game?*
2. *Does a particular action tuple* $\boldsymbol{a}$ *become the only choice to be available for ever?*
3. *Does the cost stabilise to a specific amount c?*

Remarks

Consequences of the Theorem

- It is possible to compare between game restriction rules in terms of their imposed social cost is possible.
- For a player, if the game restriction rules are known and the type of the other players are known then she can compare between her strategy specifications.

Rule synthesis

When should society intervene ?

- We associate social costs with strategies, and thresholds.
- With quantitative objectives and strategy specifications for players, we can compute at which strategy distributions society must act so that a given objective is achieved.
- Synthesis of which action is to be removed when is presented as a finite state transducer.

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Imitation

The everyday choice

Why does imitation occur ?

- Why do you wear what you do ?
- Weather decides the basic structure of clothing, but beyond that it depends on convention, self-image, etc.
- We have neither the resources nor the expertise to do a comprehensive study of what is best for us and decide.
- Following the convention is simple, saves us time and energy. It is indeed rational.

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Examples of Imitation

Is imitation viable?

- Most important premise of economics: Rational individuals optimise.
- But imitation is a common and sometimes even inevitable phenomenon (eg. choice of language).
- Is imitation justified?
  - Yes: saves time, uses less resource, doesn't do much worse than optimal in most cases.
  - No: doesn't achieve optimal in most cases.

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Basic idea

- In a large population of players, where resources and computational abilities are asymmetrically distributed, it is natural to consider a population where the players are predominantly of two kinds: optimisers and imitators.
- Asymmetry in resources and abilities can then lead to different types of imitation and thus ensure that we do not end up with 'herd behaviour'.
- Mutual reasoning and strategising process between optimizers and imitators leads to interesting questions for game dynamics in these contexts.

Imitation in Games

- May yield the optimal outcome: the tit-for-tat strategy in a repeated prisoner's dilemma.
- May be totally stupid: monkey chess.

What we study

- We consider games of unbounded duration on finite graphs among players with overlapping objectives where the population is divided into players who optimise and others who imitate.
- Since plays eventually settle down to connected components, players’ preferences are given using orderings on Muller sets.

What we show

- In this setting we address the following questions and present algorithmic results:
  - If the optimisers and the imitators play according to certain specifications, is a global outcome eventually attained?
  - What sort of imitative behaviour (subtypes) eventually survive in the game?
  - How worse-off are the imitators from an equilibrium outcome?

Imitator Strategy - Examples

- Imitate player 1 for 3 moves and then keep imitating player 4 forever.
- Imitate player 2 till she receives the highest payoff. Otherwise switch to imitating player 3.
- Nondeterministically imitate player 4 or 5 forever.

Strategy Specification - Optimiser

- One of the motivations for an imitator to imitate an optimiser is the fact that an optimiser plays to get best results.
- To an imitator, an optimiser appears to have the necessary resources to compute and play the best strategy and hence by imitating such a player she cannot be much worse off.
- However, we show that in our setting it is enough to consider optimiser strategies that are bounded memory.

Equilibrium

Let the strategy specifications of the n imitators be given by FSTs $\mathcal{R}_1, \ldots, \mathcal{R}_n$ respectively. We construct a game $(\mathcal{G}', v_0', \prec_1', \ldots, \prec_{m+1}')$ with $m+1$ players from the game $(\mathcal{G}, v_0, \prec_1, \ldots, \prec_{m+n})$ in the following steps:

Step 1 Construct the arena $\mathcal{G}' = \mathcal{G} \times \mathcal{R}_1 \times \ldots \mathcal{R}_n$

Step 2 Introduce a new player, the $m+1$th player, who owns all nodes $(v, q_1, \ldots, q_n)$ such that v was an imitator node in $\mathcal{G}$.

Step 3 Lift the preference orders of the players 1 to m to subsets of V' as follows. A subset W of V' corresponds to the Muller set $F(W) = \{v \mid (v, q_1, \ldots, q_n) \in W\}$ of $\mathcal{G}$. For every player $i : 1 \leq i \leq m$, for $W, W' \subseteq V'$, $W \prec_i' W'$ if and only if $F(W) \prec_i F(W')$

Step 4 Lift the preference order of the $m+n$th imitator in $\mathcal{G}$ to the $m+1$th player (the new player) in $\mathcal{G}'$.

Results

Theorem

Let $(\mathcal{G}, v_0, \prec_1, \ldots, \prec_{m+n})$ be a game with $m + n$ players with m optimisers playing bounded memory strategies $\sigma_1, \ldots, \sigma_m$ and the rest n imitators playing imitative strategies $\tau_1, \ldots, \tau_n$ where every such strategy is among t different types. Let W be a strongly connected component of $\mathcal{G}$. The following questions are decidable:

(i) *Does the game eventually settle down to W?*

(ii) *What subtypes of the t types eventually survive?*

(iii) *How worse-off is imitator i from an equilibrium outcome?*

Research questions: large games

- Fixed set of players to an unbounded set of players.
- Topological study of neighbourhood structures.
- Rule synthesis a la mechanism design.
- Hierarchy of interaction.
- Study of herd behaviour and runaway phenomena.
- Study of type distributions.

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