Hierarchical Reinforcement Learning: Assignment of Behaviours to Subpolicies by Self-Organization

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"He knew what he had to do. It was, of course, an impossible task. But he was used to impossible tasks. (…) The way to deal with an impossible task was to chop it down into a number of merely very difficult tasks, and break each of them into a group of horribly hard tasks, and each one of them into tricky jobs, and each one of them …"

Terry Pratchett — *Truckers* (*Bromeliad Trilogy*, book I)

Abstract

A new Hierarchical Reinforcement Learning algorithm called HABS (*Hierarchical Assignment of Behaviours by Self-organizing*) is proposed in this thesis. HABS uses self-organization to assign behaviours to uncommitted subpolicies.

Task decompositions are central in Hierarchical Reinforcement Learning, but in most approaches they need to be designed *a priori*, and the agent only needs to fill in the details in the fixed structure. In contrast, the new algorithm presented here autonomously identifies behaviours in an *abstract* higher level state space. Subpolicies self-organize to specialize for the high level behaviours that are actually needed. These subpolicies are then used as the high level actions.

HABS is a continuation of the HASSLE algorithm proposed by Bakker and Schmidhuber [1, 2]. HASSLE uses abstract states (called subgoals) both as its high level states *and as its high level actions*. Subpolicies specialize in transitions (i.e. high level actions) between subgoals and the mapping between transitions and subpolicies is learned. HASSLE is goal directed (subgoals) and this has the undesired consequence that the number of higher level actions (the transitions between subgoals) increases when the problem scales up. This *action explosion* is unfortunate because it slows down exploration and vastly increases memory usage. Furthermore the goal directed nature prevents HASSLE from using function approximators more than two more layers.

The proposed algorithm can be viewed as a short-circuited version of HASSLE. HABS is a solution to the problem that results from using subgoals as actions. It tries to map all the experienced (high level) behaviours to a (small) set of subpolicies, which can be used directly as high level actions. This makes it suitable for use of a neural network for its high level policy, unlike many other Hierarchical Reinforcement Learning algorithms.

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Chapter 1

Introduction

1.1 Learning in Layers – Solving Problems Hierarchically

A rather recent addition to the happy family of algorithms is the concept of *Hierarchical* Reinforcement Learning. In contrast with *"flat"* Reinforcement learning, this new class of algorithms is in some sense layered.

As with many terms in the field of Artificial Intelligence – for instance the buzz word "*multi agent*" – the term "*hierarchical*" has a rather wide meaning. Several different approaches have been put forward, each using Reinforcement Learning (learning by *trial-and-error*) in some way or the other. They combine different levels, scales or layers of actions (time) and/or states (space) in solving Reinforcement Learning tasks.

An Example – Game playing

A small example may illustrate the notion of hierarchies. Suppose we are playing a complicated computer game about building cities, armies and empires[1]. In this game, the ultimate goal is – of course, what else? – to rule the world, but to achieve that (high level) goal, many smaller tasks (subtasks) have to be accomplished or subgoals have to be reached.

Suppose we have already figured out that the *subtask* of building a large army will most certainly help us in achieving our ultimate goal of world domination (see fig. 1.1). We might not yet know *how exactly* we are supposed to execute the behaviour *'build large army'* perfectly, but we did learn that executing it would be the road to victory.

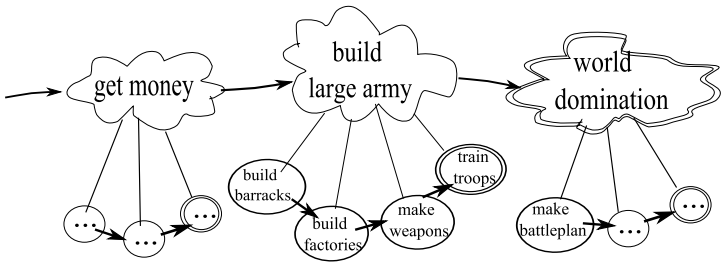


Figure 1.1: *Hierarchical planning in a strategy game.*

Or perhaps we think that we need large cities (a state or *subgoal*) that generate a high revenue but we don't know yet how to build large cities. As we play the game, we will learn how to build a metropolis and learn how to deal with all the problems of managing mayor cities. In effect we become better in reaching the state or subgoal *"have large cities"*.

On the one hand we are learning (on a high level of abstraction) *whether or not* having large cities

[1] For instance a game like *Civilization* or one of its many incarnations, clones, imitations and successors.

or building an army is an important aspect of winning the game. But on the other hand, we are learning just *how exactly* we can accomplish those smaller subgoals or subtasks.

1.1.1 Why Use Hierarchies?

There are several reasons for using hierarchies in learning and control. Hierarchies are a sensible approach, which humans use every day: we often divide the world in hierarchical structures to facilitate learning. We are able to plan better when using a hierarchy of larger and smaller subtasks (actions we want to take) or subgoals (states we want to be in) than if we had to plan everything in terms of low level actions only. Also, most of the time behaviours that are used to accomplish one subtask, can be re-used in another subtask.

Furthermore, hierarchies and behaviours facilitate exploration. A Reinforcement Learning agent usually needs to explore, and it does so *initially* by random walking (basically: *acting like a drunk*) and trying actions randomly by *trial-and-error*. After some time it will have acquired some knowledge about the environment, and its exploration will obviously be more efficient and less random. However if the agent could learn behaviours (actions that are extended over multiple time steps) which do something non-randomly, it could use these to explore faster. Instead of making random decisions at every time step, with behaviours the agent only has to make random decisions when it invokes a behaviour: the burden of random walking is shifted from the low level to the high level.

1.1.2 Different Approaches

The learning on both[2] layers can be *intertwined*: a behaviour that is not yet fully learned on the low level, can already be used at a high level. And goals that are chosen on the high level, can force exploration on the low level in certain directions.

There are lots of different ways in which to structure a hierarchical algorithm. Only the action-part could be hierarchical, resulting in approaches following the Options-framework (see section 3.3.1), but there could also be abstraction or hierarchy in the state space, resulting in layered approaches (see section 3.3.2).

The reasons for using hierarchies, and examples of several approaches that have appeared in the literature, are presented in chapter 3. An introduction to (flat, non-hierarchical) Reinforcement Learning is given in chapter 2.

1.2 HASSLE and HABS

Focus on Task Decomposition

Many (layered) approaches use the notion of task decomposition: the overall task is decomposed into subtasks (and subsubtasks, subsubsubtasks, ...) by the designer. This seems like a good approach, because humans are good at abstractions and identifying structures. But for many problems designing decompositions is cumbersome and time consuming, and we are dependent on the designer's intuition for the task decomposition. This means we that the designer needs to understand the problem in order to give a good decomposition into subtasks.

Focus on State Abstraction

Alternatively, we can focus on the *states* instead of the actions: perhaps we have an abstraction of the state space readily available, or it can be acquired easily[3]. If we have a state abstraction, why not select

[2] For now assuming there are two levels. However, there is nothing preventing us from extending this notion of 'layer' to more than two levels.

[3] If no high level abstraction of the state space is known in advance, algorithms are needed that divide the state space in appropriate subsets. Hot spots in the state space need to be identified, clustering needs to be performed, etc.

an algorithm that just uses this abstraction without bothering with designing task decompositions.

The HASSLE algorithm (*Hierarchical Assignment of Subpolicies to Subgoals LEarning*), which was the starting point for the research in this thesis, deals with exactly that problem. For understanding the new algorithm HABS, it is useful to understand HASSLE (it is therefore described rather extensively in chapter 4). HASSLE uses clusters of low level states that resemble each other as *abstract states* in a high level Reinforcement Learning algorithm. In effect it has two Reinforcement Learning algorithms at the same time: one for the high level, using the abstract states and one[4] for the low level, using the normal ("flat") states.

What makes HASSLE different from other approaches, is that is also uses its abstract states as its *high level actions*[5] for the high level. This means that HASSLE works in terms of subgoals: the agent is in a certain abstract state and wants to go to another high level state. Its high level action is this other abstract state or subgoal. On the low level it just uses its primitive actions.

Starting with Uncommitted Subpolicies

It would be very inefficient to just assign a unique subpolicy to each of the transitions from one subgoal to another. If we needed to learn all of these transitions separately, there is a fair chance that so many small problems together are tougher – and take far more time – than the large problem we started with.[6]

HASSLE uses a limited number of a subpolicies, which start totally *uncommitted*. It is not known at the start, which subpolicy will be used for which subtask(s). This small set of subpolicies together needs to cover all the required behaviours. A behaviour like moving through a corridor will probably be the same in many corridors all over the place, so we would need only one (somewhat flexible) subpolicy for many roughly similar corridors. HASSLE needs to learn the association between subpolicies and high level actions.

The system in effect has to organize itself by incrementally increasing its performance. Each learning part (*high level policy, subpolicies, the associations*) uses the still rough and unfinished other parts, to make itself a little more effective, and in turn other parts use the results to make themselves a little better. They bootstrap on each other, using still unfinished values as estimates. This is not as impossible as it might sound, for most of the basic Reinforcement Learning algorithms already use some form of bootstrapping: *using estimates of estimates*.

1.2.1 The Problem with HASSLE: an Explosion of Actions

HASSLE has an inherent flaw that prevents it from being scaled up. This will be illustrated briefly here, and it is analyzed in section 4.3. Suppose we have a state space – let's say a *'house'* – for our problem and we increase the size of the problem to something more properly called a *'palace'* (see fig. 1.2). The bigger the problem, the more high level *actions* are added to the problem (because HASSLE uses transitions between high level states as its high level actions).

This is not the case for the low level actions, because no matter how large our palace is, there is always the same (small) set of primitive actions, for instance *North, East, South* and *West*.

On the high level, not only the number of states is increased, *but also the number of actions*: the problem size grows in two ways – which is quite unusual for Reinforcement Learning! This *action explosion* on the high level is a serious problem. It is hampering the learning process, because the more high level actions there are to take, the more there are to be investigated. This makes the problem more time consuming, up and above the usual effect of the increased number of (abstract) states. It also prevents HASSLE from using neural networks as its high level policy or using more than two layers. Furthermore, it highly increases memory usage.

[4] On the low level there are several subpolicies, but only one is active at each time step.

[5] Note that on the low level, the states are different from the actions (as usual).

[6] Because if we have $\|S\|$ high level states, we would have $\|S\| \times \|S\|$ combinations of subgoals each with a different subpolicy to learn!

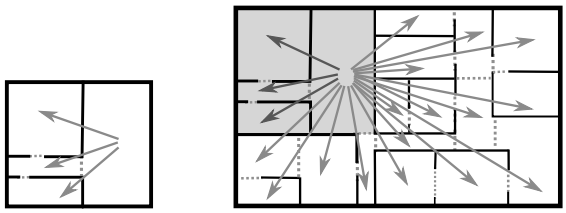


Figure 1.2: What happens when a house becomes a Palace*: the problem size (and the number of high level states) increases, so an action explosion occurs on the high level. The arrows represent transitions from one high level state (in this case a room) to another.*

In section 4.4 an attempt is described to fix this *action explosion* in HASSLE by introducing a filter. This gives HASSLE the ability to rule out transitions, which improves learning because there is less to explore. However, this rather *ad hoc* fix only remedies part of the problem.

1.2.2 A Rigorous Solution - a Brand New Algorithm: HABS

The new algorithm presented in this thesis is called HABS (*Hierarchical Assignment of Behaviours by Self-organizing*, described in chapter 5), and is derived mainly from HASSLE. It was developed to overcome the *action explosion* problem in HASSLE, and to allow neural networks to be used as function approximator for the high level policy. If this feature could be dropped and replaced by something that always uses a fixed set of behaviours as high level actions, the number of high level actions would remain constant when the problem size grows. That way we would retain the useful aspects of HASSLE, like using the abstracted state space and starting with *a priori* uncommitted subpolicies.

The solution is to *short circuit* the HASSLE algorithm by directly using the subpolicies as high level actions. But because HASSLE uses the high level subgoals as the targets when training subpolicies (i.e. *reward a subpolicy if it reaches the desired subgoal*), the short circuiting creates a new problem: how are the subpolicies to be trained, if there is no goal to train them on, because we just kicked out the notion of 'goal'?

Self-Organizing Characteristic Behaviour

The problem of an absence of subgoals is solved by introducing the notion of a "*characteristic behaviour*" of a subpolicy, which is used to train the subpolicy: *if it performs roughly as it normally would, it is rewarded*. When learning has just started, the subpolicies basically just behave meaningless[7], but because they are randomly initialized, some are slightly less bad in certain tasks than others. The feedback between what the subpolicies do, and when they are used by the high level, allows a kind of self organization.[8] This way each of the subpolicies specializes in different behaviours.

1.2.3 Shifting the Design Burden

Many hierarchical approaches focus on (designing) task decompositions. This means that the design burden lies with understanding and solving the task at a high level. The designer commits subpolicies to certain subtasks which he or she has identified, and the agent needs to fill in the blanks. In many cases this is feasible, but sometimes not enough information is available about what a good solution would be.

HASSLE and HABS on the other hand, focus on defining or identifying suitable state space abstractions. In fact, like in the case of the 'house' example, these abstractions may already be lying around somewhere – begging to be used!

[7] As with HASSLE, they start uncommitted.

[8] This approach resembles the one that is used with evolutionary algorithms. Good behaviour that is present – though still very poorly – is selected. It also has similarities with Self-Organizing Maps.

Focusing on state abstractions instead of task decompositions, imply starting with uncommitted subpolicies. Learning the commitments of subpolicies means that time is needed for learning these associations. But the trade off is that abstractions of the state space are sometimes far easier to get a hold on, saving time on designing. It can be viewed as a complementary approach to designing task decompositions.

1.3 Relevance to Artificial Intelligence

Artificial Intelligence may, as Luger and Stubblefield [9] state, be *"defined as the branch of computer science that is concerned with the automation of intelligent behaviour"*. Even though we don't really have a concise definition of 'intelligent behaviour' or 'intelligence' – let philosophers worry about that! – it is clear that *learning* is an important part of intelligence.

This thesis deals with the often occurring problem, that Reinforcement Learning does not scale well when applied to large problems. Agents that are able to learn on different hierarchical levels, can be of great use in various fields of Artificial Intelligence, because they are a promising (perhaps partial) solution to the problem of scaling.

1.3.1 Neuroscience

The proposed algorithm HABShas some similarities – in a very basic way – to the human way of problem solving. Both use abstractions and uncommitted behaviours to figure out how to solve problems. In that context work from this thesis was also presented at the NIPS*2007 workshop titled *"Hierarchical Organization of Behavior: Computational, Psychological and Neural Perspectives"* (hosted by prof. Andrew Barto) where research from the Reinforcement Learning community was brought into contact with neuroscience. Viewed from that perspective, work with self organizing behaviours could perhaps give computational confirmation for theories on behaviour acquisition.

1.3.2 Computer Games

Learning is also of relevance to the area of *computer games*. Research in computer games is a fast growing field and the game industry as a whole recently left Hollywood behind in terms of revenue (see [5]), and better AI can provide better selling games. The demand for computer games that are as realistic as possible (graphically) is high, but the disappointment is often even higher when opponents in a fantastic looking game turn out to be the dumb cousins of *Blinky*, *Pinky Inky* and *Clyde*[9]: highly predictable and often clearly scripted, unable to respond to situations not anticipated by the programmers.

Many computer games would benefit from robust Artificial Intelligence algorithms that can handle complex situations and large amounts of data. Opponents that *learn* how to play a game (and be interesting, challenging opponents), might be preferable to laboriously scripted, tweaked and tuned, hard coded non-human players. In order to reach this goal, building hierarchies and learning different strategies on different levels is almost certainly needed. If we want computer players to put up a better fight against human players, what better way to start than to imitate the human way of hierarchically dividing large problems into smaller – and therefore simpler – ones?

[9]The four infamous ghosts in Pac-Man.

Chapter 2

Reinforcement Learning

"Reinforcement Learning is learning what to do – how to map situations to actions – so as to maximize a numerical reward signal" according to Sutton and Barto [6][1]. It is the problem of finding out what the best reaction will be, given the state you are in. In more popular – somewhat biological – terms, it is *"learning by trial and error"*. In Reinforcement Learning it is common that the learning agent is not told when to perform certain actions, but has to discover this mapping from states to actions (its policy) by *trial-and-error*, by interacting with the environment.

2.1 Some Intuitions and Basic Notions

The Reinforcement Learning agent has to base its decisions on its current information. This information is usually called a 'state'. The agent also has the ability to execute actions at each time step. After executing an action, it receives a reward signal (not necessarily non-zero) which is some measure of how well the agent performs (see fig. 2.1).

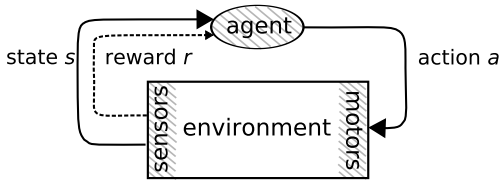


Figure 2.1: The basis of reinforcement learning*: the agent-environment interaction. The shaded areas can be considered the 'body' of the agent (including sensors and motor controls).*

The Agent

The agent is a rather simple mechanism, and the term 'agent' might be confusing since it suggests rational thought, planning, maybe even cognition or self-awareness. Nothing as fancy as this is the case however. In effect the *brain* of a Reinforcement Learning agent is nothing more than a huge table in which the *"goodness"* (expected return) of being in a certain state and executing a certain action is stored. Every time the agent needs to execute an action, it will look up its currently observed state in the table, and decide based on the values of each of the actions, which action it should choose.

The Environment

The environment is everything outside the agent, everything that the agent cannot change *arbitrarily*. Things like motor controls (for a robot) or muscles (for an animal) are in this sense considered 'outside'

[1] Their book, appropriately called *"Reinforcement Learning: an Introduction"*[6] is very good source on Reinforcement Learning (although it has nothing on Hierarchical Reinforcement Learning).

the agent and therefore part of the environment, even though they are part of its (physical) body. The agent needs to control its 'body' but this control is subject to limitations (speed, tension, etc) from the rest of the environment and the agent cannot change it without limit.

The State

The state the agent is in, is simply the vector or conjunction of all the variables (features) describing the environment (as observed by the agent with some sort of sensors). This does not mean that the agent cannot have a memory and is doomed only to react to its *current* environment. It is easy to incorporate some form of memory into the state: memory – like a piece of paper with notes on it – is considered 'external' to the agent.

The agent has a 'body' (motor controls, sensors, etc) which it can control, but it can of course also (in principle) monitor its own body. The state is therefore simply the collection of variables and their values that are known to the agent, in terms of memory or 'body' (internal) and external environment.

If the task that the agent needs to accomplish is episodic, there are 'terminal states' in the environment. If an agent reaches one of those, the episode is terminated. This means that the trial is finished and a new episode will begin (with the agent again in a starting position).

The Actions

The agent has the ability to manipulate its environment, which it does by executing actions (i.e. its motor controls). These actions are often called *"primitive actions"*. The agent has a limited set of actions, and at each time step it selects one action to execute. This action only lasts until the next time step and then terminates, after which it has either succeeded or failed (e.g. when an action "move" fails because the agent collides with a wall).

Sparse And Dense Rewards

After each action, the agent receives feedback from the environment. It might be that the agent always receives 0 when taking an action, and only receives 1 if it completes its task or reaches its goal. This would be a task with very *sparse* rewards. On the other hand the task could have *dense* rewards, meaning many non zero rewards. It might be that our agent is *Pac-Man* running around in a maze, gathering *pills*: each *pill* might be a small positive reward, but getting killed by the *ghosts* is a large negative reward. However, if *Pac-Man* would only get a 1 at the end of the game if it had survived and taken all the *pills* (and 0 on any other moment) the task would be much harder.

In many problems, it is unclear how and when exactly the rewards need to be given. Sometimes the only certainty is whether the agent eventually succeeds or fails the task, but nothing of the internal structure of the task is known so no intermediate rewards can be given.

The Learning Process, the Policy

The agent stores its knowledge about the environment in a table – or it is approximated with a function approximator. It can either save information about how good it is to be in a certain state[2], or save information about how good it is to execute a certain action in a certain state. When values are stored for pairs of a state and an action, it is often called a *Q-value* (or *Q-value function*), instead of a *value* (or *value function*).

If the term 'value' is used, it depends on the context whether the value of a state or the value of a state-action-pair (a Q-value) is used. This thesis mainly deals with Q-values (unless explicitly stated otherwise) so this small ambiguity should not pose a problem.

[2] In that case, when the agent needs to select an action to do next, it needs to look one step ahead, calculate which states it can reach, and then use those values to base its selection on. This can be used if it is always known what the effect of an action is.

Interaction – Updating The Table

The agent learns by interaction with the environment. Only three things are important for the agent: the *state* it is in, the *actions* it can take, and the *rewards* it gets (see fig 2.2). After an agent has executed an action, it experiences a reward (given by the environment). This new information can be used by the agent to update its knowledge.

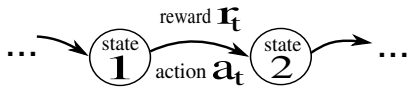


Figure 2.2: Transition between states

Updating could in principle be done by *replacing* the relevant entry in the table by the newly received reward, but this would lead to undesirable effects: if the rewards are statistical in nature, the table entries will forever continue to fluctuate highly, never converging.

It would be better to update the entry a little bit in the new (reward) direction. That way if the rewards keep fluctuating, the table entry will still converge to a stable value (only fluctuating a bit because it shifts a bit in the direction of new rewards). The update is something of the form:

$$V(s) \leftarrow (1-\alpha)\cdot V(s) \; + \; \alpha \cdot reward \tag{2.1}$$

meaning that the value V in state s in the table is shifted slightly (a factor α) in the direction of *reward* and away from the old value $V(s)$. Equation 2.1 only considers *states*, and $V(s)$ would then just be an indication of the 'goodness of being in *state*'.

If we use Q-values, the value mentioned in equation 2.1 is the value for a state-action pair, and the equation becomes:

$$Q(s,a) \leftarrow (1-\alpha)\cdot Q(s,a) \; + \; \alpha \cdot reward \tag{2.2}$$

where Q(s,a) of course represents the value (the 'goodness') of executing action a in state s.

The Future

The update scheme described above does not take the *future* effects of any taken action into account. But what if the agent finds itself in a situation like in figure 2.3?

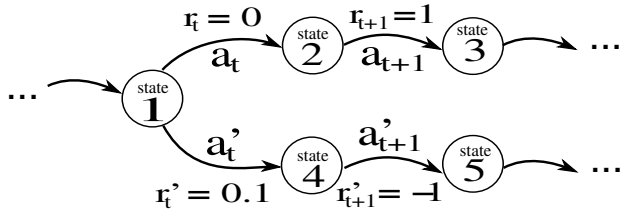


***Figure 2.3: A 'trap'**: the deceptive reward of 0.1 leads to punishment of -1 later on.*

At first is seems that taking action a_t' is better than doing a_t because it has a higher immediate return (0.1 versus 0). However, a_t' leads to a subsequent reward of -1 which is undesirable, and a_t actually would have been the better choice because it leads to a return of 1.

So how could these *traps* be avoided? It is clear that only looking at the immediate reward will not work, so the *future* must be taken into account. Reinforcement Learning algorithms accomplish this by the elegant concept of *discounting*. The idea is, that if a certain action a leads to a state from which a next action could give a high reward, the value $Q(s,a)$ needs to reflect this knowledge. This means that the value of an action should not only represent what it will return immediately, but also what can be expected later on.

Discounting is implemented by a simple constant $0 \leq \gamma \leq 1$. The primitive update rule presented in equation 2.1 then becomes:

$$Q(state_1, a) \leftarrow (1-\alpha)Q(state_1, a) + \alpha \cdot (reward + \gamma \cdot \text{VALUE}(state_2)) \tag{2.3}$$

where the term $(reward + \gamma \cdot \text{VALUE}(state_2))$ has replaced the *reward* term. $\text{VALUE}(\ldots)$ is used for now to indicate that we need some sort of measurement of how good the next state ($state_2$) actually is, but that we don't know yet what measurement to use. Different reinforcement learning algorithms use different formulas for $\text{VALUE}(state_2)$, see the sections on SARSA, Q-learning and Advantage Learning (sections 2.3.2 ~ 2.3.4).

Using this new update rule, we can see that the agent is now able to learn not to fall for the temptation of the quick reward of 0.1 (in fig. 2.3) because the *discounted reward* of the next state also plays a role. The 'goodness' of the next state, represented in $\text{VALUE}(state_2)$, is used in updating $Q(state_1, a)$. When the agent executes action a_t, it can update its knowledge about how good a_t is, with the value $0 + \gamma \cdot \text{VALUE}(state_2)$. And since $\text{VALUE}(state_2)$ in some way depends[3] on the received reward, part of this 'goodness' propagates back to $Q(state_1, a)$. The 'goodness' propagates back like ink in a glass of water.

Note that $\text{VALUE}(state_2)$ and $Q(state_1, a)$ are only estimates. Reinforcement Learning algorithms use rough estimates in calculating new estimates: this is called *bootstrapping*.

Episodic Tasks

If the agent needs to solve a task that just goes on forever, it is called "continuous". If it ends at some time or in some situation, it is called "episodic". In episodic tasks there are terminal states that stop the episode when the agent enters them. These terminal states obviously have no successor states, so discounting future rewards is not an issue there. These states are fixed points because they have no future rewards that can influence them. We can just take $\text{VALUE}(terminalState)$ to be zero.

Alternatively, we could add an *absorbing state* to the system, which can only be entered from the terminal state and where the only transition is again to this absorbing state and always with a zero reward. This way each episode ends up in the absorbing state. If we consider episodic tasks in this way, they are in fact continuous and we don't have to worry about upper boundaries (for summations etc) but can just use 'infinity'.

2.2 The Model

If our agent is going to solve a certain task, it had better retain all the relevant information from the past. Suppose we have an agent that has to retrieve some object and bring it back to a certain point. If it would forget on what part of the task it was working, it would be unable to perform its task. If the agent would only have information about its surroundings (a map, or perhaps a radar identifying walls and corridors) but not whether it has the object or not, then deciding whether to go one direction or the complete opposite would be impossible.

However, if the agent had remembered whether it had already picked up the object, the decision would be easy: if you don't have the object yet, go find it, and if you do have it, bring it to its destination. In this example the agent can easily distinguish between both cases, by looking at whether it has picked up the object or not. The same result would in this example be achieved if the agent knows of itself whether it is carrying the object or not. Both the internal state *"am I carrying the object?"* [*yes/no*] or *"did I pick up the object in the past?"* [*yes/no*] would suffice[4].

[3] If the 'goodness' $\text{VALUE}(state_2)$ did not in some way depend on the value of the rewards in $state_2$ it would not be a really good '*good*ness' function because the *only* information about the value or 'goodness' of a state (or state-action pair) that is available to the agent, actually comes from the reward signals.

[4] Provided of course that the agent does not drop the object somewhere after it has picked it up. If it could do that, the *did I pick up the object in the past?*[*yes/no*]-internal state would obviously not suffice.

Markov Property

If the state of the agent holds all the relevant information needed to carry out its task, it is said that it has the *Markov Property*. More formally the probability distribution

$$Pr(s_{t+1} = s', r_{t+1} = r \mid s_t, a_t, s_{t-1}, a_{t-1}, s_{t-2}, a_{t-2}, \ldots, s_0, a_0) \tag{2.4}$$

should for all s', s_t, a_t and r equal

$$Pr(s_{t+1} = s', r_{t+1} = r \mid s_t, a_t) \tag{2.5}$$

This means that all the (relevant) information from the past, (i.e. s_{t-1}, a_{t-1}, s_{t-2}, a_{t-2}, $\ldots$, s_0, a_0) is coded into s_t and a_t and the agent can make the same decision based on only the current state and action as it can make when knowing the entire past s_{t-1}, a_{t-1}, s_{t-2}, a_{t-2}, $\ldots$, s_0, a_0. In the earlier example of the agent retrieving an object, the past actions and positions are not relevant, only the present position and action and knowing whether you are in possession of the object are needed.

It is often the case that there is hidden information (unavailable to the agent) and the states are not Markov. However, it is still good practice to regard the states as approximating the Markov property. This is a good basis for predicting subsequent states and rewards and for selecting actions. The better the approximation is, the better these results will be.

2.2.1 Markov Decision Processes

When a Reinforcement Learning task satisfies the *Markov Property*, it is called a *Markov Decision Process*[5] (MDP). An MDP describes an environment and consists of the following items:

- A finite set of states $S = \{S_1, ..., S_m\}$
- A finite set of actions $A = \{A_1, ..., A_n\}$
- A reward function $R : S \times A \times S \to \mathbb{R}$. $R(s, a, s')$ gives the reward for the transition (action) between states s and s'.
- A transition function $P : S \times A \times S \to [0, 1]$. $P(s, a, s')$ gives the probability of going from states s to s' given action a.

In a Reinforcement Learning task, the agent tries to maximize the rewards in the long run. This means it tries to maximize R_t, the expected discounted return:

$$R_t = r_{t+1} + \gamma \cdot r_{t+2} + \gamma^2 \cdot r_{t+3} + \ldots = \sum_{k=0}^{T} \gamma^k \cdot r_{t+k+1} \tag{2.6}$$

where γ is the discount parameter ($0 \leq \gamma \leq 1$) and T is the last time step. Alternately, if we use absorbing states, we can drop the upper boundary T and get:

$$R_t = \sum_{k=0}^{\infty} \gamma^k \cdot r_{t+k+1} \tag{2.7}$$

The discount is used to determine the present value of *future* rewards. It might be that the agent is only concerned in maximizing its immediate reward in the present, never caring what the future holds. In that case γ equals 0. R_t then reduces to r_{t+1} which is the reward received for the action it has to select at time t.

[5] MDP's can be summarized graphically by transition graphs where the states are the nodes. The arrows are labelled with actions and their accompanying probabilities.

If γ has a value strictly smaller than 1, the expected reward R_t never grows to infinity[6], but the agent still takes the future rewards into consideration when selecting its actions. The closer γ is to 1, the more the agent takes into account the far future.

If γ equals 1, every reward — now or in the distant future — is equally important. This means that when the task is infinite ($T \to \infty$), the expected reward also grows to infinity, which is not desirable. So the combination of $T \to \infty$ and $\gamma = 1$ should be avoided.

2.2.2 Policies

Given a certain MDP, the agent now has to learn how to behave, what *policy* to follow. A policy π is a mapping from a state and an action to the probability of taking that action in the given state:

$$\pi : S \times A \to [0,1] \tag{2.8}$$

so $\pi(s,a)$ denotes the chance of selecting action a in state s. In every state $s_t \in S$ at time t an action $a \in A$ is selected according to the distribution $\pi(s_t, \cdot)$.

If the agent did not have a clue about what actions would be rewarding or not, the policy might look like $\pi(s,a) = \frac{1}{||A||}$, the uniform probability (where the agent in each state selects each action with the same probability).

The value $V^\pi(s)$ of state s is defined as

$$V^\pi(s) = E_\pi\{R_t | s_t = s\} = E_\pi\{\sum_{k=0}^{T} \gamma^k \cdot r_{t+k+1} | s_t = s\} \tag{2.9}$$

or more informally, as the expected return when the agent starts in s and follows policy π thereafter.We can define the value of taking action a in state s as:

$$Q^\pi(s,a) = E_\pi\{R_t | s_t = s, a_t = a\} = E_\pi\{\sum_{k=0}^{T} \gamma^k \cdot r_{t+k+1} | s_t = s, a_t = a\} \tag{2.10}$$

This quantity is often called the *Q-value* or *action value* for policy π.

The Value Function

The values V^π and Q^π can be estimated by the agent. During the interaction with the environment, the return values that the agent receives could be averaged for every state (or for every state-action-pair if we want to estimate Q^π). These averages converge to the actual values of V^π (or Q^π). This kind of approach is called *Monte Carlo*: taking averages over random samples of actual returns. For larger problems however, or problems where the episode lengths are long, this is not a suitable approach.

2.2.3 Properties of the Best Policy

For the agent to solve the task, it has to find a good policy. This policy has to achieve a lot of reward over a long time. We can define a policy π as better than or equal to another policy π' if for every state s π had a greater or equal expected return than policy π'. So we can say that

$$\pi \geq \pi' \Leftrightarrow V^\pi(s) \geq V^{\pi'}(s) \quad \text{for all states } s \tag{2.11}$$

There is always at least one best policy[7], which we call the *optimal policy* or simply π^*. The optimal state-value function is denoted as V^* and is defined as:

$$V(s)^* = max_\pi V^\pi(s) \quad \text{for all states } s \tag{2.12}$$

[6] Provided that the rewards are finite.

[7] We can always construct a better (or equal to the current best) policy by taking for every state s the $\arg\max_{\pi \in \Pi} \pi(s,a)$ with Π the set of all policies we have available. If there is more than one optimal policy, they all share the same state-value function.

The same can be done for the optimal action-value function:

$$Q(s,a)^* = max_\pi Q^\pi(s,a) \quad \text{for all states } s \text{ and actions } a \tag{2.13}$$

There is a fundamental property of these (Q-)value functions that is used extensively in Reinforcement Learning. This is the Bellman (optimality) equation:

$$\begin{aligned} V^\pi(s) &= \max_a Q^{\pi^*}(s,a) \\ &= \max_a E_{\pi^*}\{R_t \mid s_t = s, a_t = a\} \\ &= \max_a E_{\pi^*}\{\sum_{k=0}^{T} \gamma^k \cdot r_{t+k+1} \mid s_t = s, a_t = a\} \\ &= \max_a E_{\pi^*}\{r_{t+1} + \gamma \sum_{k=0}^{T} \gamma^k \cdot r_{t+k+2} \mid s_t = s, a_t = a\} \\ &= \max_a E\{r_{t+1} + \gamma \cdot V^*(s_{t+1}) \mid s_t = s, a_t = a\} \\ &= \max_a \sum_{s'} P(s,a,s') \left(R(s,a,s') + \gamma \cdot V^*(s')\right) \end{aligned} \tag{2.14}$$

For Q^* the last two formulas in the above derivation would be:

$$\begin{aligned} Q^\pi(s,a) &= E\{r_{t+1} + \gamma \cdot \max_a Q^*(s_{t+1}, a') \mid s_t = s, a_t = a\} \\ &= \sum_{s'} P(s,a,s') \left(R(s,a,s') + \gamma \cdot \max_a Q^*(s', a')\right) \end{aligned} \tag{2.15}$$

These results are called Bellman optimality equations and they describe the properties of the optimal Q- or V-function.

2.3 Decent Behaviour: Finding the Best Policy

If the structure of the problem (the MDP) is known, techniques from the field of Dynamic Programming (DP), like value iteration or policy iteration [6], could be used. On the other hand we could use Monte Carlo methods [6] (estimating the values from the returns of episodes) for which no knowledge of the underlying MDP is needed (*model free*). Temporal Difference Learning is the best alternative when the model is unknown.

2.3.1 Temporal Difference Learning

Temporal Difference Learning is a combination of both Monte Carlo and Dynamic Programming ideas and uses the bootstrapping from DP and the power of Monte Carlo methods to learn from experience without the need to know the underlying dynamics of the environment.

The results of the Bellman equation are transformed into an update rule, by shifting the current estimate of $V(s_t)$ towards the estimate for $(R(s,a,s') + \gamma \cdot V^*(s'))$ (from equation 2.14). This estimate is the actual experience $(r_{t+1} + \gamma \cdot V(s_{t+1}))$ of the agent:

$$V(s_t) \leftarrow (1-\alpha)V(s_t) + \alpha(r_{t+1} + \gamma \cdot V(s_{t+1})) \tag{2.16}$$

where α is the learning rate that indicates how much the estimate is shifted to the new value.

The idea is, to on the one hand making the policy greedy with respect to the current value function (policy improvement) and on the other hand making the value function consistent with the current policy (policy evaluation). Policy improvement is done by adjusting the policy such that it follows the current

values greedily. Policy evaluation is done by using the current policy and update the $V(s)$ for every s based on the rewards.[8]

When the agent learns $V(s)$ for every s, it has to look ahead 1 step every time it needs to select an action. The values of $V(s_{t+1})$ for every possible action a need to be evaluated, and this is only possible if the agent knows which state is the result of taking which action. If this is not possible, the alternative is to learn and store $Q(s,a)$ instead. The algorithms SARSA, Q-Learning and Advantage Learning are all examples of the latter.

2.3.2 SARSA

Instead of updating $V(s)$ we can also choose to update $Q(s,a)$ resulting in the update rule:

$$Q(s_t,a_t) \leftarrow (1-\alpha)Q(s_t,a_t) + \alpha(r_{t+1}+\gamma \cdot Q(s_{t+1},a_{t+1})) \tag{2.17}$$

where α is the learning rate.

This update rule is called SARSA[9]. It is an *on policy* method, meaning that it improves Q^π for the current policy π and at the same time changes π towards the greedy policy. The action selection is not strictly greedy, but highly prefers the (current) best action. By sometimes selecting a lesser action, the policy can explore its environment. SARSA is illustrated in algorithm 1 in pseudo code.[10]

Algorithm 1: **SARSA**

```
initialize Q(s,a) arbitrarily;
foreach (episode) do
    t = 0;
    initialize s_0;
    while (episode not finished) do
        agent is in state s_t;
        agent selects action a_t;          // using policy derived from Q, e.g. ε-greedy
        if (t > 0) then                                              // update rule eq. 2.17
            Q(s_{t-1},a_{t-1}) ← (1-α)Q(s_{t-1},a_{t-1}) + α(r_t + γ·Q(s_t,a_t)) ;
        end
        agent executes action a_t resulting in new state s_{t+1} and reward r_{t+1};
        t ← t + 1;
    end
end
```

2.3.3 Q-learning

SARSA uses $Q(s_{t+1},a_{t+1})$ — the value of the specific action that was chosen — for its updates. Another option is to use the $\max_a Q(s_{t+1},a)$ instead. In that case the agent does not update its knowledge with the action that it has actually done, but according to what would have been the best action (given its current knowledge). Equation 2.17 then becomes:

[8] This idea stems from Dynamic Programming and is called Generalized Policy Iteration. In policy iteration (DP) these two processes alternate, but in value iteration (DP) they are intertwined and each step only one iteration

[9] SARSA is named after the 5-tuple of values that it uses in its update: $s_t, a_t, r_{t+1}, s_{t+1}, a_{t+1}$

[10] Note that the time in the update in the pseudo code is shifted back one time step in comparison with the update equation to indicate that you can obviously only update $Q(s_t,a_t)$ when you know s_{t+1} and a_{t+1}, so while at time step t, the update $Q(s_{t-1},a_{t-1})$ is executed.

$$Q(s_t, a_t) \leftarrow (1-\alpha)Q(s_t, a_t) + \alpha\left(r_{t+1} + \gamma \cdot \max_a Q(s_{t+1}, a)\right) \tag{2.18}$$

where α is the learning rate.

The pseudo code[10] for Q-Learning (see algorithm 2) is nearly identical to that of SARSA (see algorithm 1): only the update rule has changed. However, to illustrate that knowledge of the new action a_{t+1} is not necessary at the moment of update (since the maximum is used instead of the actually selected new action), the update and the selection of the new action are swapped.

Algorithm 2: **Q-Learning**

```
initialize Q(s,a) arbitrarily;
foreach (episode) do
    t = 0;
    initialize s_0;
    while (episode not finished) do
        agent is in state s_t;
        if (t > 0) then                                   // update rule eq. 2.18
            Q(s_{t-1}, a_{t-1}) ← (1-α)Q(s_{t-1}, a_{t-1}) + α(r_t + γ·max_a Q(s_t, a)) ;
        end
        agent selects action a_t;          // using policy derived from Q, e.g. ε-greedy
        agent executes action a_t resulting in new state s_{t+1} and reward r_{t+1};
        t ← t + 1;
    end
end
```

Since Q-Learning is not dependent on the *new* action (a_{t+1}) that is selected, it is called an *off policy* method. The learning in state s_t is done independently of the action a_{t+1}. It only depends on action a_t and the resulting state s_{t+1}.

2.3.4 Advantage Learning

Advantage Learning([14], [15]) was proposed by Baird III. [11] Its update rule is:

$$A(s_t, a_t) \leftarrow (1-\alpha)A(s_t, a_t) + \alpha\left(\max_a A(s_t, a) + \frac{r_{t+1} + \gamma \cdot \max_{a'} A(s_{t+1}, a') - \max_a A(s_t, a)}{k}\right) \tag{2.19}$$

where α is the learning rate, and k the scaling factor ($0 < k \leq 1$). When $k = 1$ this equation reduces to the Q-Learning update rule (equation 2.18). The pseudo code[10] for the Advantage-Learning algorithm is acquired by simply substituting the update rule (equation 2.19) for the Q-learning update in algorithm 2. Because of the scaling, Advantage-Learning often works better with function approximators than Q-Learning.

2.4 *"Here Be Dragons"* – the Problem of Exploration

Reinforcement Learning does not use examples of good solutions[12], so the only option for the agent is to explore its world. It needs to figure out by itself what actions on average lead to high or low returns.

[11] Advantage *Learning* is often – at least in internet tutorials on Reinforcement Learning like [8] – confused with Advantage *Updating*[13]. Advantage Updating however uses both a value function V and an advantage A. The confusion arises because for Advantage Updating the A is 0 for the optimal action and < 0 for the other actions, but this is not the case for Advantage Learning. However, this property is often erroneously claimed for Advantage Learning.

[12] Using examples of good (or bad) solutions for training is called *supervised* learning.

If the agent never explores but always greedily selects the option it thinks best, it will probably turn out to be sub-optimal. This is because the initial knowledge about what to do in what state, is often incorrect[13], and if the agent only followed this incorrect signal, it would just always do the same sub-optimal action, i.e. repeating the same error for ever and ever.

If the agent would only (or mostly) explore, it would of course see lots and lots of interesting things, and would learn a lot about the world which it is in. But in the end it would just be trying to investigate the entire environment, resulting in very detailed knowledge about all the uninteresting and unrewarding places.

It is evidently true that in the end – when the agent has explored everything in a static environment – it could just greedily choose its actions. However, this is a highly inefficient approach, and the agent would basically just have executed some dynamic programming algorithm: just iterate through every state and action for a great many times, until the entire problem is solved. Exploring your (static) environment by just wandering around nearly infinite amounts of time *does* get you a perfect model of your environment, but it might take *a while* and it only works in unchanging environments!

Balancing Exploration and Exploitation

Since we are only interested in an optimal (or nearly optimal) solution to the problem, the *details* of the uninteresting and unrewarding regions are of no real concern to the agent. It only has to have a rudimentary knowledge of them, since it has to know that in those regions no good solutions are to be found – and they need to be avoided – but that's it. As far as the agent is concerned, it could just as well say *"here be Dragons"*[14]. No more information is needed than that it is a area it does not want to go to, and it would be highly inefficient to let the agent learn all the ins and outs of these uninteresting regions. That time is better spent investigating more promising avenues.

The agent also needs to exploit what it already knows most of the time. If it sometimes explores, it will follow paths it deems good (for now) but sometimes take a wrong turn *deliberately* to see if it is actually better than the agent thought it was. To balance exploration and exploitation this way, some selection scheme is needed.

2.4.1 ε-Greedy Selection

By far the most simple selection method is ε-*greedy exploration*. It just selects the best action (greedy) with a large probability, but given a small value $\varepsilon \in \langle 0, 1 \rangle$ it selects actions randomly. The agent selects its action $a_{selected}$ according to:

$$a_{selected} = \begin{cases} \arg\max_{a' \in actions} Q(s, a') & \text{with probability } (1 - \varepsilon) \\ \\ \text{random} & \text{with probability } \varepsilon \end{cases} \tag{2.20}$$

where *random* denotes that an action is selected with uniform probability from the set of actions. It is possible to decrease ε slowly to 0 in order to reach convergence.

2.4.2 Boltzmann Selection

Boltzmann selection is a form of soft max selection. It is designed to overcome the problem with ε-greedy methods, that the action that has the highest value is always selected many more times than the second (or third, ...) highest even if they don't differ much. It might be desirable to investigate all the actions in some way proportionate to their Q-values, to insure that values that are only slightly less than

[13]For instance because at the beginning all entries in the table are randomly initialized, or arbitrarily set to zero.

[14]The expression "*Here be Dragons*" was used by ancient cartographers to denote parts of the world, which one knew nothing – or almost nothing – about. Instead of leaving unexplored areas of the map empty, cartographers were in the habit of drawing monsters and dragons there. Hence the expression "*Here be Dragons*"

the current maximum, are not ignored as with ε-greedy methods, because in that case actions other than the maximum only have probability $\frac{\varepsilon}{||actions||}$ to be selected.

The Boltzmann selection rule works as follows: each action a is selected with a probability according to the distribution P_{Boltz}, where P_{Boltz} is defined as

$$P_{Boltz}(s,a_i) = \frac{e^{Q(s,a_i)/\tau}}{\sum_{a' \epsilon actions} e^{Q(s,a')/\tau}} \tag{2.21}$$

where s is the current state, a_i is the action under consideration, $P_{Boltz}(s,a_i)$ gives the probability of selecting a_i in s and τ is called temperature, which determines the selection strength. If necessary, τ can be changed during learning.

Boltzmann selection results in nearly equal probabilities when the Q-values are nearly equal, but a large selection pressure for the maximum Q-value when the distance to the others is large. In fact, the probability of selecting the maximum (i.e. acting greedy) comes arbitrarily close to 1 when the difference becomes large enough.

2.5 Generalization – Neural Networks as Function Approximators

Reinforcement Learning was designed with look-up tables in mind, but when problems get larger, tables may not always be a good idea. Tables have no generalization capacity: they either contain a certain value, or they don't — there is no middle ground.

When a Reinforcement Learning problem has many states, it can take vast amounts of time and memory to learn a good policy, simply because every state has to be visited a number of times to get a good approximation of the value function. Each state is unique as far as the look-up table is concerned and has to be learned all by itself, even though some states might be very similar to each other, and closely resembling states might have closely resembling Q-values.

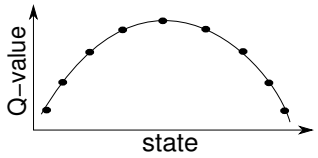


Figure 2.4: Table versus Function Approximator*: An example where one parabola ($a \cdot x^2 + b \cdot x + c$) with only three values (a, b and c) approximates nine values (the black dots).*

Function approximators are often used when a problem grows too big to handle with discrete tables (for the Q-values) or when generalization is desired. These approximators use certain structures and patterns in the problem space to compress the table to a smaller set of values that is easier to learn and store in memory (see for example fig. 2.4) because the learned function approximates the values between the known data points[15].

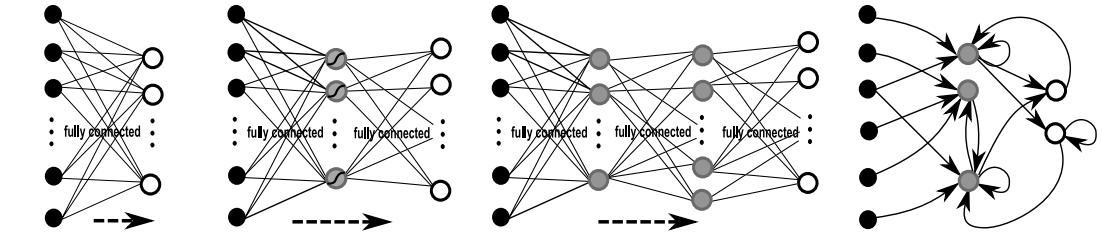
2.5.1 (Artificial) Neural Networks

Artificial neural networks are often used as function approximators. There are numerous different architectures, but the most important feature common to all is that they consist of – loosely biologically inspired – neurons and connections with their associated weights.

Neural Networks come in many flavours, from the most simple Linear Neural Network (see fig. 2.5(a)) that only has an input and an output layer, to the most complicated Recurrent Networks (see fig. 2.5(d)) where every neuron can in principle be connected to any other neuron. For the purpose of this thesis

[15] This is related to how flexible a function approximator is and what types of functions it can represent, but that is not the topic of this thesis.

however, only two of the simple types are relevant: the Linear Neural Network and the Multilayer Neural Network (see fig. 2.5(b)).



(a) linear neural net (b) one hidden layer (c) two hidden layers (d) recurrent neural net

Figure 2.5: (a) linear neural net*: the black circles represent inputs, the white circles are the outputs.* ***(b) and (c) multilayer neural nets with one and two hidden layers****: the gray circles represent hidden units.* ***(d) recurrent neural net****: the arrows represent the flow of information during computation.*

2.5.2 The Linear Neural Network

The linear neural network can be considered as an unthresholded Perceptron [10]. Its structure is extremely simple (see fig. 2.5(a)). The network consists of *n* input nodes and *m* output nodes, and each input node is connected to all output nodes. Usually an extra node that always has the value 1 is added. This node called the bias[16].

Every connection has a scalar value (a weight) associated with it. The input is fed into the network by giving the input nodes values. Then for each output node, the sum of the inputs multiplied with their associated weights is calculated.[17] The output of the network is simply the vector consisting of the values of the output nodes. All this is called the propagation phase.

The network consists of an input vector $\vec{x}$ and a matrix $\mathcal{W}$ of weights, and an output vector $\vec{y}$. The weights matrix $\mathcal{W}$ has dimensions $(|\vec{x}|+1)\times|\vec{y}|$. The extra column contains the weights for the bias node. The output of the network is calculated as follows:

$$\vec{y} = \vec{x}\cdot\mathcal{W} \qquad \text{with weights matrix } \mathcal{W} \text{ and inputs } \vec{x} \tag{2.22}$$

Note that this matrix multiplication in effect calculates the beforementioned weighted sum.

By adjusting the weights, the network is able to act as a function approximator. The network can be trained to give a certain output given a certain input. However since the network architecture is extremely simple, the functions it can approximate are also very simple.

Delta-Rule (Backpropagation)

The training phase is also called the *backpropagation phase*. For a given set of examples *D* and desired outputs *T* a *training error* is calculated. For convenience we only consider the case with 1 output node, but a network with more than 1 node can simply be treated as a collection of networks with 1 output node[18] because the output nodes are independent of each other.

Usually the error is defined as:

$$E(\vec{w}) = \frac{1}{2}\sum_{d\in D}(t_d - y_d)^2 \qquad \text{with } t_d \in T \tag{2.23}$$

[16]The bias gives the network more flexibility as it adds an input independent value to the weighted sum (since the value of the bias is always 1, this added value is only dependent on the weight connecting the bias to each output).

[17]If the network is a Perceptron, the outputs are not simply the weighted sum, but a value of 1 or -1 (or 0) depending on whether the weighted sum is above or below a certain threshold.

[18]with only one output node, the calculation $\vec{y} = \vec{x}W$ reduces to $\vec{y} = \vec{x}\vec{w}$ with $\vec{w}$ being the vector with the weights

The weights need to be adjusted in the direction that will minimize this (or any other) error. To get this direction, the derivative of E with respect to every component of $\vec{w}$ needs to be taken. This derivative (gradient) $\nabla E(\vec{w})$ gives us the steepest ascent along the error surface E:

$$\nabla E(\vec{w}) = \left[\frac{\partial E}{\partial w_0}, \frac{\partial E}{\partial w_1}, \cdots, \frac{\partial E}{\partial w_n} \right] \tag{2.24}$$

The vector $\nabla E(\vec{w})$ specifies the direction of steepest increase of E, so if we want to adjust the weights to minimize the error E, we need to adjust the weights vector $\vec{w}$ with a factor $-\alpha \nabla E(\vec{w})$ (where α denotes a learning rate).

Therefore the weights update for each individual weight w_i is:

$$w_i \leftarrow w_i - \alpha \cdot \nabla E(\vec{w})_i = w_i - \alpha \cdot \frac{\partial E}{\partial w_i} \tag{2.25}$$

The gradient $\nabla E(\vec{w})$ can easily be calculated:

$$\frac{\partial E}{\partial w_i} = \frac{\partial}{\partial w_i} \frac{1}{2} \sum_{d \in D} (t_d - y_d)^2 = \sum_{d \in D} (t_d - y_d)(-x_{id}) \quad (x_{id} \text{ is the input } i \text{ for example } d) \tag{2.26}$$

And thus the weight updates become:

$$w_i \leftarrow w_i + \alpha \sum_{d \in D} (t_d - y_d) \cdot x_{id} \tag{2.27}$$

The procedure outlined above is called the *batch* version because first all the examples are propagated and all the errors summed up, and only then all the weights are adjusted.

Incremental Gradient Descent

When the weights are adjusted after each individual example, it is called *stochastic* or *incremental gradient descent*. We get incremental gradient decent if we replace the update in equation 2.27 with:

$$w_i \leftarrow w_i + \alpha \cdot (t_d - y_d) \cdot x_{id} \tag{2.28}$$

and (obviously) use equation 2.29 instead of equation 2.23 as the error for training example d:

$$E_d(\vec{w}) = \frac{1}{2}(t_d - y_d)^2 \qquad \text{(where } d \text{ is a training example)} \tag{2.29}$$

2.5.3 Neural Networks with Hidden Layers

Often the network architecture is expanded by introducing one or more layers of so called *hidden* nodes or units (see fig. 2.5(b) and (c)). The input layer is then connected to the first hidden layer, the first to the second, and so on, and the last hidden layer is connected to the output layer. This allows the neural network to represent highly non-linear decision surfaces, and in theory approximate any continuous function, given enough hidden units.

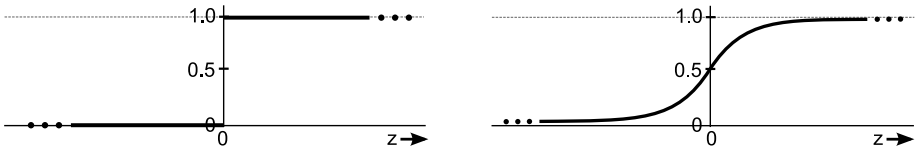
The nodes in the hidden layer and output layer calculate the weighted sum of the inputs as usual, but the hidden layer does not output these sums as in the linear case, but first applies a threshold function to this sum, and then outputs the result. This threshold function can be an *all-or-nothing* threshold (see fig. 2.6(a)), but usually a more smooth (and differentiable) roughly *S*-shaped function is used, such as the sigmoid function (sometimes called a 'logistic function').

The sigmoid function (σ) is a function that goes asymptotically to 0 for $z \to -\infty$ and to 1 for $z \to +\infty$ and ascends fast in the region near 0 (see fig. 2.6(b)). σ is defined as:

$$\sigma(z) = \frac{1}{1 + e^{-z}} \tag{2.30}$$

The sigmoid function is used very frequently because it behaves similar to the *all-or-nothing* threshold function when $z \ll 0$ or $z \gg 0$ but also has a very simple derivative which can be calculated very fast:

$$\frac{\partial \sigma(z)}{\partial z} = \sigma(z) \cdot (1 - \sigma(z)) \tag{2.31}$$



(a) *all-or-nothing* threshold (b) sigmoid

Figure 2.6: (a) all-or-nothing threshold-function*: in this case the threshold is at* $z = 0$*, at which point the value jumps from 0 to 1.* ***(b) sigmoid function****: going asymptotically from* $-\infty$ *to* $+\infty$*.*

Backpropagation

To train a multilayer network, an algorithm similar to that for the linear network can be used. First the error needs to be defined. Because these networks are often used with multiple outputs, the error that was earlier defined in eq. 2.29, needs to be extended. We now only look at the *stochastic case*, but the *batch* version is similar[19].

$$E_d = \frac{1}{2} \sum_{k \in \, outputs} (t_k - o_k)^2 \quad \text{with } t_d \in T \tag{2.32}$$

Now to keep things 'simple', we'll use the following conventions:

- x_{ji} is the i^{th} input to node j
- w_{ji} is the weight associated with the i^{th} input to node j
- $net_j = \vec{w} \cdot \vec{x} = \sum_i w_{ji} x_{ji}$ (i.e. the weighted sum of inputs for unit j)
- o_j the output computed by node j (i.e. $\sigma(net_j)$ if the layer uses a sigmoid)
- t_j the *target* output for node j
- $next(j)$ for the set of nodes whose immediate inputs include the output from unit j

As before with the linear case, we use error E_d (eq. 2.32) to calculate the weight updates:

$$\Delta w_{ji} = -\alpha \cdot \frac{\partial E_d}{\partial w_{ji}} = -\alpha \cdot \frac{\partial E_d}{\partial net_j} \cdot \frac{\partial net_j}{\partial w_{ji}} = -\alpha \cdot \frac{\partial E_d}{\partial net_j} \cdot x_{ji} \tag{2.33}$$

The Output Nodes

Since net_j only appears in o_j, we can continue for the output unit weights (using the chain rule):

$$\Delta w_{ji} = -\alpha \cdot \frac{\partial E_d}{\partial o_j} \cdot \frac{\partial o_j}{\partial net_j} \cdot x_{ji} \tag{2.34}$$

The third term on the right of this equation is the derivative of the sigmoid (equation 2.31), so:

$$\frac{\partial o_j}{\partial net_j} = \frac{\partial \sigma(net_j)}{\partial net_j} = o_j \cdot (1 - o_j) \tag{2.35}$$

Here we see the benefit of using the sigmoid function. It has a very simple (not to mention fast to compute!) derivative. If the network uses linear functions for the outputs (as is often the case for function approximation) then $\frac{\partial o_j}{\partial net_j} = 1$. Note that other differentiable functions could also be used as threshold functions.

[19] For the batch version we need the extra $\sum_{d \in D}$ in eq. 2.32 to account for batching over all training examples $d \in D$

The second term in equation 2.34 is more complicated. We can substitute equation 2.32 for E_d:

$$\frac{\partial E_d}{\partial o_j} = \frac{\partial}{\partial o_j}\frac{1}{2}\sum_{k \in outputs}(t_k - o_k)^2 \tag{2.36}$$

And since $\frac{\partial}{\partial o_j}(t_k - o_k)^2$ is zero if $j \neq k$ (because o_j and o_k are different variables for $j \neq k$) we can drop the sum and rewrite to:

$$\frac{\partial E_d}{\partial o_j} = \frac{\partial}{\partial o_j}\frac{1}{2}(t_j - o_j)^2 = \frac{1}{2}\cdot 2\cdot(t_j - o_j)\cdot\frac{\partial(t_j - o_j)}{\partial o_j} = -(t_j - o_j) \tag{2.37}$$

When we substitute equations 2.37 and 2.35 in 2.34 we get an equation for the value of the weight update in the output units:

$$\Delta w_{ji} = \alpha\cdot(t_j - o_j)\cdot o_j(1 - o_j)\cdot x_{ji} \tag{2.38}$$

or if we use linear functions for the outputs ($\frac{\partial o_j}{\partial net_j} = 1$ in that case), instead of sigmoids:

$$\Delta w_{ji} = \alpha\cdot(t_j - o_j)\cdot x_{ji} \tag{2.39}$$

The Hidden Nodes

For the hidden units, we can use the above derivation up to and including equation 2.33. The weights w_{ji} can only influence the network outputs indirectly, because (in our case) i is in the input layer[20] and j is in the hidden layer.

$$\begin{aligned}
\Delta w_{ji} &= -\alpha\cdot\frac{\partial E_d}{\partial net_j}\cdot x_{ji} \\
&= -\alpha\sum_{k \in next(j)}\frac{\partial E_d}{\partial net_k}\cdot\frac{\partial net_k}{\partial net_j}\cdot x_{ji} \\
&= -\alpha\sum_{k \in next(j)}\frac{\partial E_d}{\partial net_k}\cdot\frac{\partial net_k}{\partial o_j}\cdot\frac{\partial o_j}{\partial net_j}\cdot x_{ji} \\
&= -\alpha\sum_{k \in next(j)}\frac{\partial E_d}{\partial net_k}\cdot w_{kj}\cdot\frac{\partial o_j}{\partial net_j}\cdot x_{ji} \\
&= -\alpha\sum_{k \in next(j)}\frac{\partial E_d}{\partial net_k} w_{kj}\cdot o_j\cdot(1 - o_j)\cdot x_{ji} \\
&= -\alpha\cdot o_j\cdot(1 - o_j)\sum_{k \in next(j)}\frac{\partial E_d}{\partial net_k}\cdot w_{kj}\cdot x_{ji}
\end{aligned} \tag{2.40}$$

The quantity $-\frac{\partial E_d}{\partial net_k}$ is often called the δ_k or *error term* associated with unit k and is gotten from neurons in the next layer (i.e. the layer closer to the end of the network, because the backpropagation starts with the *outputs* and propagates the error back to the *inputs*).

We know from equation 2.33 that $\Delta w_{ji} = -\alpha\cdot\frac{\partial E_d}{\partial net_j}\cdot x_{ji} = \alpha\cdot\delta_j\cdot x_{ji}$ so if a neuron in the next layer is an output, we use $\delta_k = (t_k - o_k)\cdot o_k\cdot(1 - o_k)$ from equation 2.38. If it is not an output (e.g. if the network consists of multiple layers of hidden neurons) we use equation 2.40 for hidden neurons.

The Backpropagation Algorithm

When we put all of the above together, the result is the (pseudo)code for the BACKPROPAGATION algorithm, as displayed in algorithm 3. Usually the algorithm runs until some termination criterion is reached, for instance that the overall error is below a certain threshold, or it is stopped after a fixed number of iterations.

[20] This can also be generalized to more layers.

Algorithm 3: **BACKPROPAGATION and FORWARDPROPAGATION**: for a network with one hidden layer with sigmoids. Outputs that can either be linear or sigmoids. The incremental gradient descent version is given.

BACKPROPAGATION ::

Data: a set of training examples and their target outputs $< \vec{x}, \vec{t} >$
Result: the network with updated weights

while $(termination\ criterium\ not\ reached)$ **do**
  foreach $(< \vec{x}, \vec{t} >\ in\ the\ set\ of\ training\ examples)$ **do**
    calculate all outputs o_m using FORWARDPROPAGATION;
    foreach $(k \in output\ nodes)$ **do**
      if $(outputs\ use\ sigmoids)$ **then** $\delta_k = (t_k - o_k) o_k (1 - o_k)$;
      else $\delta_k = (t_k - o_k)$; // linear outputs
    end
    foreach $(h \in hidden\ nodes)$ **do**
      $\delta_h = o_j (1 - o_j) \sum_{k \in next(j)} \delta_k w_{kj}$;
    end
    $w_{ji} \leftarrow w_{ji} + \alpha \delta_j x_{ji}$; // update the weights
  end
end

FORWARDPROPAGATION ::

Data: one training example and its target output $< \vec{x}, \vec{t} >$
Result: the new values of the (hidden and) output nodes

foreach $(h \in hidden\ nodes)$ **do**
  $o_h = \sigma(\sum_{i \in input\ nodes} x_i w_{hi})$;
end

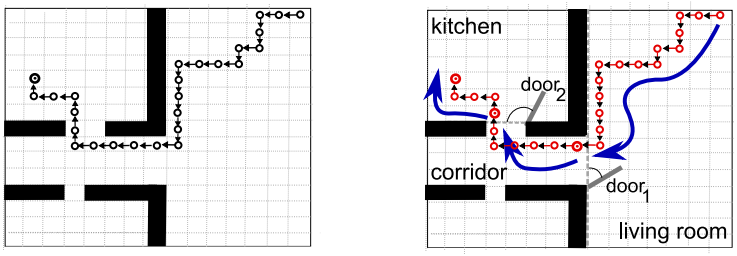
foreach $(m \in output\ nodes)$ **do**
  if $(outputs\ use\ sigmoids)$ **then** $o_m = \sigma(\sum_{h \in hidden\ nodes} o_h w_{mh})$;
  else $o_m = \sum_{h \in hidden\ nodes} o_h w_{mh}$; // linear outputs
end

Chapter 3

Hierarchical Reinforcement Learning

The use of hierarchies in Reinforcement Learning is one of the strategies for dealing with large state spaces (among others are the use of abstraction or function approximators or coarse tiling). The idea is, to somehow improve on the normal 'flat' Reinforcement Learning algorithm by giving the agent the ability to execute actions that are *extended in time* instead of only taking actions that have a duration of one time step.

Suppose we are training a robot to navigate through a house (see fig.3.1(a)). The house is just a grid, using small squares (for instance the floor tiles) and the states are all the possible positions of the robot on this grid. The robot could be at position $(10, 24)$ which would mean *10 tiles to the north, and 24 to the east, starting from the origin-tile*.



(a) primitive actions (b) temporally extended actions

Figure 3.1: ***(a) primitive actions****: circles and arrows denote visited states and primitive actions.* ***(b) temporally extended actions****: the same situation as in (a), now divided into rooms (acting as high level states). The large arrows are the temporally extended actions.*

There is usually only a small number of primitive actions, whereas there are lots[1] of states. The set of primitive actions might consist of the four cardinal actions of *moving to the next tile North, East, South or West* or perhaps only the three actions of *step forward one grid cell, turn left* or *turn right*. The problem of navigating from one place in the house to another place, is just a question of which (possibly very long) sequence of primitive steps the robot has to take.

Temporally Extended Actions

It is possible to define *temporally extended actions*, which are sequences of primitive (atomic) actions. We can then treat these temporally extended actions as if they are one atomic action. As an illustration, let's suppose, that we are able to come up with a more abstract representation of the navigation problem because we can use some of the underlying structure. We might be able to group related tiles together, for instance in a *kitchen*, a *living room*, etc. (see fig.3.1(b)), or perhaps we can identify *hot spots* like doors.

[1] In fact, when the problem gets bigger and "lots" becomes "lots and lots and lots", we want hierarchies!

Subgoals

We can use these newly found *rooms* as *subgoals*[2]. The robot starts in the *living room* and has to reach the *kitchen* to achieve its goal, and it needs to go through the *corridor* to reach the *kitchen*. Now the sequence of steps is much smaller (when viewed on this more abstract level): first it needs to select the subgoal *"corridor"* and then *"kitchen"* and after that the subgoal that consists of moving toward the *"goal"*. Now we need one policy that learns on the level of subgoals, and several smaller policies that learn how to reach the subgoals. In effect, we have introduced a higher layer.

Task Decompositions

On the other hand we could decompose the overall task of reaching the goal into several smaller subtasks (creating a task decomposition). Now we have subtasks for reaching the corridor when starting in the living room (or in the kitchen), for reaching the living room from the corridor, and for reaching the kitchen from the corridor. The agent now learns in what order these subtasks need to be executed and how the subtasks are to be performed.

Options

Another alternative is, to define two macros (called *"options"*): one option to go to $door_1$ and one for $door_2$, because we have identified both doors as "hot spots" in the problem. We could now just add these options as new actions to the set of actions (augmenting the primitive actions). Now the agent could first select the option to $door_1$, after that (or perhaps after some primitive steps) select the $door_2$-option and then use some primitive steps to reach the goal.

Since the options are added directly to the primitive actions, there are no new layers introduced. Nevertheless the options *are* temporally extended, and are therefore hierarchical in action and time. Note that we could also introduce options that do not have any goal, but are simply (perhaps even random) sequences of actions like *"move North twice, and then East"*.

3.1 Problems in "Flat" Reinforcement Learning

Before we embark on a search for solutions, it is best to find out *why exactly* Reinforcement Learning becomes hard when the problem grows larger, and *how* hierarchies could be of any benefit.

3.1.1 Curse of Dimensionality

The first obvious reason why Reinforcement Learning gets hard when the problem size grows is the infamous *curse of dimensionality*. This is the problem that the number of states in a problem grows exponentially with each new dimension that is added.

For instance, if we have a grid-world[3] consisting of a line divided in 10 pieces and an agent that can walk along that line (each time choosing to go left or right) then the problem consists of 10 states (one state for each piece the agent can be in). Suppose we introduce the second dimension, and make it a grid measuring 10 by 10 squares. Now the agent can be in 10×10 different states[4]. But when we introduce a third dimension, the agent suddenly has 10^3 states (cubes) to wander in (see fig. 3.2).

So basically the size of the problem grows exponentially when a new dimension (i.e. an extra sensor or variable) is added. This is obviously undesirable, but it cannot really be avoided, because real-world problems often have many dimensions (that is probably what makes them *real world* instead of *toy* problems in the first place). The curse of dimensionality therefore makes it harder for the agent to

[2] The way the hierarchy is introduced here, resembles the way HASSLE uses subgoals.

[3] More correctly called a *line-world*.

[4] Under the assumption that the actions that the agent can take are also extended, so it can actually go to these new parts of its world.

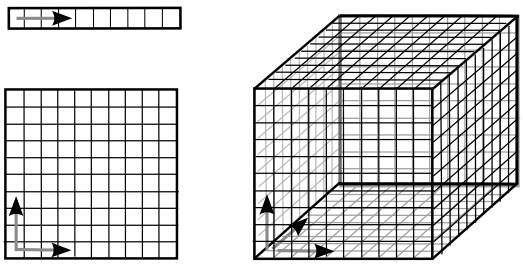


Figure 3.2: Curse of Dimensionality*: each new dimension exponentially increases the state space.*

learn about the mechanics of its environment, because the environment quickly becomes astronomical, growing exponentially with every dimension that is added to a problem.

3.1.2 No Knowledge Transfer

There are regularities in many state spaces: parts of the state space resemble other parts, and good actions in one part might also be good actions in other parts. But apart from function approximators, we have no way of re-using this knowledge, or to transfer it from one part of the policy to another part.

A function approximator (e.g. a neural network) painstakingly needs to figure out that it can generalize over certain patterns, but it would be better to be able to just state that a certain part of the policy can be re-used somewhere else.

3.1.3 Slow Exploration Due to Random Walking

A third problem is that the agents in the beginning of its learning phase just randomly walks because it has never even seen its goal yet. This obviously is a big problem in tasks with sparse rewards, where the agent only acquires non-zero rewards when it achieves its goal (or perhaps some subgoals in the task). In problems with a large state space, this random walk which the agent executes when it has not yet reached its goal, grows increasingly large.

In one dimension, when we have a random walk over the integers which starts in $0 \leq z \leq a$, the expected time before the random walk leaves the interval $\langle 0, a \rangle$ is $z \cdot (a - z)$. When we apply this to the Reinforcement Learning case where the agent starts 'somewhere in the middle' (i.e. not close to either end of the interval $\langle 0, a \rangle$) and when we only reward the agent when it reaches the end (for instance with -1 and 1 for reaching 0 or a) then the expected time is quadratic in the size of the interval. Longer intervals give quadratically longer times. For more dimensions a random walks were simulated[5] showing the same quadratic relation between expected distance and time.

So with increasing problem size, the estimated time before the agent reaches its target for the first time increases disproportionately with relation to the distance in the state space. This means that for larger problems, the time before the agent actually starts to learn, scales badly.

3.1.4 Signal Decay over Long Distances

A problem related to that of random walks is that of the propagation of the reward signal over long distances. It needs to be propagated back because not only the final action before the goal, but also actions before that probably contributed to acquiring the goal reward. But when a problem scales up these past actions will become more and more distant. And the more distant these actions are, the more iterations it will take to propagate the reward signal all the way back, because essentially the reinforcement learning algorithm looks ahead one step when selecting an action (choosing from the Q

[5] A square grid (size n) was created and a 'random walking' agent was put in the grid. Then the random walk was simulated until the agent arrived at a predefined coordinate (always near the lower right corner at $(n-2, n-2)$). The average time was calculated just letting the agent walk his random walk a number of times. The results were - not surprisingly - that the same roughly quadratic relation holds

values of all actions possible in a certain state) and it updates one step back (updating the Q value for the past state and taken action with the new information available about the value of the resulting state).

So iterating back is linear in the distance between actions. It does not matter if subsequent episodes – after an episode where the goal was reached – actually do reach the goal. It is enough for them to reach some part of the problem space where the new information (from reaching the goal previously) has already been propagated to. After reaching the goal for the first time this would only be in some state(s) near to the goal, but as more and more episodes reach these states with updated information, the 'front' progresses further and further away from the goal.

Suppose we are in state *B* (see fig. 3.3), having two actions to choose from, where action *x* will lead us to state *C* (which is the best move towards the eventual goal) and action *y* will return us to state *A*, the state we just came from in the previous step and *z* will bring us from *A* to *B* again.

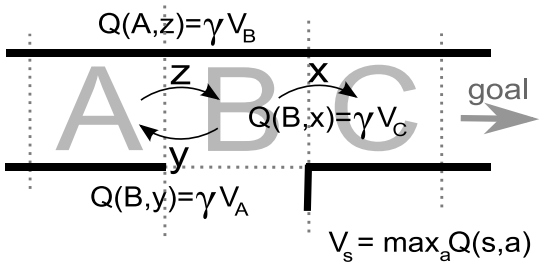


Figure 3.3: maze(-like) task after (Bellman)-equilibrium is reached.

Let us denote the maximum Q value in a state *S* with V_S, so $V_S = \max_a Q(S,a)$ (this is assuming some kind of Q-Learning algorithm is used and the only non-zero reward is given when the (sub)goal is reached). Now when the equilibrium is reached (i.e. the Bellman-equations hold) then the value of taking action *x* in *B* is $\gamma \cdot V_C$ (or lower if the actions are non deterministic and the agent sometimes fails to accurately execute what it has selected). But the value of state *A* is $V_A = \max_a Q(A,a) = \gamma \cdot V_B = \gamma^2 \cdot V_C$. This means that action *y* in state *B* has $Q(B,y) = \gamma^3 V_C$. Which in turn means that the difference between *x* (best) and *y* (worst) action in *B* is $(\gamma - \gamma^3) \cdot Q_C$ and $(\gamma - \gamma^3) \approx (1 - \gamma^2)$ when $\gamma \to 1$.

A difference in the order of $(1-\gamma^2)$ between best and worst actions (γ near 1) might not be a problem when a tabular representation is used for the Q-values, because a table stores the Q-values perfectly, but it certainly poses a huge problem when function approximators are used. The smaller the difference, the more fine grained the approximator needs to be. In terms of neural networks[6], this means that more and more hidden neurons are needed, and longer and longer training times are the result – and because the networks are larger, each iteration of the forward- or backpropagation algorithm itself also takes more time.

If the reward signal has to cover a lot of distance, the discount value γ needs to be high (otherwise the reward signal would decrease too rapidly to zero). In maze like problems (where the only reward not equal to zero is given when a goal state is reached, as in the example above) the difference between best and worst action is in the order of $(\gamma - \gamma^3)$!

Using Advantage Learning might somewhat alleviate this problem because it does not use the Q-values but scales their relative differences (see section 2.3.4) but in general the problem remains: long chains of actions need a high discount, but a high discount makes discriminating between actions difficult.

No Middle Ground

We observe that for long distance propagation (i.e. in large problems) a high γ is needed because otherwise the value would vanish too fast. But in maze-like tasks γ cannot be too high because then function approximators cannot any longer discriminate between the best and worst actions.

[6] the most common approximators used

So given a certain function approximator, these two conflicting demands result in an upper boundary in the propagation length and therefore in the problem size. Places where (non zero) rewards are received cannot be too far apart because either the function approximator can't handle it (with high γ), or the expected reward in distant states decreases to zero too fast (small γ).

3.2 Advantages of Using Hierarchies

3.2.1 Exorcising the Dæmon of Dimensionality

Using task decompositions will usually reduce the state space on each level of the hierarchy, because at each node in the task decomposition, only a subtask needs to be solved, which most of the times will involve only a subset of the variables or a subset of the entire state space. So task decompositions are helpful in reducing the size of the problem, and therefore in exorcising the Dæmon of Dimensionality, as Dayan[41] eloquently put it.

3.2.2 Subpolicy Re-use

The use of extended (or high level) actions can be compared to the reasoning that a programmer uses, when she introduces functions and methods in her program. Introducing functions allows her to write larger and more complex programs, no longer being constrained to putting together long strings of basic operations. In the same way defining (or learning) sequences of actions that can be grouped together, and treated as if they were one atomic unit, improves Reinforcement Learning because it allows for explicit use of similarity (see fig. 3.4).

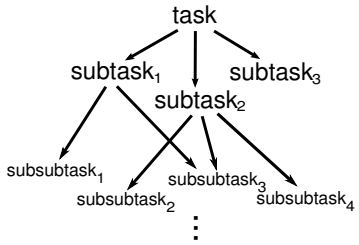


Figure 3.4: Task decomposition*: dividing a task into several smaller subtasks, allowing re-use.*

If the designer knows that a certain subtask or subgoal occurs more than once, there is no need to learn them separately. Knowledge gained from one instance can directly be used elsewhere in the problem where this particular subtask also occurs.

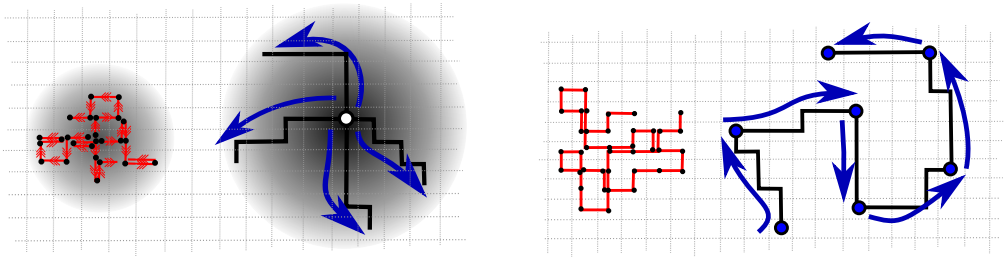
3.2.3 Faster Exploration – Moving the Random Walk Burden Upwards

Hierarchies introduce behaviours (sequences of primitive actions), and in principle these behaviours do something non-randomly. The idea is, that behaviours move the agent in a non-random way through the state space. So if the agent has a set of behaviours, but does yet not have the solution for the overall problem, it can execute a behaviour, and move purposefully (i.e. not needing random walking) for the duration of the behaviour. After the behaviour is terminated, another behaviour can then be selected (at random because the problem was not yet solved). In effect the random walk exploration is moved from the low level of primitive actions to the high level of behaviours.

The high level actions (or behaviours) are now considered as atomic actions in a new *high level* random walk, only this time the actions are temporally extended, and the agent walks with larger steps (albeit still randomly selected). For this to work, the hierarchical actions need to be more effective (i.e. travelling further, doing more, etc) than what the primitive actions can accomplish in the same time with random walking. Since the estimated time needed to cover a distance d is $O(d^2)$ this means that the distance covered in a certain time t is $O(\sqrt{t})$. The hierarchical behaviours need to be at least as

good as this (on average) to be able to cover as much "ground" – or more general, to cover more of the problem space – as the random walk using primitive actions.

When the hierarchical actions are *meaningful*, that is, they are not completely random and they do actually achieve something (on average they move the agent a certain non-zero amount through state space), then their use has a benefit over selecting the same number of primitive actions. Instead of selecting these primitive actions completely random, there is only one randomly selected (hierarchical) behaviour, and the primitive actions that it executes form a reasonably structured larger behaviour.



(a) random walks on average (b) one long random walk

Figure 3.5: Random walks on average*: four random walks on the "primitive action level" versus four random walk on the "high level". All paths through the state space are of equal length, but the random walks on the high level each have only one random decision point (the dots) whereas the random walks on the primitive action level each have 6 decision points, therefore on average they reach further.* ***One long random walk****: walks of equal length (35) through the state space, but the random walk on the high level has far less random decision points (the dots) than that on the primitive action level.*

This principle is most easily illustrated when we would try the following experiment a number of times (see fig. 3.5(a)). An agent has a small number of time steps to explore (in fact just enough for one behaviour, let's say 6 time steps).

If the agent has no behaviours at its disposal, it needs to randomly walk on the level of primitive actions. This will result in covering an expected distance that is the square root of the available time. But if it can select its actions from a set of different behaviours it will only make one random choice (i.e. which behaviour to execute), and after that, it will move purposefully. So on average, the area that the behaviour level random walk will visit is larger than the area that the primitive random walk will visit.

On longer timescales, this means that the agent can make greater leaps through the state space and cover more "ground" than a flat learner (see fig. 3.5(b)). This allows for faster (random walk) exploration. This means that, *if* behaviours are available or can be learned early on in the learning process, the agent can use these behaviours to cover more "ground" in the state space, which increases the probability of solving the problem faster[7]. It is even possible to do some exploration when executing a behaviour, as long as it does not completely destroy the non-random character of the behaviour.

3.2.4 Better Signal Propagation

Large steps of temporally extended actions allow the algorithm to propagate the rewards faster. A chain of actions is faster traversed if you are allowed to regard several atomic actions together as one large step, and that way a state from which that large step was taken now receives meaningful reward signals faster than if it had taken several small steps (and reached the same goal).

3.2.5 Different Abstractions for Different Subpolicies

Using subpolicies for subtasks, allows for different state spaces (or abstractions) for each of the subpolicies. This is in fact a spin off that we get for free when we use subpolicies. For certain subtasks, a certain

[7] Obviously under the condition that the problem can be solved in terms of the available behaviours.

variable might not be relevant at all[8]. It is then possible to use different state space representations for different subtasks by leaving out variables (sensor readings) that are not relevant to the particular subtask. This is for instance used in MAXQ, and it comes quite naturally with task decompositions.

But hierarchies and subtasks allow another – more radical – possibility: to use completely different state representations for different subpolicies that execute the same behaviour. That way the agent is allowed to select the best representation for each part of the state space. That way it can switch its representation (by switching subpolicies) but continue doing the same task. It now simply has two or more subpolicies that are trained on the same task, but use different representations of the state space.

Of course all the different representations could be bundled into one large (and redundant) state representation, but that would result in exponential growth in size, and again the Curse of Dimensionality. Splitting representations to different subpolicies, and allowing the agent to learn when to switch, does not have that problem. An example of this approach is described in [3] and was proposed at the same workshop where work from this thesis was also presented.

3.3 An Overview of Hierarchical Techniques

3.3.1 Options – Hierarchy in Time and Action

As mentioned before, the most popular approaches are *adding options* and *introducing layers*. An option is basically just a sequence of primitive actions, that is *added* to the set of actions the agent can select from. The primitive actions themselves can also be viewed as options, i.e. options that only invoke one primitive action. They are sometimes called *one-step options*. Options therefore have a hierarchical structure in time, because the options are decomposed into smaller options, all the way down to the primitive actions. There is no state abstraction or hierarchy or use of different abstractions of the state space for different layers.

Options are known under many different names, some of them being *macro-operators, macros, skills, temporally extended actions, behaviours, modes* or *activities*, but the basic idea remains the same. Some Options approaches are described briefly section 3.4.

In principle, options should be able to execute not only primitive actions, but also to call other options as 'subroutines', although not every approach that uses options also uses this rather complex structure.

The Options Framework – Between MDPs and Semi-MDPs

Sutton et al. [17] proposed to use Semi-MDPs (SMDP)[9] to extend the usual reinforcement learning structure (the MDP, see section 2.2.1) to temporally extended actions.

Options require that we extend the definition of MDPs to SMDPs which include actions taking more than one time step, so that the waiting time in a state is equal to the duration of the invoked option. These options can be completely fixed (i.e. *optionX* is *"always go North twice, and then East"*) or they could themselves be stochastic policies. Since options are just considered actions (although extended) and are added to the action set, options are ideally suited for re-use. An option that is itself a stochastic policy could be specialized in solving a certain sub-problem, but could also be successful in solving similar sub-problems somewhere else in the problem space.

An option consists of three components: a policy $\pi : S \times A \to [0,1]$ (same as in equation 2.8) that gives the probability of selecting an action from A in a given state from S, an initiation set $I \subseteq S$, and a termination condition $\beta : S \to [0,1]$. An option $\langle I, \pi, \beta \rangle$ is available for execution in state s_t if and only if $s_t \in I$. The termination criterion β allows for stochastic termination of the option.

The primitive actions are considered special one-step options. They are always available when action a is available: ($I = S$, assuming that a is always available, of course), and always last exactly one step, so always have a termination probability of 1 ($\forall s \in S : \beta(s) = 1$).

[8] because it never changes during execution of this subtask

[9] SMDPs are used to model multi-step and continuous time discrete-event systems.

We can create policies with options, the same way that we normally formulate policies (see section 2.2.2). More formally, when we let O be the set of options, we can formulate the options-policy μ: $S \times O \rightarrow [0,1]$ where option $o \in O$ is selected for execution in state $s_t \in S$ at time t according to the distribution $\mu(s_t, \cdot)$, and terminates after k steps in state $s_{t+k} \in S$. In this way the policy μ over options, just defines a "flat" policy π. In fact flat policies are just a special case of the more general policies over options.

Note that the policy that results after "flattening" might not have the Markov Property (see section 2.2) because the selection of a primitive action at a certain time t does not only depend on the current state s_t, but on information available when the option started several time steps ago. This is also the case, when a timeout is introduced, which allows the agent to stop the policy after a certain number of time steps has elapsed, *even though the termination criterion* β *is still not met*.

Options Are Not Hierarchical In The State Space

Options extend the set of actions, and they are hierarchical in the sense that an option calls other options (or at least primitive actions – one-step options) when invoked. But the state space is not broken down into hierarchical parts. In essence, an option augments the agent by giving it more (temporally extended) actions to choose from. But the selection of a primitive action or an option are still based on the same observed state, there is only abstraction in the actions, not in the states. This does not seem to be the way humans do it though.

It would seem that much of the information that is available is completely irrelevant when you need to make a decision about a destination that is far away and what option to choose to go there. On the other hand this 'local' information would be extremely relevant when we are near or goal.

3.3.2 Multiple Layers – Hierarchies in State Space

The problem of 'fine tuning' versus the 'wider perspective' that was sketched informally above, lies at the heart of the second approach. Instead of just augmenting the set of actions with larger actions (i.e. 'options'), the state space is hierarchically decomposed. The problem is viewed on two (or more) levels with differing 'resolution' or scope.

On a more global (higher) level, only globally relevant information is contained in the (abstract or high level) state. On the lower level the more local, more detailed information is present. This ensures that on a higher level the agent is not bothered with trivial small questions, but that on the lower level still all the information is available with the same high resolution as in the corresponding flat Reinforcement Learning problem.

To achieve these layers, the problem is decomposed into a set of hierarchical sub problems which are (hopefully) smaller and easier to solve. This implies that a hierarchy of MDPs is created. Each MDP has its own set of states, and the higher level usually has a smaller state set of more abstract states. Some algorithms use a selection of states from the lower level (for instance bottlenecks in the state space) while other algorithms construct entirely new 'higher level' states which are abstractions of some lower level features.

It should be noted, that abstracting on the higher level, will often result in suboptimal policies. It is often the case that the optimal policy is no longer available in the set of policies that is still possible within the constraints of the hierarchy. The policy can of course still be optimal with respect to this restricted set. Often this is a trade-off between getting a good – though not optimal – solution *fast*, and finding the optimal solution after *a very long time*. It is up to the designer to ensure that its hierarchy still allows for nearly optimal – if not optimal – policies.

Generic Framework

Layered approaches differ far more than approaches using the Options-framework, because they not only have temporal hierarchies (temporally extended actions), but also allow for abstractions and therefore

hierarchies in the state space. Any structure trying to unify them, is therefore doomed to be only a very coarse and general high level description[10].

A learning problem (with given state space S for a "flat" learner) is decomposed in subtask layers $\{L_1, \ldots, L_n\}$ where each layer is defined as: $L_i = (S_i, O_i, T_i, M_i, h_i)$ with:

- S_i is the (abstract) state space for layer i
- O_i, the set of actions
- T_i, the set of training examples
- M_i, the machine learning technique that is used
- $h_i \mapsto O_i$, the 'hypothesis' which is the result of running M_i on T_i

Given the diversity in layered approaches, it is hard to denote any more common features.

3.4 Relevant Work with Options

Some relevant algorithms are presented here that use the Options-framework. The order is not chronological order, but rather in (roughly) ascending complexity. Its purpose is to give a very brief (and by no means complete) overview of the many different ways in which *Options* can be used.

3.4.1 MSA-Q, Multi-step Actions

Schoknecht [19] together with Riedmiller [20] have proposed a very simple but useful kind of options. The idea is to define 'multi-step' actions of degree n as $A^{(n)} = \{a^n | a \in A^{(1)}\}$ (where $A^{(1)}$ consists of the primitive actions). Multi-step options are options that consist of just repeating the same primitive action n times. The rationale behind adding these multi-step actions is that the agent can make greater steps through the search space during its random walk exploration.

When a multi-step action is executed, the discounted reward is not only applied to the multi-step action that was taken, but also to all the states that were visited during execution, because executing the multi-step action amounts to executing the primitive action in each of the visited states.

Comments

This way of distributing the (discounted) reward can be considered a form of offline learning which is mixed with online learning. Because multi-step actions have a fixed length n only a fixed amount of space is needed to store the states that the multi-step action visits. This principle does not seem to be limited to this approach.

One could re-use one multi-step even further and see if it also contains other smaller options. Obviously an n-step action not only contains n primitive steps but also two $n-1$-option steps, three $n-2$-option steps, and all other intermediates between one n-step and n one-step actions.

This approach could probably be used in virtually any hierarchical approach that *adds* options, though for systems that use hierarchical layers and decompositions, it is often unsuitable. In those systems the primitive actions (lowest level) are not mixed with higher level extended actions and often subpolicies have their own goal conditions. So one cannot always extract 'smaller' options or primitive actions from the extended action that was executed, and thereby re-use the experience. It is of course still possible to mix *offline* and *online* learning in hierarchies because one can always store the entire trace of the current extended action.

[10] The structure presented here is very similar to the approach that Stone and Veloso [26] proposed, though they restricted their (abstract) state spaces for the different layers to subsets of state features/variables from the original state space, so each layer has an input vector (state) that is composed of selected features out of the original state space. Their layers are learned independently and sequentially, the lowest layer first, and each learned layer then provides the actions for the next layer.

Multi-step actions are especially good for reducing the 'random walk exploration' phase in the beginning of learning. Since the options are available right from the start, the agent can use them immediately to move purposefully through the state space, reducing the burden of random walking.

3.4.2 Macro Languages

David Andre [18] has proposed some sort of Macro Language to describe options. This language consists of programming language-like statements like IF and DO . . . UNTIL. The idea is, to try and build these deterministic macro's for each percept (observational variable) that is perceived by the agent during learning. These learned macro's could then be used by the agent in another, similar, environment to speed up learning.

Comments

The problem is that building macro's needs lots of memory because the past needs to be stored. A path needs to be distilled out of the track-record and translated into statements in the language. Much searching through the past seems necessary. Also, using the (stored) past to mine for paths, restricts the algorithm to problems which don't have a continuous (or nearly continuous) state space.

3.4.3 Q-Cut

The Q-Cut algorithm [21] by Menache et al. is a graph-theoretic approach to automatic detection of sub-goals. Using a max-flow/min-cut algorithm and a map of the process history (trace) it identifies bottlenecks in the state space by looking for cuts that divide the problem space. When a bottleneck is found, the algorithm uses replay of experience to learn a policy to reach this bottleneck. The resulting policy is then added as an option.

Comments

Q-Cut extensively uses the discrete nature of the problem because the max-flow/min-cut algorithm works on tree-like structures, so (nearly) continuous state spaces represent a huge problem.

3.4.4 Predecessor-Count

Goel and Huber [22] have proposed an approach[11] similar to that of Q-Cut, but instead of using a graph-theoretic approach, they use the difference in count of the number of predecessors as a measure of whether a state is a bottleneck. A significant increase signifies a bottleneck. Policies are learned to the bottlenecks, and added to the set of actions.

Comments

Unlike Q-Cut, the Predecessor Count seems able to handle large or continuous spaces. The only problem might be that the 'predecessor' function has spikes. This will for instance occur when large (fully) connected areas are connected to each other by very small 'doorways'. The function approximator that needs to estimate the predecessor count might have a hard time approximating the sharp spikes and discontinuities, but this depends heavily on what kind of approximator is used.

3.4.5 Discovering Subgoals Using Diverse Density

McGovern and Barto [24] have proposed yet another method[12] that is similar to Q-Cut and the 'predecessor count'. Their rationale is that when an agent just randomly explores, it is likely to remain within

[11] Without giving it a name, so for clarity I have baptized it 'Predecessor-Count'.

[12] . . . yet another *unnamed* method. Hence for convenience I will call it 'Diverse Density' after the criterion the use for discovering bottlenecks.

the more strongly connected regions of the state space. An option on the other hand should connect separate strongly connected regions, because that is what a bottleneck is in essence. By adding options to these bottlenecks, these separated regions become more closely connected, allowing the agent to more uniformly explore its environment. They reject using the frequency of visit as a measure of how much of a bottleneck some state is, because they deem it too noisy and it is not at all clear how such an approach could be generalized to very large state spaces (when approximators are most likely used)[13].

As an alternative they propose to use diverse density learning. This is a learning approach to the problem of multiple-instance learning. In multiple instance learning [23], we are faced with the problem that we only know that the training example can be represented by one *of a set of given feature vectors* instead of the normal situation that you know that a given feature vector *is* the representation of the training example. The sets of possible features are called bags, and if a bag contains at least one positive instance (i.e. a feature vector that works for the target concept) then it is considered a positive bag, otherwise it is a negative bag. The goal is to learn the concept from the bags that are presented. McGovern and Barto consider the mining of trajectories for bottlenecks an instance of the multiple-instance learning problem. A trajectory is viewed as a bag, and individual observation vectors are considered as the feature vectors. A positive bag corresponds with a successful trajectory; negative bags are unsuccessful trajectories.[14] After some interaction with the environment (and a lot of storing of trajectories) the bags are made, the concept (for that subgoal) is learned, and the option is added.

Comments

Since this 'diverse density' method relies on many examples of successful trajectories, it only kicks in later in the learning process. It does not help in the initial stage of exploration, but is designed to increase the rate of convergence, at least it is in the experiments conducted by McGovern and Barto. It might be interesting to investigate instances of this 'diverse density' approach which are tailored to boosting the exploration phase. But this is like the 'chicken-and-the-egg' problem, because a criterion for success is needed to classify the bags, and what other measure of success can be used than the reaching of the goal of the experiment? So it is clear that this approach can speedup the convergence by simply adding options that leap to bottlenecks that are on the successful trajectories, but it would seem that this approach is less suited to attack the exploration phase itself.

The use of memory (storing trajectories) might make it unsuitable for large state space, because the 'bags' will greatly increase in size.

3.4.6 acQuire-macros

Amy McGovern [25] proposed 'acQuire-macros'. This algorithm looks at peaks in the temporal history of the rewards, and these peaks are then used to form trajectories in a continuing task. When acQuire-macros finds a peak in the reward, it examines the saved clusters of visited states[15] and sees if the cluster that is visited with the peak in reward is a frequently visited region. When such a much-visited space is found, an option is posited that has reaching this space as its goal, and learning continues.

The aQuire-macros algorithm is specially designed for problems with large or continuous state spaces. The trajectories are stored, but they are greatly compressed by using a clustering algorithm.

3.5 Relevant Work with Layers

In this section, several layered approaches to Hierarchical Reinforcement Learning are described, beginning with the most popular (MAXQ and variants, HAM and HEXQ) and after that some approaches

[13] this is a question one can ask for the 'predecessor count' and 'Q-Cut' method, which both rely heavily on discrete states

[14] 'successful' is problem-specific. It might for instance be all the trajectories that eventually reach a subgoal, or perhaps only those that reach it within a predefined time period, etc

[15] This approach is designed for real values continuous states, so it is not possible to register visits to discrete states. Therefore clusters are formed (with k-means or similar algorithms), and visitations to these clusters are registered.

that are more similar to HASSLE, or that are relevant for the design of the new algorithm.

3.5.1 MAXQ, MASH and Cooperative HRL

The first approach, called MAXQ Value Decomposition (MAXQ for short) is arguably the most popular approach to hierarchical reinforcement learning. It was first proposed by Dietterich in 2000 ([27]). MAXQ uses a decomposition of the target MDP into a hierarchy of smaller MDPs and uses a decomposition of the value function of the target MDP into an additive combination of the value functions of the smaller MDPs. MAXQ provides no way of *learning* these decompositions, so it is up to the designer to identify a set of individual subtasks which are deemed relevant to solving the overall task. For each of these subtasks (and of course for the overall task as well) it then needs to be specified which subtasks or primitive actions it can employ. Actions that are not relevant are simply not included in the action set of a given (sub)task. This information can be summarized in a 'task graph' (a directed acyclic graph). Each node is a (sub)task, the leaves are the primitive actions, and the edges denote which subtasks a task may use.

More formally, the MAXQ decomposition takes a given MDP M and decomposes it in a finite set of subtasks $M_0, M_1, \ldots, M_n$ (where M_0 functions as the 'root', i.e. solving MDP M_0 solves the original problem M) and each subtask M_i consists of a tuple $< T_i, A_i, R_i >$. Here T_i denotes the set of termination states, meaning that the execution of the subtask ends there. A subtask can only be executed in states $\notin T_i$. A_i is the set of actions (i.e. a selection of primitive actions and/or other subtasks), and R_i is a *pseudo-reward function*. This function assigns 0 to all non-terminal states, and typically also to the terminal states that are considered (sub)goal states. The pseudo reward for entering a non-goal terminal state is negative. This function is only used during the learning process.

Primitive actions are simply considered 'primitive subtasks' that always terminate with reward 0, *but* can also always be executed[16].

The subtasks can have formal parameters[17] and subtasks with different parameters are considered different subtasks. These parameters can therefore also be viewed as being part of the name of a certain task. For each parameter, a different subtask is learned. The subtasks are pushed on a stack (similar to stacks in normal programming languages). Because of this stack the hierarchical policy can be non-Markov with respect to the original MDP, since the contents and order of the stack provide a means to store some extra information that was not available in the flat MDP.

A hierarchical value function $V^{\pi}(< s, K >)$ is then defined which gives the expected reward for policy π starting in state s with stack-contents K. Also $V^{\pi}(i,s)$, the projected value function of policy π on subtask M_i is defined. This is the expected reward of executing π starting in s until M_i terminates. The MAXQ value decomposition tries to decompose $V(0,s)$ in terms of the projected value functions of the subtasks.

The learning happens using a Reinforcement Learning like update rule on a quantity called the *completion function* $C^{\pi}(i,s,a)$ which is the expected discounted cumulative reward for completing subtask M_i after invoking the subtask M_a in state s. This completion function makes it possible to express the Q function recursively as $Q^{\pi}(i,s,a) = V^{\pi}(a,s) + C^{\pi}(i,s,a)$.

At each time step t during learning the value $V_t(i,s)$ needs to be calculated, and therefore a search through the entire decomposition three. While this is not a problem for small trees, this is computationally intensive for larger ones.

Only the global structure of MAXQ is described here, and much of the detail is left out. But this description gives a sense of the way MAXQ works. The programmer decomposes the problem into a tree of subtasks (with their terminal states and pseudo-reward functions) which she deems useful. The structure of the problem is therefore (roughly) given in terms of tasks and subtasks. The agent needs to learn the value function for each of the subtasks (and thereby for the entire task).

[16] So T_i does not specify the complement of the states in which the primitive subtask can be selected.

[17] So the subtasks defined above are in fact *unparameterized subtasks*.

Multi Agent Approaches: MASH, Cooperative HRL

Several researchers have created multi agent extensions for MAXQ. Mehta and Tadepalli [28] have proposed MASH ("Multi Agent Shared Hierarchy") which is a multi agent extension to HARL ("Hierarchical Average-reward Reinforcement Learning"), which in turn is their extension of MAXQ.

Ghavamzadeh, Mahadevan and Makar [29] have also extended the MAXQ framework to a multi agent situation, which they call "Cooperative-HRL"[18]. The idea is, to define certain levels as "cooperative". These "cooperation levels" are the levels on which the agents receive information about each other. The benefit of only letting the other agents know what you are doing on a higher level, is that the lower levels are independent. On the lower level it does not matter what the other agents are doing, the agent only has to complete its own subtasks.

Comments

Moving communication towards higher levels is of course only practical when cooperation is only needed at the higher level, and the task can be decomposed in subtasks that don't depend on the other agents. If, for instance, the agents are game characters which are to attack two enemies using magical spells, it might be that their coordination is essential on the lowest level because their combined attacks are more effective than when uncoordinated.

Suppose that two spells cast together (coordination at each low level time step) are stronger than when each each agent is just attacking the enemy as if he was alone (only coordination at the level of "*let's get that enemy now*" but not at each individual time step). This can be the case when a mighty spell ("*fire!*") is cast by one character, but at the same time the other casts the opposite ("*water!*"). This will certainly result in lots of steam, but besides that the actions have effectively canceled each other out.

So Cooperative-HRL can most effectively be used when cooperation can be pushed upward the task graph, because each level that requires cooperation is a level that has an explosion in the number of states (because the states now become the Cartesian product of the states of all the agents).

This approach is not restricted to MAXQ like frameworks. It can as easily be incorporated in for instance the HASSLE or HABS framework or any other system where higher layers denote higher abstractions and longer temporally extended actions.

3.5.2 HAM - Hierarchy of Machines

Parr and Russell [34] have presented an approach to Reinforcement learning which is based on constraining the learning policies by hierarchies of partially specified machines. HAMs are nondeterministic finite state machines, whose transitions may invoke lower-level machines. The machines can be fully specified (in which case no learning takes place and all work needs to be done by the designer) but more useful is the specification of the general organization of behaviour into a layered hierarchy.

The machines are specified by a set of states, a transition function and a start function (which determines the initial state of the machine). There are several types of states: *action* states execute an action in the environment, *call* states execute another machine (using it as a subroutine), *stop* states stop execution and return control to the previous call state, and *choice* states are places where the machine non-deterministically selects a next state.

Learning this policy is achieved by (for instance) a variation on Q-Learning. The Q-table contains $(state, machine)$-pairs instead of just the states, and there is a cumulative discount for the gained rewards.

Comments

The hierarchy of machines in effect reduces the search space, because it constrains the policy to those policies that are possible within the constraints of the machines. This reduction decreases the "random

[18] Prefixed with "COM" for their further extension which incorporates communication in the framework.

walk exploration" phase leading to faster knowledge about the environment.

HAM can be considered the middle ground between reinforcement learning systems and teleo-reactive approaches like RL-TOPs. It specifies plans (partial machines) like RL-TOPs does, but on the other hand it incorporates these machines in a MAXQ-like hierarchical structure.

3.5.3 CQ, HEXQ and concurrent-HEXQ

In 2002 Hengst [31] proposed HEXQ, which tries to find (and exploit) repeating sub-structures in the environment. HEXQ tries to automate the process of finding a task hierarchy instead of relying on the programmer to define it, as MAXQ requires. His HEXQ approach is similar to his CQ algorithm [30] that was proposed a few years earlier.

The idea is to sort the variables in the state vector according to the frequency of change. Faster changing variables belong to a lower level and slower changing variables to a higher level. The algorithm will then look for transitions that are not invariant with respect to the slower changing higher level variables. Such transitions are considered 'exit points' where a lower level subtask may enter a new domain and is finished. As an example, the "pickup" action in Dietterich's taxi-domain is given. In the taxi-world, the fastest changing variable is the position, and the "*has passenger*"-variable changes much more infrequently. The "pickup-action" may or may not succeed in picking up a passenger at the pickup-location (because she may or may not be there) but if it succeeds, the slower changing variable ("*has passenger*") is changed. Therefore the 'pickup'-action at the 'pickup-location'-state is a state-action pair that could lead to a new situation on the level of "*has passenger*", so this state-action-pair is considered an "*exit*".

More precisely, an *exit* is a state-action pair where (1) the transition is non-Markovian with respect to the state at this level (2) a higher state variable may change or (3) the current subtask or overall task may terminate. The *exits* are viewed as the subgoals for the subtasks at the next level, so that in fact a sub-MDP is defined for each exit where the corresponding exit-state and action lead to a single terminal state. This MDP can then be solved by dynamic programming or reinforcement learning. This policy is then added as a subtask to the next layer.

The original HEXQ-algorithm first completely finishes the decomposition and then goes on to learn using Reinforcement Learning. However, it might not be desirable to wait with the learning process until the decomposition is finished, but to intertwine learning and decomposing. This is what concurrent-HEXQ ([32], [33]) does. It specifies a mechanism for fixing bad decompositions on the run, and allows learning while the decomposition is not yet complete.

HEXQ and its concurrent modification both use multiple layers in their hierarchy. The layers consist of the state variables and are ordered with respect to their frequency of change. From this ordering and the application of some statistics (to identify non-Markovian 'exits') the algorithm builds a MAXQ-like subtask graph.

3.5.4 HQ-Learning

Wiering and Schmidhuber [35] take a different approach to hierarchical reinforcement learning with their HQ-Learning algorithm. Instead of adding options or introducing layers, they regard the problem as a multi-agent problem.

A sequence of agents (each with their own policies) is defined, and each of these agents will be used once in solving the problem. Each of the agents is allowed to select its own subgoal (the observation it wants to see) and tries to reach this, starting with the first agent. After the agent finishes, control is passed over to the next agent and the process repeats itself, until the overall goal is reached (or timeout occurred). After this[19], each agent adjusts its own policy *and* its *HQ-table* (the subgoal selection policy which each agent has) using Q(λ)-learning[20].

[19] Wiering et al use off-line learning, but they note that in principle on-line learning during the episodes could also be used

[20] Q(λ)-learning is Q-learning with a trace. Visited state get a small (decaying) trace which is used in the updating rule to increase convergence. See for instance [6] for details.

The strength of this architecture is that this HQ-structure is able to solve partially observable Markov decision problems (POMDP), i.e. problems where the same state (input) occurs in different parts of the environment, and different actions are required at these different places. This means that the agent would need memory to differentiate between two states with the same input but with different history[21].

So where exactly is this memory in the HQ-algorithm? It is implicit in the sequence of the agents. Because the problem space is divided between the agents, it is possible that two agents learn to behave differently on the same state (input) because these same inputs occur at different stages of the problem, and each agent just specializes in solving its small part. So the memory is located in the pointer to the active agent.

This implicit memory is formed when the agents learn what subgoal to select (i.e. learning their HQ-tables). Better combinations of sequential subgoals will result in higher rewards and will tune the HQ-tables to the sequence needed to solve the problem efficiently. This way the agents cooperate without explicit communication.

Learning of the lower level Q-values is rather straightforward, only with the exception that agents can also learn from the Q-values of the next agent: when control is handed over to the next agent (because the previous agent has reached its subgoal) this action also needs to be rewarded. This means that the Q-values for each agent are global, they are expected rewards for the entire task, not for completing the localized behaviour of the agent in question. The HQ-values are updated in the same way as the Q-values for the lower level. The values in the HQ-table represent the discounted reward that the agent can expect when choosing a certain subgoal.

The authors note a disadvantage of the sequential nature of HQ. This is a structure that will work on achievement tasks where a goal needs to be reached (for example navigation tasks, search tasks, etc) but not for maintenance tasks where a desirable state needs to be kept for a longer time. In the latter case, HQ will eventually run out of agents. Other architectures than the sequential one might provide a solution here.

Comments

It is interesting to note, that the hierarchy in HQ is not used to speed up learning as such, but to facilitate learning problems that require memory, i.e. that require the agent to keep track of certain events in the past. This can be seen from the fact that HQ generates a policy that can solve POMDP tasks which need memory. Approaches like MAXQ, HEXQ or RL-TOPs on the other hand, give policies for problems that could also be solved (albeit much more slowly) using a flat reinforcement learning algorithm.

In effect HQ makes a MDP from the POMDP by introducing a sequence and therefore a memory. The problem could probably also be made MDP by changing the states to include (some) history, but when it is not clear what history to incorporate and what not, the risk of greatly enlarging the problem space by adding many extra variables to the state is high. HQ does not add variables to the problem but introduces a sequence of agents which then cuts the problem into pieces that are themselves MDPs and an overall structure (sequence and HQ-table) which is also a MDP.

Because policies (agents) are sequential, HQ cannot get any gain in the exploration phase from re-use of behaviours. It does not matter whether nearly all of the subtasks are identical, each agent has to learn this task on its own. This hampers learning in tasks where hierarchy is usually employed, i.e. where much ground needs to be covered in different parts of the search space that resemble each other.

3.5.5 Feudal-Learning

Dayan and Hinton [40, 41] proposed an algorithm that resembles the medieval feudal fiefdom. Managers are given total control over their sub-managers, and can order them to do tasks and give punishments

[21] This is often explained using the traffic light problem: suppose someone gives directions saying: *"go left after the second traffic light"*. Given the fact that both lights look exactly the same, only the use of memory (counting traffic lights) will bring you to your destination.

and rewards just as they desire, but on the other hand managers have to obey their superior manager, and so it goes all the way to the top.

The development is guided by two principles. The first is *reward hiding*, which means that managers have to reward (or punish) their sub-managers for doing what they are commanded *whether or not* that satisfies the commands of super-managers or is according to the overall goal. Sub-managers just need to do as they are told, and are therefore rewarded if they achieve what was commanded even if this does not help the manager that commanded it in furthering its goals.

The second principle is *information hiding*. The manager does not need to know the details of what its sub-managers are doing. It also does not need to know what the goals of its superior are. Only the command that needs to be executed is known to a manager, and its superior needs to know how to evaluate the results in order to be able to give the appropriate reward.

At each layer managers are assigned to separable parts of the state space. Each layer of the hierarchy views the state space at a coarser level, so higher levels have smaller numbers of managers[22]. Each manager adds the commands from its superior to its state, so behaviours for different commands can be learned. At each level there is only a limited set of predefined commands available (except at the lowest level, where the primitive actions are used).

In their example task, these commands are just the instructions to move into one of the four cardinal directions. This is because their task is navigation in a maze where the coarser view of the state space is accomplished by taking blocks of two by two lower level states together as one higher level state, all the way up to the top where there is only one abstract state left.

Comments

Feudal-Learning can speed up the exploration phase because the lower level managers will learn their behaviours early in the exploration phase. These learned behaviours are then used by the higher levels to make larger steps, thereby increasing the distance that can be covered by random walk exploration.

Feudal-Learning does – at least in the simple form presented – not re-use behaviours. This could of course be fixed by implementing some sort of structure that associates behaviours with managers that can use that behaviour. This would obviously mean extra structures to learn.

Because the structure of the states is known on all levels, the reward conditions for the actions ("commands") on each level are available. This is important to note, because it means that for each state (on a certain level) the states that can be reached directly, can be considered as its subgoals. So each manager can be viewed as a collection of behaviours (reactions to commands from the superior) for reaching the subgoals (the adjacent higher level states). Considered this way, Feudal-Learning is a hierarchical structure where all the subgoals are identified before learning starts, due to the fact that the problem space is known on different levels of detail.

When this information is freely available, there is no reason not to use it, but in cases where the state space is not known, Feudal-Learning is of no use, because the hierarchical structure is not model free. Unless of course (part of) the structure can be learned during execution.

3.5.6 RL-TOPs

Ryan, Pendrith and Reid [36, 37] have proposed a hybrid system which combines teleo-reactive planning and reinforcement learning, called RL-TOPs. They observe that for many robotic tasks, the state space is vast because there are so many (sensory) inputs. Task decompositions like the ones used in MAXQ or HEXQ and similar approaches are based on geometric considerations, and seem, according to Pendrith and Reid, ill-equipped to deal with high-dimensional sensory information without simple uniform geometry. A better approach would be a *subsumption*-like architecture as proposed by Brooks[38]. A *subsumption architecture* has several separate (hierarchical) learning modules which learn their tasks independently. The problem they identify with Brooks' architecture is that it is *hand coded*.

[22] Only one manager on top.

Ryan et al. propose a system that resembles a subsumption architecture but uses a planner instead of hand coding on the upper level. They use a planner called Teleo-Reactive Planning System (TR) which is based on the notion of a *teleo-operator* (TOP) $a : \pi \rightarrow \lambda$. The TOPs are (temporally extended) behaviours a that have a pre-image π and post-condition λ (both conjunctions of predicates from the planner's state description language). This means that if a is executed while π is true, eventually λ will become true.

Teleo-reactive plans are represented as *TR-trees*. The nodes are state descriptions (the root is the goal) and connections between the nodes are labelled with actions, denoting that if the action shown is executed in the lower node, then the result is the upper node. TR-trees are continuously re-evaluated, and the action corresponding to the shallowest *true* node is selected.

The TOPs have another important function besides planning. They can also be used as descriptions of reinforcement learning problems for the lower level. The behaviours (i.e. subpolicies) that are specified by the TOPs are learned using reinforcement learning, and the success or failure of a TOP is defined by its post-conditions.

Comments

RL-TOPs depends on a planner for its upper layer, which means that knowledge from the domain is needed, because suitable (pre- and post) conditions for the lower level behaviours need to be specified. For this, the dynamics of the domain needs to be known. This is a disadvantage because in many reinforcement learning domains the environment is not deterministic or even completely unknown.

3.5.7 Self-Organization

The work of Takahashi and Asada [39] needs to be mentioned here. Their algorithm does not have a name, only the description "Behavior acquisition by multi-layered reinforcement learning". They propose a multi-layered system that organizes itself, to avoid all the work that needs to be done when the designer needs to specify each and every subtask and subgoal.

They regard the neural networks[23] that represent the subpolicies, as experts. The behaviours self-organize as each one tries to become an expert on a different part of the problem space (the sensory input).

The higher level state is defined as a vector consisting of $\max_a Q(s,a)$ for each behaviour (s of course being the current state of the robot), so the higher level state vector actually is an indication of how close each behaviour is to reaching its own subgoal, when the robot is in state s. This is because the value $\max_a Q(s,a)$ is a measure for how close state s is to the subgoal of a policy (at least in navigation- and search tasks etc).

The higher level uses these behaviours as its actions, but not in the usual way. The outputs of the higher level are vectors, and each element of the vector specifies the desirability for one of the behaviours. This vector could be seen as a vector of Q-values for all behaviours available in a given higher level state. So in each high level layer the input to a module is a vector consisting of a measure of progress for all the behaviours one level below, and the output is a vector that denotes the expected returns for the behaviours.

The behaviours need *subgoals* to be able to learn anything useful at all, but since no external goals are given and no task decomposition is made *a priori* , the system needs to identify its own subgoals. This self organization is done by trying to distribute the subgoals of the behaviours roughly uniformly. This means that the subgoals need to be assigned and updated online.

The value function $\max_a Q(s,a)$ is used as a measure of distance of a certain behaviour to the goal. Each behaviour then needs to shift its subgoal (a state in the state space) to a region where the maximum of the Q-values of other behaviours is low – and if needed, new behaviours are added or behaviours too close to each other deleted. So the subgoals move around and become separate from each other, each covering another part of the state space.

[23] They use Recurrent Neural Networks, but the same holds if other function approximators are used.

Comments

Because the behaviours each try to avoid subgoals close to each other, they will strive to cover the problem space uniformly. This means that no (or only little) *a priori* information needs to be incorporated. If *a priori* information is available, it can be used to fix the goals of some of the behaviours, and these behaviours can then be treated just as the others that don't have a fixed goal.

The problem with this approach is, that there is no real feedback between the higher and lower level, so there is no feedback to drive the subgoals in useful directions. Subgoals are forced apart (as if they repel each other) because they seek areas where the other value functions have low values, but this means that many of them could end up in utterly uninteresting parts of the problem space, because they are driven outward instead of towards useful subgoals that are perhaps near to other subgoals.

Chapter 4

HASSLE — Using States As Actions

The HASSLE algorithm was proposed by Bakker and Schmidhuber [1, 2]. It stands for *Hierarchical Assignment of Subpolicies to Subgoals LEarning*. HASSLE is the starting point for the new algorithm proposed in this thesis.

4.1 HASSLE — *Hierarchical Assignment of Subpolicies to Subgoals LEarning*

HASSLE is a layered structure that uses higher level states (subgoals) that are abstractions of the lower level state space. Furthermore it has *a priori* uncommitted subpolicies that are dynamically assigned to the task of reaching subgoals.

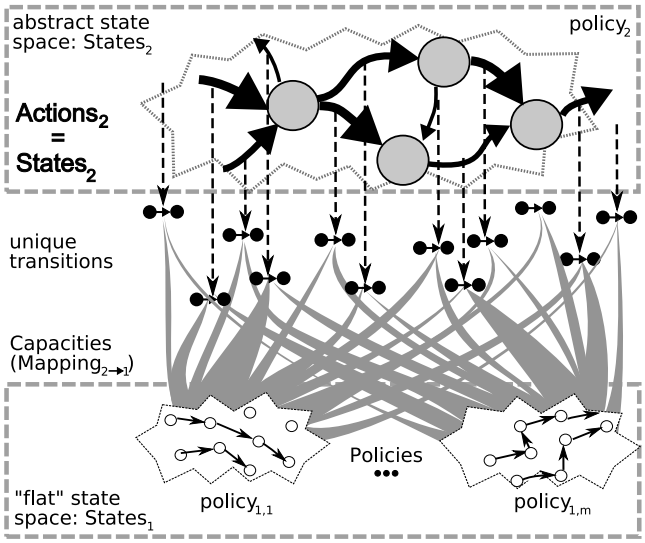


Figure 4.1: The hierarchical structure of* HASSLE: the large gray circles are the high level states (subgoals). The Q-values of the high level policy are indicated by the thickness of the black arrows between the subgoals. The •→• represent the (unique) transitions between subgoals. The Capacities ($Mapping_{2\rightarrow 1}$) are represented by the thick gray lines (thicker means higher capacity). Notations correspond to those in section 4.1.1.***

On the higher level, these subgoals act both as the states for the Reinforcement Learning policy, and also as the actions. This means that every abstract higher level state is a subgoal, but not that there is one unique policy for each subgoal. There is a fixed number of subpolicies available and the algorithm learns which subpolicies are best assigned to reaching a certain subgoal (starting from another subgoal). For each subpolicy, HASSLE keeps track of its ability or capacity to execute a transition between two subgoals. This is done by a mechanism called *Capacities* (see fig. 4.1).

4.1.1 HASSLE Formalized

HASSLE for an agent using 2 layers[1] (layer $i \in \{1,2\}$) is described by the following:

- sets of states $States_i = \{state_{i,1}, state_{i,2}, \ldots\}$:
  $States_1$ is the set of primitive states and $States_2$ is a set of abstract (higher level) states (i.e. subgoals). Obviously $|States_2| < |States_1|$. $States_1$ can in principle be continuous. $States_2$ needs to be discrete (see section 4.3.1).

- sets of actions $Actions_i = \{action_{i,1}, action_{i,2}, \ldots\}$:
  $Actions_1$ is the set of primitive actions, and $Actions_2 = States_2$ (because subgoals are used as high level actions).

- sets of (sub)policies $Policies_i : \forall_k \left(policy_{i,k} \in Policies_i : States_i \times Actions_i \rightarrow \mathbb{R} \right)$:
  Standard Reinforcement Learning (e.g. Q-learning or Advantage Learning) policies[2] for each layer. The top layer has only one policy (so $Policies_2 = \{policy_2\}$), but the lower level has m_1 policies ($Policies_1 = \{policy_{1,1}, \ldots, policy_{1,m_1}\}$), each specializing in different subtasks. Only one (sub)policy is active at each level at each time step.

- *AgentInternals*: variables describing the observations and memory of the agent: $currentState_1 \in States_1$, $currentState_2 \in States_2$, $currentAction_1 \in Actions_1$, $currentAction_2 \in Actions_2$ and indicators for detecting timeouts on both levels: $timeout_1$ and $timeout_2$ and which policies are active on each layer.

- $Mapping_{2\mapsto 1} : States_2 \times Actions_2 \rightarrow Policies_1$:
  A mapping from pairs of higher level states and actions to lower level policies that are able to execute the requested action in the given state. This is a way to get an appropriate subpolicy when a certain transition between subgoals (a high level action) is selected.

- stop conditions $Stop_i : AgentInternals \rightarrow \{terminate, continue\}$:
  Determine whether a (sub)policy has reached termination conditions.

$$Stop_1 = \begin{cases} terminate & \text{If } timeout_1 \vee (S \Rightarrow S' \wedge S, S' \in States_2 \wedge S \neq S') \\ continue & \text{otherwise} \end{cases}$$

$$Stop_2 = \begin{cases} terminate & \text{If } timeout_2 \vee \text{the agent reaches its goal} \\ continue & \text{otherwise} \end{cases}$$

- reward functions $Reward_i : AgentInternals \rightarrow \mathbb{R}$:
  $Reward_1$ is 1 if the agent reaches the $state \in States_2$ that was selected by $policy_2$, and 0 otherwise (during execution, after reaching the wrong subgoal or after timeout). Note that this is an internal reward, i.e. is not received from the environment.

$$Reward_1 = \begin{cases} 0 & \text{If } Stop_1 = continue \\ 1 & \text{If } Stop_1 = terminate \wedge S \Rightarrow G \wedge S, G \in States_2 \\ & \quad \wedge \ G \text{ was selected by } policy_2 \\ 0 & \text{If } Stop_1 = terminate \wedge S \Rightarrow X \wedge S, G, X \in States_2 \\ & \quad \wedge \ G \text{ was selected by } policy_2 \wedge G \neq X \\ 0 & \text{otherwise} \end{cases}$$

  For $Reward_2$ the accumulated rewards that the environment gives the agent during execution of a subpolicy, can be used because they are related to solving the overall problem[3].

$A \Rightarrow B$ indicates a transition from state A to B.

[1] The unmodified form of HASSLE is not extensible to more layers, see section 4.3.3. This somewhat mathematical formulation is not in the original papers. Nevertheless I think it will make the structure of HASSLE clearer.

[2] These policies also require that the *previous* state and action on both levels is stored.

[3] in sparse reward tasks this will amount to 0 when the goal is not reached, and $reward_{goal}$ when it is reached.

Control Flow

The high level policy $policy_2$ runs until $Stop_2$ indicates termination. During execution, $policy_2$ selects actions G to execute (i.e. a subgoal to goto) when in a subgoal S. Using $Mapping_{2\mapsto 1}(S,G)$, a subpolicy $policy_{1,active}$ is selected which must accomplish the transition $S \Rrightarrow G$.

Control is passed to $policy_{1,active}$, which is executed until $Stop_1$ indicates termination. At each time step during execution the subpolicy $policy_{1,active}$ is updated with a 0 reward according to reward function $Reward_1$.

After $policy_{1,active}$ terminates ($Stop_1 = 1$) it is rewarded according to $Reward_1$. For reaching the correct subgoal it is rewarded, for reaching a wrong subgoal or failing to reach anything at all (timeout) it is punished.

The agent then substitutes the subgoal X that it has reached for subgoal G that it was trying to reach (see section 4.2.3 for explanation).

The higher level $policy_2$ is updated according to the $Reward_2$ (the accumulative reward) for selecting (and reaching) subgoal X in subgoal S and the $Mapping_{2\mapsto 1}$ is updated with the new information that $policy_{1,active}$ was (not) able to reach subgoal G[4].

The control flow is illustrated in algorithm 4 and the relation between all the components in fig. 4.1.

4.1.2 Capacities

HASSLE needs to learn the associations ($Mapping_{2\mapsto 1}$) between $level_2$ states and actions and $level_1$ policies, when we want to be able to reuse policies and avoid needing a new policy for each new combination of a $level_2$ state and action.

HASSLE was originally proposed with a specific way to do the mapping $Mapping_{2\mapsto 1}$, called *"Capacities"* or *"C-Values"*. The idea is that for every policy its capacity to do a transition from one subgoal to another, is learned. This is done by storing an average performance.

Every $level_1$ policy ($policy_{1,j}$) has its own table of so-called *Capacities*. The entries in these tables are values that denote the capacity of the policy to perform the required behaviour, i.e. to reach $level_2$ action A when in $level_2$ state S.

When in a $level_2$ state, the $level_2$ policy selects a new $level_2$ action and then needs a $level_1$ subpolicy to execute this action. It can select a $level_1$ subpolicy based on the Capacities of all the $level_1$ subpolicies, for instance with Boltzmann exploration. If this selected $level_1$ subpolicy indeed succeeds in reaching the selected $level_2$ action, its Capacity is increased, otherwise it is decreased.

The Capacities are of the form:

$$C_{2\mapsto 1} : States_2 \times Actions_2 \rightarrow \mathbb{R}^{\|Policies_1\|} \tag{4.1}$$

where $C_{2\mapsto 1,k}(hlState, hlAction)$ denotes the capacity of $subpolicy_{1,k}$ to accomplish $hlAction$ in subgoal $hlState$. A selection mechanism $Select : \mathbb{R}^N \rightarrow Policies_1$ (for instance Boltzmann selection) can be used to select one of the policies, thereby insuring exploration and exploitation: $Mapping_{2\mapsto 1} = Select \circ C_{2\mapsto 1}$.

The Capacities are updated after the subpolicy terminates. As a measure of performance, the exponential function $\gamma_C^{\Delta t}$ with $0 \leq \gamma_C < 1$ is used to ensure that the performance is between *zero* and *one* and that a longer execution time (Δt) means a lower performance. The Capacities are updated according to the following equations:

$$\begin{aligned} C_{i,act}(start,goal) &\leftarrow C_{i,act}(start,goal) + \Delta C_{i,act}(start,goal) \\ \text{with} \quad \Delta C_{i,act}(start,goal) &= \begin{cases} \alpha_C^r \cdot \left(\gamma_C^{\Delta t} - C_{i,act}(start,goal)\right) & \text{success} \\ \alpha_C^f \cdot (0 - C_{i,act}(start,goal)) & \text{failure} \end{cases} \end{aligned} \tag{4.2}$$

where i is the level, act is the number of the active subpolicy, $start$ and $goal$ are the current high level ($level_2$) state and selected action (subgoal), where α_C^r and α_C^f denote the learning rates for *reaching*

[4]This could be extended to updating $Mapping_{2\mapsto 1}$ with the fact that $policy_{1,active}$ was able to accomplish the transition $S \Rrightarrow X$, but this is not done in the original HASSLE algorithm.

Algorithm 4: **HASSLE in pseudo code**: updating the policies is done with standard Reinforcement Learning. **RLUPDATE (…)** updates a policy with your favourite Reinforcement Learning algorithm (see section 2.3). "NULL" indicates that a value is unknown.

HASSLE ::

while $(\neg Stop_2)$ **do** // Policy_2 (high level) loop
  $accumReward_2 = 0$; // For high level reward
  agent is in subgoal $S \in States_2$;
  $policy_2$ selects subgoal $G \in Actions_2$; // Select HL-action "goto G"
  $Mapping_{2\mapsto 1}(S,G)$ selects $policy_{1,active}$; // Select $(\text{sub})\text{policy}_{1,active}$
  while (TRUE) **do** // $(\text{Sub})\text{policy}_{1,active}$:(low level) loop
    agent is in state $s_t \in States_1$;
    $policy_{1,active}$ selects primitive action $a_t \in Actions_1$;
    agent executes action a_t and receives reward $receivedReward_t$;
    $accumReward_2 \leftarrow accumReward_2 + receivedReward_t$; // Accumulate
    if $(Stop_1)$ **then BREAK**; // Terminate subpolicy
    else RLUPDATE ($policy_{1,active}, s_{t-1}, a_{t-1}, 0, s_t, a_t$**)**; // Sparse rewards
  end
  // See whether $\text{subpolicy}_{1,sel}$...
  determine current subgoal $S^+ \in States_2$; // ...needs to be...
  if $(S^+ = G)$ **then RLUPDATE (**$policy_{1,active}, s_{t-1}, a_{t-1}, 1, s_t$, NULL**)**; // ...rewarded
  else RLUPDATE ($policy_{1,active}, s_{t-1}, a_{t-1}, 0, s_t$, NULL**)**; // ...or punished
  substitute S^+ for G; // For explanation: section 4.2.3
  update $Mapping_{2\mapsto 1}(S,S')$; // Update mapping...
  RLUPDATE ($policy_2, S^-, S, oldAccumReward_2, S, S^+$**)**; // ...and policy_2
  $S^- \leftarrow S$; // Save vars for...
  $S \leftarrow S^+$; // ...next iteration
  $oldAccumReward_2 \leftarrow accumReward_2$;
end

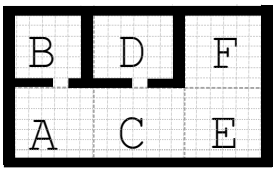
RLUPDATE (POLICY p**, STATE** s_{t-1}**, ACTION** a_{t-1}**, REWARD** r_t**, STATE** s_t**, ACTION** a_t**) ::**

switch $(favourite\ Reinforcement\ Learning\ algorithm)$ **do**
  case (*Q-Learning*) // See section 2.3.3
    update $Q(s_{t-1},a_{t-1}) \leftarrow (1-\alpha)Q(s_{t-1},a_{t-1}) + \alpha(r_t + \gamma \max_a Q(s_t,a))$;
  end
  case (*Advantage-Learning*) // See section 2.3.4
    update $A(s_{t-1},a_{t-1}) \leftarrow$
    $(1-\alpha)A(s_{t-1},a_{t-1}) + \alpha\left(\max_a A(s_{t-1},a) + \frac{1}{k}\left(r_t + \gamma \max_{a'} A(s_t,a') - \max_a A(s_{t-1},a)\right)\right)$;
  end
  ⋮
end

the selected $level_2$ goal (i.e. *success*), or not (i.e. *failure*) and $\gamma_C^{\Delta t}$ (with $\gamma_C \leq 1$) is a measure of success. Shorter Δt mean that the agent accomplishes the transition faster and this leads to a higher capacity. Using two different learning rates (α_C^r and α_C^f) allows for more fine tuning. If the subpolicy terminates because it reached its designated goal, it is updated towards $\gamma_C^{\Delta t}$ (equation 4.2–success), but if it reached another subgoal (or if it failed to get out of the current subgoal) the average performance is moved towards 0 (equation 4.2–failure)

4.1.3 A Simple Example

To get an idea of how HASSLE works, we'll examine a simple example (fig. 4.2). Suppose we have an agent which inhabits a *grid world* and which can move in the four cardinal directions. The high level consists of clusters of lower level states.



$$
\begin{aligned}
states_2 &= \{room\,A, room\,B, \ldots, room\,F\} \\
actions_2 &= \{(goto)\ room\,A, (goto)\ room\,B, \ldots, (goto)\ room\,F\} \\
Actions_1 &= \{North, East, South, West\} \\
States_1 &= \{all\ grid\ cells\}
\end{aligned}
$$

Figure 4.2: The Example Grid World

Simplification – No Reuse

We use as many policies on $level_1$ as there are combinations of $level_2$ states and actions (there is only one high level policy, so $Policies_2 = \{policy_2\}$). This means we won't have any policies that are re-used – ignoring one of the strengths of hierarchies, but it will make the structure clearer. So for each pair of subgoals $(room\,X, (goto)\ room\,Y)$ we have a unique policy: $policy_{1,X \Rightarrow Y}$, so the subpolicies are:

$$Policies_1 = \{policy_{1,X \Rightarrow Y} | X \in States_2 \text{ and } Y \in Actions_2\}$$

Where $policy_{1,X \Rightarrow Y}$ denotes the subpolicy that is used for executing the action "$(goto) roomB$" when in the subgoal *roomB*. Each of those $level_1$-subpolicies $policy_{1,X \Rightarrow Y}$ uses $States_1$ and $Actions_1$ as its states and actions.

Since we use a unique subpolicy for each combination of subgoals, $Mapping_{2 \mapsto 1}$ is defined as:

$$Mapping_{2 \mapsto 1}(room\,X, (goto)\ room\,Y) = policy_{1,X \Rightarrow Y}$$

where $X \in States_2$ and $Y \in Actions_2$ are the subgoals.

Learning

If the agent finds itself in room (subgoal) A, it can select one of the actions[5] $(goto)\ room\,B$, $(goto)\ room\,C$, ... Suppose it selects $(goto)\ room\,F$. It then uses $Mapping_{2 \mapsto 1}$ to select a subpolicy that can execute this desired transition. In our simplified case, there is one unique subpolicy for each subtask, so the subpolicy $policy_{1,A \Rightarrow F}$ is selected and the agent hands over execution to this subpolicy. The subpolicy will execute some primitive actions (i.e. from $Actions_1$) and after some time either reach a new high level state, or it times out[6]. During execution, it will (perhaps) receive some rewards, which are summed up (*summedReward*) and will be used for the high level after termination.

After the subpolicy terminates, the agent can observe in what high level state it is, and can actually use this information to determine what high level action it has executed (as explained in section 4.2.3). Suppose our agent actually wandered into room C instead of the – unreachable – room F. The agent can now update the high level policy $policy_2$ with the transition from room A to room C, as if it had actually selected "goto room C" (for example with equation 2.18, Q-learning):

$$Q(room\,A, room\,C) \rightarrow (1-\alpha) \cdot Q(room\,A, room\,C) + \gamma \cdot summedReward$$

[5] For convenience, we don't allow going to the subgoal the agent is already in. This slightly speeds up learning, and keeps the structure more clear.

[6] If no timeout is used, a (bad) subpolicy could go on literally forever by just staying inside the high level state (*looping*).

The subpolicy $policy_{1,A \Rightarrow F}$ always gets a zero reward *during* execution (when it is not terminated), and since it failed in its task to reach room F, it also gets a zero reward (or possibly a negative reward) for its final action. If it would have succeeded, it would have received a positive reward for its final action.

Re-using Policies

Since we used a unique subpolicy for each transition between two subgoals, there are $|States_2| \times |States_2|$ policies, each of which has to be learned separately. With this quadratic relationship, that the number of subpolicies that need to be learned will soon become too large. But by assigning unique subpolicies to each transition, we ignored the fact that many transitions look remarkably similar (for instance $A \Rightarrow B$, $C \Rightarrow D$ and $E \Rightarrow F$ — all entering a room to the north through a small passage) and could probably be executed by the same subpolicy, provided it has enough flexibility.

Some mechanism is needed to learn which combinations of subgoals are associated with which policy, so $Mapping_{2 \mapsto 1}$ needs to be learned. This means that the agent needs to try several of the available subpolicies to see which one(s) are best suited for which high level steps. The mechanism that HASSLE uses, is the Capacities-mechanism (section 4.1.2).

$subpolicy_1$	(to) A	B	C	D	E	F
(from) A	–	**0.97**	0.20	0.01	0.02	0.01
B	0.01	–	0.05	0.03	0.04	0.02
C	0.04	0.03	–	**0.76**	0.02	0.01
D	0.03	0.01	0.03	–	0.00	0.01
E	0.00	0.02	0.01	0.01	–	**0.89**
F	0.00	0.01	0.00	0.01	0.01	–

$subpolicy_2$	A	B	C	D	E	F
A	–	**0.27**	0.17	0.01	0.02	0.00
B	0.00	–	0.01	0.03	0.00	0.02
C	0.20	0.01	–	**0.33**	0.22	0.01
D	0.03	0.01	0.00	–	0.00	0.01
E	0.00	0.02	0.15	0.01	–	**0.99**
F	0.03	0.01	0.03	0.01	0.00	–

⋮

$subpolicy_7$	A	B	C	D	E	F
A	–	0.00	0.01	0.00	0.01	0.02
B	0.03	–	0.03	0.03	0.02	0.01
C	**0.99**	0.01	–	0.01	0.01	0.02
D	0.00	0.01	0.00	–	0.00	0.04
E	0.03	0.00	**0.91**	0.00	–	0.03
F	0.01	0.01	0.05	0.01	**0.41**	–

Table 4.1: Snapshot of a Capacities-table*: note that impossible transitions like $A \Rightarrow F$ can still have non-zero values, because Capacities start at random values, so it takes time to learn that a subpolicy is incapable of reaching a certain (unreachable) subgoal. Also note that a subpolicy can be good in one task, but still perform on other tasks (although mediocre).*

A limited number of subpolicies is created (7 in our example) and each of these gets a Capacities table (randomly initialized) that indicates the capacity of the subpolicy to execute a transition from one subgoal to another. After some time, it might look like table 4.1.

Let's look again at our example and suppose that the agent is in room E and selects high level action *"goto room C"*. This time it needs to figure out which of the 7 subpolicies it will select for this transition. For this it uses the mapping from pairs of high level subgoals to subpolicies ($Mapping_{2 \mapsto 1}$) which is the

combination of a selection mechanism[7] and the Capacities ($Mapping_{2\mapsto 1} = Select \circ C_{2\mapsto 1}$).

The Capacities of the subpolicies for the transition $E \Rightarrow C$ can be found in the table (4.1), and are $0.01, 0.15, \ldots, 0.91$ (the underlined values). Suppose that $subpolicy_7$ is selected (perhaps because of its high capacity): this subpolicy then takes control of the agent and after some time Δt it terminates because the agent ends up in room E. The capacity of $subpolicy_7$ for going to room E starting in C needs to be updated. The current value (0.91) is moved towards the current performance $\gamma_C^{\Delta t}$. So if (for example) the agent reached the new subgoal in 5 steps (Δt) and $\gamma_C = 0.99$, then the performance is $\gamma_C^{\Delta t} = 0.99^5 = 0.951$ and the update is (equation 4.2 in section 4.1.2):

$$\begin{aligned} C_{i,act}(start, goal) &\leftarrow C_{i,act}(start, goal) + \alpha_C^r \left(\gamma_C^{\Delta t} - C_{i,act}(start, goal)\right) \\ &\leftarrow 0.91 + 0.01 \cdot \left(0.99^5 - 0.91\right) \end{aligned}$$

After all this, the high level policy can select a new subpolicy to execute.

4.2 A Closer Look at HASSLE

4.2.1 Properties of the HASSLE State Abstraction

Bakker and Schmidhuber don't exactly specify the properties of the state abstractions[8] that they use for HASSLE, but it is clear that abstract states are generated according to the following:

(1) consistent mapping (to): states close together map to abstract states near each other (or the same abstract state)

(2) consistent mapping (from): neighbouring abstract states correspond to nearby regions in the original State Space

(3) a transition in abstract state space is a meaningful change in the original State Space

(4) abstract state space needs to be significantly smaller than the original state space

These properties specify that the mapping that is used to derive the abstract state space, preserves the underlying "geometric" structure (not constrained to a mere "spatial" geometry) in the original state space. This means that points close together in the state space should be near each other in abstract state space, and vice versa (see fig. 4.3).

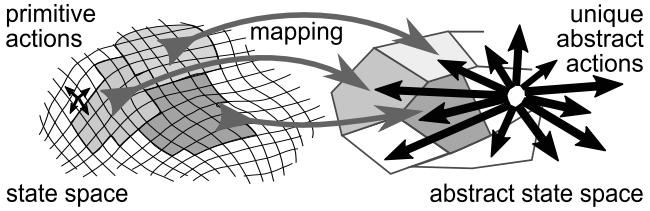


Figure 4.3: Abstract states as used in* HASSLE*: the mapping from state space to an abstract representation preserves (some of the) underlying structure of the state space.

This way the abstract state space and the state space have the same internal structure (at least on a coarse level). Note that it is up to the designer to recognize this structure and use it to create a suitable abstraction of the state space. If no structure exists (or remains hidden) creating an abstraction

[7] For instance Boltzmann or ε-greedy selection.

[8] They use a clustering algorithm to derive their abstract states. But using a clustering algorithm to create abstract states fits with the properties specified here.

of the state space is of little use, because transitions in the abstract state space will then most often *not* correspond to something useful in the original state space, making the abstract state space in question useless for planning or learning on a high level.

How To Get The Higher Level States?

Until now the higher level states are treated as given, but how could these states be acquired? This question is not the subject of this thesis, but it cannot go unnoticed. One suggestion would be to use *pre-fabricated* states, derived from some heuristic that the designer programmed. Other alternatives would be to let the agent discover these higher level states by itself. This could be accomplished if the agent was equipped with some clustering algorithm, for instance ARAVQ (Adaptive Resource Allocation Vector Quantization, [11], [12]).

The higher level states could then be constructed either *before* learning takes place (by just randomly exploring the environment), or the clustering could occur *during* the learning phase. This last option would mean that during learning, new clusters (and therefore new high level subgoals) are discovered and that these new discoveries should from that moment on also be used in the learning process.

4.2.2 Assumptions Behind the Capacities

The Capacities are the bookkeeping mechanism for mapping transitions between subgoals to subpolicies. It is based on two (hidden) assumptions. The first is that there is only a limited amount of different behaviours in the problem. The second that these behaviours can be classified in terms of transitions from subgoal to subgoal.

The first assumption makes sense because if there is (virtually) no similarity between any of the transitions, then as many subpolicies would be needed, as there are transitions. If there is only a limited amount of substantially different behaviours, there is only need of a small set of subpolicies that together cover all the needed behaviours. This gives rise to another property for the HASSLE state abstraction:

(5) there is a limited amount of groups of similar transitions between abstract states

The second assumption is reasonable because we need some way of identifying these behaviours. If the behaviours would not be related to transitions from subgoal to subgoal, the whole idea of trying to describe the problem on a higher, more abstract level, is futile. This second assumption is equal to the third property (*"a transition in abstract state space is a meaningful change in the original State Space"*) that was deducted above (section 4.6).

4.2.3 Error Correction: Replacing Desired with Actual Higher Level States

By replacing the selected high level actions with the actions it actually experienced, HASSLE can make more efficient use of its experience. Suppose that a subpolicy (starting in state *startState*) reached $level_2$ state *reachedState* which is different from the selected subgoal (action *goalState*). HASSLE can simply *replace* the taken action *goalState* (which failed) with the actually executed action (which is of course the reached subgoal *reachedState*). So the high level ($level_2$) policy can lean as if nothing bad happened and as if it had selected *reachedState* all along. This also works for the Capacities, *but* it won't work for the ($level_1$) subpolicy that was selected to reach *goalState*, since obviously this policy has to be trained for what it had been selected to do, and not for what it accidentally did.

The use of subgoals as actions therefore has an interesting side effect: after an action is executed and the new state is reached, it is known which *action* was performed. This knowledge allows for error correction by substitution of what was selected by what is actually experienced. This makes learning on the higher level far more efficient.

Primitive Actions and Motor Errors

When the agent has sensory information that is detailed enough, it can correct for motor errors. If it happened to execute an action other than the one it intended (because of an error in the motor controls), it could calculate what primitive action it *actually* did. This calculated action can then be used for the Reinforcement Learning update, instead of the action it had selected (but did not execute).

However, for the *primitive actions* this would usually not be feasible. The primitive actions are dictated by the structure of the environment and the limitations of the motor controls, but often the information that the agent gets about the environment, is not detailed enough to allow calculation of the action that was taken, and there is no way to correct for motor errors. Only in situations where the sensor inputs contain enough information, this deduction can be made.[9]

Furthermore, most of the time error correction for the primitive actions is not *needed*. If the motor controls are not too error-prone, an occasional error will not greatly hamper learning. So using some sort of error correction would introduce extra calculations which would only result in a very small gain in learning speed. If the motor controls are (for instance) 95% accurate, at most a 5% gain would be possible.

Inherent Error Correction in HASSLE

HASSLE starts with uncommitted subpolicies and needs to learn the Capacities (matching high level actions with subpolicies). Therefore its subpolicies are very different from the primitive actions. HASSLE will make mistakes very often during learning. This could happen when a policy with a high capacity for the given state and subgoal failed to deliver on its promise, or when a policy with a low capacity for the given task was selected because of exploration for the Capacities.

This means that unlike the primitive actions it does not have (at least at the start of learning) a high accuracy for its high level actions. Subpolicies need time to specialize for certain subtasks. This means that the probability that a high level action goes wrong is far greater than the probability that it actually works out alright. HASSLE therefore *needs* error correction to compensate for the many errors its uncommitted subpolicies and its untrained mapping (Capacities) will make in the beginning.

This more efficient use of the available information is made possible because the subgoals are an artificial construct created by the designer. That means that the structure is known to the agent, and it is possible to calculate what kind of high level action it actually did in this artificial structure. Note that this is often not possible in the "flat" state space because it has an *unknown* external environment that we have not created ourselves.

4.3 Problems with the HASSLE Architecture

HASSLEis taken as a starting point for the development of an algorithm that uses self-organizing.[10] This is done because HASSLE already starts with uncommitted subpolicies, has a focus on state abstraction instead of task decomposition and uses local reward functions (see section 4.6).

HASSLE uses subgoals (i.e. abstract states) as its actions on the higher level(s). This certainly seems like a good choice, because having a goal is better than only having some behaviour. Goals can be used in planning, but when we want to use behaviours in that way, the resulting state for each behaviour is needed, so implicitly we would still be using goals.

This idea can be seen in many other approaches listed in sections 3.4 and 3.5. Subtasks or options are defined in terms of reaching some goal state or set of states. So whichever way we look at it, the notion of using (sets of) states as subgoals is a common feature. However, for HASSLE it is also the feature that gives rise to the problems described in this section.

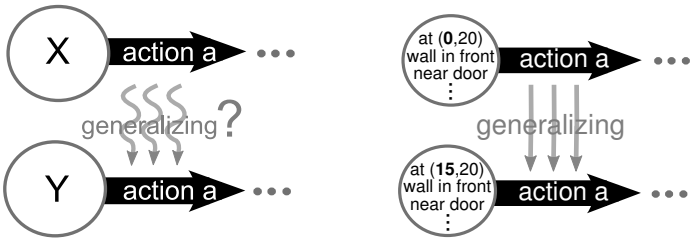
[9] For instance when the agent is a robot that uses a high resolution positioning system like *GPS*.

[10] The self-organizing approach (section 3.5.7) was not known to me at the start of this research. It only came up after I had modified (read: *completely mangled*) HASSLE to create HABS. It turned out that the self-organizing approach has some interesting similarities to HABS.

4.3.1 No Generalization on the High Level

The structure of the Hassle algorithm makes it unsuitable for use of generalization (using function approximators) on the higher level. Function approximators (see section 2.5) are often used when a problem grows too large to handle with discrete tables (for the Q-values).

The ability to generalize depends on how the states are represented.[11] If for instance the states the agent can be in, only have an arbitrary unique number or label as their description, there is no way to compare two different states (see fig. 4.4(a)). That means that if an action works good in one state, there is no pattern to generalize upon to determine whether it will also work in another state. There is simply no common ground between the two states, because their only description contains too little information. So there is little room for generalization in that case.



(a) no generalizing (b) possibility for generalizing

Figure 4.4: (a) No possibility for generalizing*: The best action in state X is action a, and the best action in state Y is also action a. But there is too little information in the state description.* ***(b) Possibility for generalizing****: The best action in both states is action a, but since there is much information about the environment incorporated in the state, a function approximator could for instance learn that being near a door and a wall is relevant for selecting action a.*

However, if the state description contains much information, there is a far greater possibility that a function approximator can extract relevant features and patterns (see fig. 4.4(b)). On that basis a good estimate of the usefulness of a certain action in a certain state can be given.

Generalization on the Lower Level

HASSLE can use a state representation with many variables for its lower level, so it does not have the problem described above. This is just like normal reinforcement learning without any hierarchies. The lower level can always use function approximators. More so, it actually *needs* function approximators because tables have no generalizing capabilities, so using approximators provides a way of re-using subpolicies in other parts of the state space.

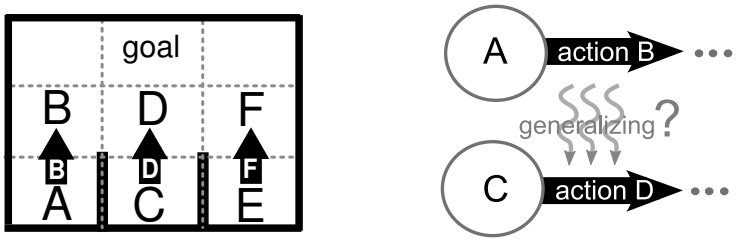
Generalization on the Higher Level — The Q-Values

For the high level Q-values, a different state always[12] means a different action, because the high level states (subgoals) are used also as *actions*. A good subgoal in one part of the problem space, is obviously completely useless in another part, because it is too far away – even though the actual behaviour that is required, might be essentially the same.

The problem lies with the fact that the answer needs to be *absolute*, because subgoals that were useful in similar situations *elsewhere*, are in principle not applicable in the subgoal under consideration. Subgoals are *absolute* designations, they refer to fixed points in the high level state space, and are therefore only applicable in the neighbourhood of those points. Two similar situations in different places

[11] Generalization obviously also depends on the type of approximator. An approximator with good generalizing Capacities is assumed here.

[12] Actually: *nearly* always, because in two adjacent subgoals A and B, the best action can actually be the same subgoal C, bordering on both A and B.



(a) different subgoals (b) no generalizing

Figure 4.5: Different subgoals, different actions*: In high levels states (subgoals) A, C and E the best high level actions (subgoals) are B, D and F.* ***No generalizing****: even though the best high level action ("move North") is the same in all cases, the high level actions (B, D and F) differ.*

in the state space, that require similar behaviour, nevertheless require different subgoals (absolute points) to go to. This means that, even though both high level actions (going to subgoals) can in fact use the same subpolicy, a different high level action is taken. This is illustrated in fig. 4.5(a).

This situation (fig. 4.5(b)) is even one step worse than the case illustrated above (in fig. 4.4(a)). There it was assumed that the actions would be the same in similar situations, *but* the state description contained too little information. In the case of HASSLE, not only do the high level state descriptions contain no information other than a nominal one, but also the *actions* that need to be taken in similar situations, have different designations as well!

The problem of too little information and the absolute nature of each of the high level actions, makes it extremely hard for a function approximator to do the job, because in essence it must emulate a table *without any repetitive features or structure, each entry is unique*. Using an approximator to approximate such a table would probably take much more time than just learning the table itself.

Generalization on the Higher Level — The Capacities

When HASSLE needs to be applied on a large problem, not only the Q-values-table grows. The Capacities-tables also grow quadratically with the number of subgoals (see fig. 4.6).

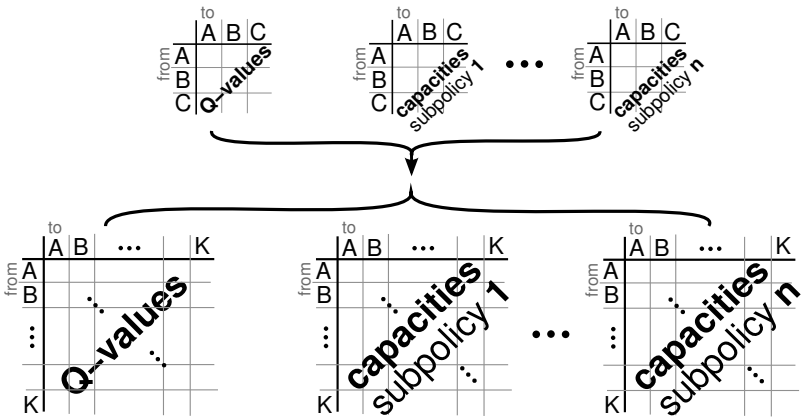


Figure 4.6: **HASSLE** ***– scaling****: both the Q-values table and Capacities tables grow quadratically.*

It would appear at first glance that for the Capacities tables the same problem would hold as with the Q-values and the uniqueness of subgoals. The Capacities map higher level transitions (from subgoal to subgoal) to subpolicies and because each higher level step is unique, it would seem that the Capacities suffer the same problem.

But this is not entirely true – there is a small but significant difference. When a high level action is needed, the Q-values-table needs to return a subgoal, given a certain subgoal (state) as input, but the Capacities table(s) only need to return a subpolicy. As noted before, the *subgoals* are only applicable in

certain neighbourhoods, but *subpolicies* are applicable everywhere (though of course they can specialize in certain regions). The Capacities only have to return one subpolicy out of a small set of subpolicies, and these subpolicies *can be* re-used everywhere in the problem space. Similar situations in different parts of the problem space, can in fact yield the same subpolicy.

It will often be possible to identify patterns in the transitions between subgoals, that always require a certain subpolicy. If for example the situation is as in fig. 4.5(a), then even though subgoals A, C and E are in completely different parts of the problem space, still the step from A to B, from C to D and from E to F are essentially the same and can result in similar Capacities for each of the transitions. So there is room for function approximation here, as long as the high level states are not represented with arbitrary numbers, but with meaningful state representations (coordinates, proximity to walls or doors, etc), in which case we have arrived at the situation depicted in fig. 4.4(b) again.

Even though the Capacities can probably be generalized when good (higher level) state representations are used, the higher level Q-function cannot use generalization even in principle because of its goal directed and absolute nature. Therefore HASSLE cannot use function approximators on its high level, but is constrained to use tabular representations.

4.3.2 Action Explosion on the High Level

When a problem scales up, there is in general no upper limit for the number of lower level states, and therefore also no upper limit for the number of abstract higher level states for HASSLE. At first one might wonder what the difference is between this problem on the higher level and on the lower level, since the problem size increases on both levels, so why would this increase in problem size be a problem specifically for the higher level, more than it is for the lower level? *The difference lies in the fact that on the higher level states and actions are the same.* The abstract states, the subgoals, are used both as states and as actions for the high level policy.

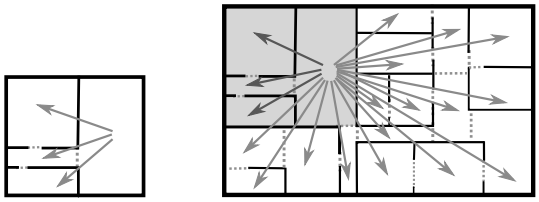


Figure 4.7: Action explosion*: the problem size increases, and therefore the number of higher level states. An action explosion occurs on the higher level of* HASSLE*.*

So on the higher level, not only the number of states is increased (as is normal when a Reinforcement Learning problem scales up) but also the number of actions (quite unusual), so the problem size grows in two ways (see fig. 4.7). This can also be seen in fig. 4.6 where is illustrated that the Q-values table and the Capacities tables will grow quadratically.

Since states are used as the actions (subgoals), the number of actions increases with the number of states. This could greatly decrease performance for problems with larger numbers of states ($> 10^3$), since there are more actions (subgoals) to explore (the Q-values-table needs to be filled in accurately), out of which probably only a few are accessible from the current state.

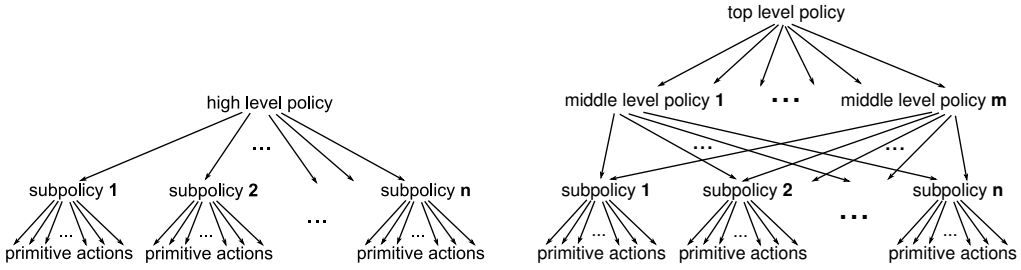
This *explosion* of the number of actions on the higher level is a serious problem, it is hampering the learning process, because the more high level actions there are to take, the more have to be investigated, making the problem more time consuming, up and above the effect of the increased number of states. This means that the time needed for exploration increases vastly and as a second problem the memory required to store the Q-values and Capacities tables, increases quadratically. This *action explosion* is the central reason for introducing a new algorithm that gets rid of this problem in a radical way.

4.3.3 Three or More Layers Impossible

If the problem size scales up, it will result in significantly more low level states. This means that either the number of abstract states stays the same but they grow in size, or their number increases. Increasing the size of the abstract states will make those subproblems harder, and we will get trouble using *simple and small* function approximators for our low level subpolicies, so that can only be done to a very limited amount.

When the problem gets bigger it will results in more abstract states. In general this would not be a problem in itself, because we could introduce a new layer. That way all the subtasks on all the layers remain at a reasonable size. But HASSLE is not suitable for use with more than two layers.

The idea behind introducing extra layers (see fig. 4.8) would be to enable the re-use of partial solutions (subtasks) elsewhere in the problem and to manage the growing number of (abstract) states when the problem scales up.



(a) two layers (b) three layers

Figure 4.8: (a) Two layers*: the number of layers is determined by the number of (sub)policies.* ***(b) Three layers****: in the middle layer policies all share the underlying subpolicies. An alternative is, to give each middle layer policy its own set of unique subpolicies, but this leads to large numbers of subpolicies and prevents re-use.*

Unfortunately, since HASSLE uses absolute actions on the middle level, there is no possibility for re-use. The policies[13] on the middle layer are using fixed subgoals as their actions, so a middle layer policy that is a solution in one part of state space, is completely useless somewhere else, because it refers to subgoals that are specific to other regions of the state space.

Note that three or more layers *can* of course still be used, but there will then be no re-use of (middle layer) subpolicies and all the layers above the lowest cannot use function approximators. This would create a structure that is similar to Feudal-Learning (section 3.5.5). As noted there, Feudal-Learning also does not re-use its policies and has all of its subgoals predefined. So HASSLE is unsuitable for three or more layers.

4.4 An Attempt to Fix HASSLE – Defining Filters

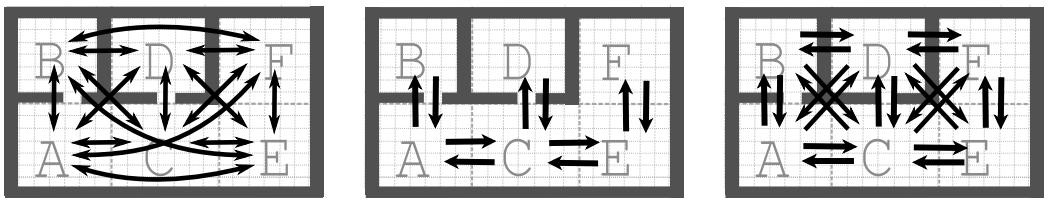
HASSLE uses subgoals both as states and as actions, the problem is viewed (on the high level) as a fully connected graph where the subgoals are the nodes and the edges are the transitions from one subgoal to another (see fig. 4.9(a)).

As a first solution to the problem of action explosion, we could simply eliminate certain subgoals when in certain abstract states – at least if that information is available. We might conclude *a priori* that some subgoal X is just not reachable, so the agent is not even allowed to try it. A filter could be defined to drastically reduce the number of edges available in a each of the subgoals. To this effect, we could add the following rule the formal description of HASSLE in section 4.1.1:

[13] When expanding to three layers, there will probably be sets of (sub)policies both on the lower and the middle layer, and one policy on top.

- $Filter_i : States_i \rightarrow \wp(Actions_i)$, a filter that gives the available actions at a given state.

where i is the layer, and $States_i$ and $Actions_i$ are the sets of states and actions for that level (same notations as in section 4.1.1).



(a) fully connected (b) restrictive filter (c) liberal filter

***Figure 4.9: (a) Fully connected graph**: on the high level, the subgoals and their accompanying high level actions ("goto subgoal …") form a fully connected graph. Note that all arrows are unique actions. **(b) Restrictive filter**: Only subgoals that are connected (i.e. can actually be reached), are allowed by $Filter_2$ for the higher level. **(c) Liberal filter**: subgoals that are near to each other, are allowed by $Filter_2$. Both filters create a smaller model of the higher level problem. The black arrows depict which subgoals are reachable from a given subgoal.*

With this *a priori* filter we can specify sets of available subgoals (high level actions) such that $Filter_2(someSubgoal) = availableActions \subseteq \wp(Actions_i)$ (see fig. 4.9(b) and (c)). Note that this also allows us (in principle) to specify restrictions on which primitive actions can be used in what states by defining $Filter_1$ in a suitable way (if all primitive actions are always available, $\forall s \in States_1 : Filter_1(s) = Actions_1$).

This approach is not very interesting, because by *a priori* excluding actions at certain subgoals, the (high level) graph of the problem is simply preprocessed and reduced beforehand. In the reduced graph learning is obviously faster, because there is less to learn compared to the original graph.

4.4.1 Automatically Creating *A Priori* Filters

It is up to the designer to specify the entire filter – and since filters are proposed as a fix for the action explosion when the problem size grows large, that would be a labourious task to do manually.

If we would settle for a less efficient filter, we could do it automatically (given a suitable state representation on the higher level). When we are solving a navigation task, each subgoal can have coordinates, and those can be used to determine automatically whether two subgoals are too far away to be adjacent, for instance because they are further away than the agent can travel before timeout occurs, etc. This would generally result in more liberal filters (like in fig. 4.9(c)) because the heuristics that are used to prune the graph will most often be coarser (but faster) than manual pruning.

4.4.2 Learning Filters

Designing or automatically generating *a priori* filters might be a possibility in simple cases, but this would not be the case in general. If the problem is too large, constructing the entire filter is too much work, and if we already know so much about the problem already, why try to solve it using Reinforcement Learning, instead of just doing it ourselves, or maybe use a planning algorithm?

The point is, that if we don't exactly know the underlying structure of the problem (and the problem is large), then filtering out subgoals is no option for us, and for the creation of a suitable filter we are stuck with Reinforcement Learning-like solutions where agents have to determine *by themselves* what the structure of the problem is, and that certain subgoals are just unreachable and that it is a waste of time even trying them.

However when a problem is large, and no reward signal is present for a long time because no goal is reached yet, the agent will consider each of the subgoals a valid option, even though most of these

'options' are in fact never reached at all. This is because the subgoals that *have* been reached, don't yield any non-zero rewards (because the goal is too far away and the positive rewards are not yet propagated back). These reached subgoals (with zero reward) *cannot* be distinguished from the unreachable subgoals (with zero reward) using only the rewards.

The normal Reinforcement Learning algorithms will be of no help here. We could try to punish the agent for failing to reach a subgoal, but in the beginning the agent is probably not able to reach any other subgoal at all, because its behaviours are not yet learned. So basically we would be decreasing the expected reward *for every subgoal* including the good ones, which is not very useful at all. This would just leave us in the same situation that we started in: no knowledge whatsoever about good or bad actions.

Subsets of the Action Set

Some sort of filter is needed to create a bias towards successfully reached subgoals, even when they haven't yielded any non-zero rewards yet. HASSLE needs to be augmented with some sort of record keeping device which keeps track of successful high level actions (i.e. reachable subgoals). This *success rate* needs to be used in our filter, so we can add a more specific rule to HASSLE (formalized in section 4.1.1) instead of $Filter_i$:

- $ActionFilter_i : States_i \times successRate \rightarrow \wp(Actions_i)$, a filter that gives the available actions at a given state, as a probabilistic function of the success rates.

where i is the layer, and $States_i$ and $Actions_i$ are the sets of states and actions for that level, so a subset of $Actions_2$ is given by $ActionFilter_2(state_t, successRates_t)$.

Since the agent is not very successful in doing anything at all in the beginning, the learned filter needs to be somewhat flexible and fuzzy. We cannot simply keep track of a list of subgoals that we could reach in the past (and in that way just building up the $Filter_2$ defined earlier). We need to bias successful high level actions, *but* we cannot remove subgoals that were unsuccessful (so far) entirely.

Keeping track of the success rate for reaching a certain subgoal can be done in a table with the same size as the Q-values table. Values in that table represent a running average[14] of the success rate. The frequency with which subgoals are selected, might look something like fig. 4.10 after some learning.

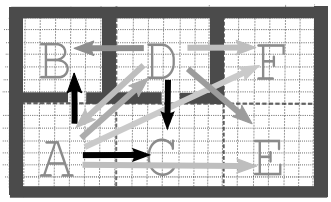


Figure 4.10: Learned Filter*: The arrows depict the selection frequency (only shown for starting in subgoal A or D). Darker represents higher frequency. Transitions like going from A to F will gradually get a lower frequency.*

One way to use these success rates as a filter, is to probabilistically determine a subset of the actions each time an action needs to be selected. The action is selected from this subset. Membership is determined probabilistically as a function[15] of the success rate. Using success rates boosts exploration towards those subgoals that are apparently easier reached than other subgoals (that are perhaps completely unreachable).

[14] A running average, because subpolicies change in behaviour and failure in a too distant past should not count negatively on current performance.

[15] The probability cannot be equal to the success rate itself, because that would mean that a subgoal that is not reached yet (success rate of 0) does never get into the subset. Hence some function of the success rate is needed.

Boosting Boltzmann Selection

Alternately the success rate can be used to bias the selection mechanism, and we don't need to make subsets of available actions. By adding the success rate (suitably scaled or transformed) to the Q-values when Boltzmann selection is executed, the selection can be biased towards those subgoals that were successful in the past. Equation 4.3 in section 2.4.2 then changes into:

$$P_{Boltz}(s, a_i, successRate) = \frac{e^{(Q(s,a_i) + \sigma(s,a,successRate))/\tau}}{\sum_{a' \in actions} e^{(Q(s,a') + \sigma(s,a,successRate))/\tau}} \quad (4.3)$$

where $P_{Boltz}(s, a_i, successRate)$ gives the probability of selecting a_i in s, s is the current state, a_i is the action under consideration, *successRate* is the success rate and $\sigma(s, a, successRate)$ is some function transforming the success rate into a good "boost".

Boosting selection does not work when using ε-greedy selection, because it only looks at the maximum Q-value. Incorporating success rates (in some way) into this selection does not have the desired effect because ε-greedy selection focuses on one (the best) action, whereas we want to bias the exploitation to *all* the successful subgoals. Because Boltzmann selection (soft-max selection in general) selects proportional to some function of the Q-values, it is suited for boosting successful subgoals during selection.

When the *ActionFilter* is used to produce a subset, there is no restriction on what selection mechanisms can be used, because it does its work *before* action selection starts.

The various filters proposed here are needed for the initial stages of learning. When the agent has no meaningful information yet about the goal (no non zero rewards yet) a filter can bias learning towards those *few* subgoals that are reachable, instead of equally dividing its time – and *wasting* it – on all subgoals. When information about the target becomes available, the filter is redundant, because the Q-values then implicitly carry information about subgoals that can be reached successfully and will eventually lead to the goal. The effects of the filter can be reduced after more information becomes available, or it could be removed entirely.

4.4.3 Filters – What Remains Unfixed?

The *a priori* filter proposed here requires that information about reachability is available *before* learning, which means that the agent needs to have information about the results of each action (i.e. selecting a subgoal) when determining if a state should be selected. This solution therefore imposes a restriction: the entire environment needs to be known to the agent: it is not *model free*[16].

The advantage of a *learned* filter is that it does not need *a priori* information about the entire environment, because reachability is learned during exploration. But still the agent needs knowledge about what subgoals there are in its world. This is however not related to this fix, but to the structure of HASSLE in general.

On the other hand there is a new problem: the agent now has one extra table that grows quadratically (Q-values, Capacities and *success rates*)! So even though the agent can now learn what is reachable, it still needs to do this by exploring, so in the beginning there still remain quadratically many subgoals to explore. Only after some time the effect of using success rate kicks in and exploration will be biased to more successful transitions. Therefore this fix is only a partial solution to the exploration produced by the explosion of actions and it does not solve the accompanying memory problems but even increases them slightly[17].

[16] The reinforcement learning techniques described in chapter 2 are all *model free*. They can be used even when the only information available to an agent is its current state, its actions and possible rewards. So model free means that no (probabilistic) model of all the transitions and states is needed.

[17] The main memory problem comes from the Capacities tables, because there is one for each subpolicy, whereas there is only on (high level) Q-values table and only one success rate table.

Furthermore, filters are also no solution for the lack of generalization and the inability to use more than two layers, because they are related to the absolute and unique nature of the subgoals, not to the number of subgoals. For this reason only a few experiments are done (and only with boosting Boltzmann selection) as a proof of concept.

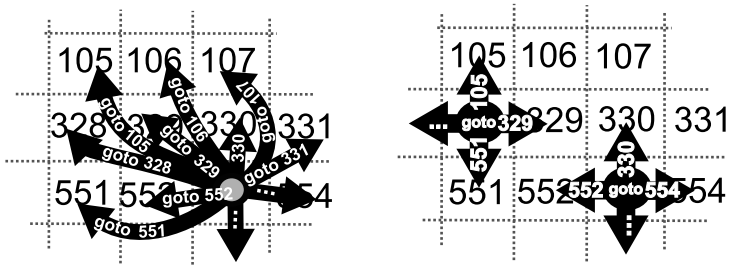
4.5 Identifying the Underlying Problem in HASSLE

There is a recurring theme in all the problems identified in section 4.3. Generalization is not possible because each of the actions is unique (because subgoals are used as actions). For the same reason it is not possible to use three or more levels because each of the subpolicies of the second or higher layer has subgoals as actions, so each of those subpolicies is only applicable exactly in that part of the state space where those subgoals are. Furthermore using high level states as actions results in a quadratic growth of the high level Q-values table (and specifically for HASSLE. also a quadratic growth in the Capacities tables) because the number of actions grows with the number of high level states (an *action explosion*).

These problems with generalization, more layers and action explosion are not specifical for HASSLE only. They will occur in every algorithm that like HASSLE uses high level subgoals directly as actions. *Defining behaviours in terms of subgoals to be reached, is an absolute way of defining behaviours*: the behaviours are determined by a fixed point in the state space (either the flat state space, or a higher level abstract state space).

4.5.1 Analyzing Primitive Actions

This absolute nature of the behaviours is in stark contrast with the way that the primitive actions are defined. An agent does not usually have a thousand actions labelled $action_1, \ldots, action_{1000}$ when it is in a grid world consisting of a thousand cells (like in fig. 4.11(a)). Most of the time it only has a small set of *primitive actions*.



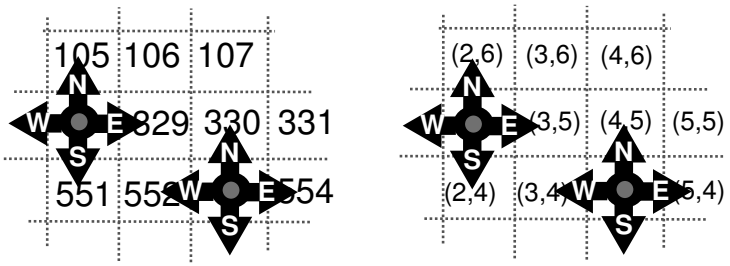
(a) 1000 primitive actions (b) only neighbouring states

***Figure 4.11: (a) 1000 primitive actions**: primitive actions defined in an absolute way. There are 1000 actions if there are 1000 cells. **(b) only neighbouring states**: primitive actions defined in an absolute way, but limited to neighbouring states. There are still as many actions as there are states, but in each states only a few actions are available.*

For a start, the largest part of these other cells are probably not even reachable from a certain cell, so the set of actually useful actions in a given cell is very small (see fig. 4.11(b)). But still this is not the most natural way to define primitive actions, because it would mean that for each cell the connectivity needs to be stored explicitly, because each primitive action would still be unique, each pointing to one specific cell.

Instead, an agent usually has (and the environment provides) only a very limited set of actions, for instance *North*, *East*, *South* and *West* or the ability to rotate and move straight ahead. This would still be of little use when the states themselves are not related to those actions (but for instance labelled or numbered arbitrarily, as in fig. 4.12(a)) because explicit storage would then still be needed. Even though

we seem to have only four primitive actions in this situation, we still would need to store that from cell 328, we would end up in cell 105 if we executed the action *North*.



(a) four primitive actions

(b) four primitive actions with appropriate coordinate system

Figure 4.12: (a) four primitive actions*: the actions North, East, South and West (N,E,S,W) are introduced. For each cell the results of the actions need to be stored.* ***(b) primitive actions with coordinate system****: actions are now related to a coordinate system which is part of the state features. Results of actions are implicit in the coordinate system.*

So what is commonly done, is using some sort of coordinate system. This need not consist of spatial coordinates[18], but could also be more abstract notions like "*agent has object*" or "*door is open*". So what we usually do, is something like depicted in 'fig. 4.12(b), where the primitive actions (*North, East, South* and *West* in this case) are used together with states that have coordinates.

This way there is no need to explicitly store all effects of all primitive actions. These effects are implicitly present in the coordinate system and the way the primitive actions are defined with respect to the coordinates. If the primitive action *North* is executed, the effect is something that is relative to the state where the agent came from. Provided there is no wall, the new state of the agent is just one cell to the north of the previous state.[19] If we started in state (2,5) and executed the primitive action *North (i.e. add 1 to the y-coordinate)*, we would end up in state (2,6).

Primitive Actions Are Relative

The primitive actions are not defined in a goal-directed way or absolute way in terms of specific *points* in the state space that they should go to. Instead they are defined in terms of what they *do*, relative to the state they are invoked in. This is possible because *under the hood* the environment has a certain geometry. The geometry is most obvious for grid-world like tasks, but it is not constrained to those kind of tasks, as noted before. This geometry defines directions in the environment, and because of the geometry it is possible to define meaningful difference vectors in the environment (or state space).

These difference vectors can then be used to define the primitive actions, and there is no need for using explicit subgoals as absolute targets for the primitive actions. Actions are coupled with certain difference vectors, and although the agent may not be flawless, and sometimes fails to execute the action correctly and fails to achieve the usual difference, the coupling nevertheless works because there is at least a statistical connection between an action and a vector.

In all four cases (fig. 4.11 and fig. 4.12) the environment is *the same* but our description of the environment differs, with greatly varying results. Absolute ways of defining the primitive actions would lead to an explosion of actions (as many primitive actions as there are states) and enormous storage needs. Defining actions relative to some (smart) coordinate system results in virtually no storage at all, and incidentally also introduces the possibility for generalization (see section 4.3.1)! Both generalization

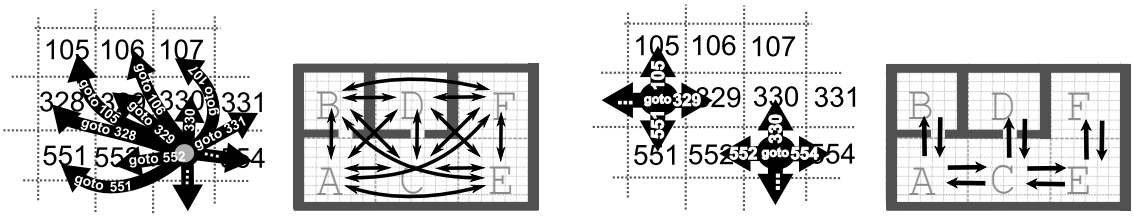
[18] However, because spatial tasks are intuitive, they are mostly used as examples here.

[19] Even when the environment is non-deterministic, and the agent does not know what the results of its actions will be, still the actions in the environment are defined relative to the state in which they are executed, the only difference being that the resulting states are defined by probabilities.

and relative definition of primitive actions are made possible by using good, information rich state descriptions.

4.5.2 Behaviours Should Be Like The Primitive Actions

Subpolicies (behaviours) in HASSLE are constructed similar to the primitive actions when they are defined in a unique way. The high level subgoals that HASSLE uses, are defined in an absolute way (fixed points in state space). In fig. 4.13(a) and (b), this situation is illustrated.



(a) absolute actions (b) fully connected (c) neighbouring states (d) a (restrictive) filter

Figure 4.13: absolute actions, fully connected*: see fig. 4.11(a) and fig. 4.9(a) for more information.* ***neighbouring states, filters****: see fig. 4.11(b) and fig. 4.9(b) for more information.*

The results after applying (*a priori* or learned) filters, is comparable to what we get when we restrict the (still unique) primitive actions to only neighbouring states (see fig. 4.13(c) and (d)).

A good situation was obtained for the primitive actions, when they were defined relative to some sort of coordinate system in the state space. This is something HASSLE does not do, and it is the reason why HASSLE needs a new action for each new subgoal (action explosion) and cannot use generalization on the higher level (and related to this, why it cannot work with more than two layers).

This is not something that is restricted to HASSLE, because it is related to the use of high level abstractions of the state space. When such abstractions are used, the abstract states *cannot* be used as high level actions in an efficient way.

So if we want to design an algorithm that does not run into problems with generalization or exploding numbers of high level actions, it seems that we need to define high level actions in a relative way, similar to the way primitive actions are defined.

4.6 Comparing HASSLE To Other Approaches

HASSLE is compared (see table 4.2) to the other algorithms mentioned in the previous chapter. The various Options-approaches are so similar in structure (and so different from the layered approaches), that a comparison with the general options-framework suffices.

Abstract States

HASSLE focuses on giving a good abstract representation of the state space. It not only breaks up action (and time) hierarchically, but also decomposes the state space in a hierarchical manner. This allows for smaller state spaces at higher levels.

The state abstraction in HASSLE is not constrained to subsets of features from the state space. Arbitrary abstractions allow the designer to use his or her knowledge from the problem domain to define a suitable abstract structure for the higher level states, for instance using a clustering algorithm, or perhaps some entirely different heuristic (or just a selection of features from the lower level after all – if that is most suitable).

	Options	MAXQ	HAM	HEXQ	HQ	Feudal	RL-TOPs	Self-org.	HASSLE
high level uses abstract states	–	+	–	+	–	+	+	±[20]	+
abstract states composed of low level state features only		+		+		–	+	–	–
high level policy can (in principle) use neural network		±	±[21]	–[22]	–	+		+	–[23]
focus on task decomposition	[24]	+	+	+	+[25]	–	–	–	–
subpolicies used *directly* as actions by high level policy	+	+	+	+	[25]	–	+	+	–
subpolicies start uncommitted	±[26]	–	–	–	±[25]	–	–	+	+
subpolicy rewards independent of reward for overall task	±[26]	–	–	–	–	+	+	+	+

Table 4.2: Comparison of different Hierarchical Approaches*: a "+" means that a feature is present, and "–" that it is not. The approaches are listed in the same order as they were described above.*

HASSLE stands out because it uses higher level abstract states which can be arbitrary instead of just a clustering of lower level states or a selection of features out of the lower level states. In this aspect it resembles Feudal Learning. Other approaches create higher level states out of selections of variables (features) in the lower level state, as MAXQ, HEXQ, RL-TOPs and others do.

Like HQ-Learning it uses a Q-values table for the higher level to select high level states as subgoals to go to. HASSLE and HQ both learn which subgoals to select, but HQ uses low level states as its subgoals, while HASSLE uses its abstract higher level states for this.

Using Neural Networks On The High Level

In section 4.3.1 it was shown that HASSLE cannot function with function approximators (including neural networks) on its high level. This is due to the fact that the subgoals are used as high level actions. For generalization, some structure and predictability is needed, but the HASSLE high level actions will always be different in different subgoals (even if their *content* is the same).

Many other hierarchical approaches also have trouble using neural networks on their high level.[27] They are (with the exception of Feudal Learning and the Self-Organizing approach) all dependent on some elements that pose problems for function approximators because of their highly discrete nature. The Self-organizing approach on the other hand is explicitly constructed with a neural network on both layers.

[20] The high level uses a vector composed of low level values for its decisions, but each behaviour has a low level subgoal.

[21] HAM has a Q-values table that contains $(state, machine)$-pairs. It all depends on how the machines are constructed.

[22] The exit states are used as subgoals. These are fixed locations in the state space, making generalization hard (see section 4.3.1).

[23] See section 4.3.1 for full discussion.

[24] There is no task decomposition (Options are added to the set of primitive actions).

[25] HQ has a completely fixed execution order, so it has a *fixed* task decomposition! Each of the subpolicies (viewed as agents) is always executed in the same order. It is not specified though what each of the tasks should be, so it is fair to say that the subpolicies are at least partly uncommitted.

[26] Depends on the specific options approach.

[27] At least, as far as can be deduced on the basis of the descriptions presented in the literature.

MAXQ uses a hierarchical value function $V^{\pi}(< s,K >)$. This function gives the expected reward for policy π starting in state s with stack-contents K. But this makes it dependent on the subtasks that are still on the stack and these are (probably) discrete. Small variations in the stack contents will often results in entirely different policies. This could make function approximation difficult.

The Q-values table that HAM uses, contains information on both the *state* and *machine* because it consists of *state*,*machine*-pairs. These machines act as the high level actions in HAM. Whether or not HAM can use a neural network for its high level machine, rather depends on how these machines are constructed. If they rely on information about positions in the state space (i.e. absolute) then there will be problems similar to those HASSLE runs into. However, if these machines can be constructed in terms of *doing things* instead of *reaching subgoals*, a function approximator like a neural network will probably be able to cope with them.

HEXQ depends on identification of 'exit' states, and these exits serve as subgoals for subtasks. This makes HEXQ goal directed and a function approximator would probably run into the same problems as with HASSLE.

HQ learns subgoals for each of its sequential agents. This means it will have problems similar to HEXQ. Only if the problem is somehow repetitive, generalization is possible, because more than one agent would in that case have a similar subgoal in a similar situation.

Feudal Learning has only a limited set of orders on each layer, and these orders need not be dependent on specific (absolute) subgoals.[28] This means that Feudal Learning could in principle use a neural network on a higher layer.

RL-TOPs does not even use a Reinforcement Learning policy on its high level, but uses a planning system instead. Planning and calculation are not the strong points of neural networks.

The ability to use function approximators (more specifically neural networks) for the Reinforcement Learning policy on the high level, would be very useful. Neural networks are often used when a tabular representation is infeasible because the problem is too large. If a Hierarchical Reinforcement Learning algorithm (for its high level) depends on information that has an absolute nature, or that requires too much precision (e.g. stack contents) however, it is hard to combine it with a neural network.

(No) Focus On Task Decomposition

The HASSLE algorithm does not define a task decomposition (like for instance MAXQ or HEXQ) but only needs the definition of abstract states for use on the higher level. The focus in HASSLE is not on decomposing the task into subtasks, but on defining suitable abstract states (and therefore on ordering the state space hierarchically). In this reduced state space a learning algorithm can learn to solve the problem, using standard Reinforcement Learning techniques.

This means that the focus is on designing good abstractions rather than task decompositions. A decomposition of the task is useful when the designer knows roughly how the task needs to be solved and what kind of subtasks are needed. The focus on state space abstraction on the other hand is useful when a solution to the problem is not so clear but when some state abstraction is evident.

This is rather similar to the way RL-TOPs works. RL-TOPs defines teleo-operators (TOPs) with pre- and post-conditions. A planning algorithm then searches for a solution (but the subpolicies are trained with Reinforcement Learning). The decomposition is only implicitly present in the way the TOPs are defined.

HASSLE can work in non-deterministic and unknown environments because of the use of reinforcement learning on all levels, while those unknown environments pose a problem for RL-TOPs because planning in is difficult in an environment with unknown rules. But aside from that, HASSLE could to a large extent be reformulated in RL-TOPs terms, because behaviours in HASSLE can be viewed as TOPs that have the high level state (or better: all the low level states falling under the same high level state) that they start in as a pre-condition and likewise have the destination high level state as postcondition.

[28] In fact, Feudal Learning is rather similar to an *a priori* fixed version of HABS, see section 5.1.2.

(Not) Directly Adding Subpolicies As Actions – Capacities

HASSLE uses the Capacities mechanism as a link or interface between the subgoals and the subpolicies. Most of the other approaches use their subpolicies *directly* as behaviours and incorporate them directly in task decompositions. Feudal Learning is the only other exception: it uses a system of *managers*: they receive orders and themselves give orders to lower submanagers.

These managers form an interface between the orders and the execution of behaviour. The managers are free to decide how to execute the orders, and it could be that the manager has a selection of suitable behaviours to choose from. The managers in Feudal-learning therefore act in a similar fashion to the HASSLE Capacities.

There is no specific reason why adding subpolicies directly is better or worse than using some sort of interface mechanism (like the HASSLE Capacities or the managers in Feudal-learning). Using an extra interface mechanism means that there is more to learn, but on the other hand it provides flexibility, because the (learned) interface provides the ability to fine tune. An interface could receive the same order (the same high level behaviour) in two different situations, and in one case translate it to one subpolicy in one case and to another subpolicy in another case, because the interface observes more fine grained information and sees differences that are abstracted away on the higher level. This keeps the higher level decision and learning simpler.

On the other hand this fine tuning could also happen inside the subpolicy. If the subpolicies that are used are flexible enough, they are able to provide the same differentiation of one high level action into many similar (but not entirely equal) behaviours. In the same way as with an extra interface, this would keep the higher level decision and learning simpler.

Uncommitted Subpolicies

HASSLE uses its Capacities-mechanism to organize its *a priori* uncommitted subpolicies. Each subpolicy starts out uncommitted, but using the Capacities, subpolicies can specialize and organize themselves. This is unlike most of the other approaches. MAXQ for instance has a predetermined task decomposition and certain subtasks are assigned to certain subpolicies, so it is known before the learning phase which subpolicy should accomplish which subtask.

The self-organizing algorithm is the other exception: it tries to find low level states which cover the state space and that way tries to organize its subpolicies. It is structured to self-organize the behaviours over the state space, and each behaviour shifts to regions of the state space where the maximum of the Q-values of other behaviours is low. HASSLE uses *a priori* information in the form of high level states, unlike the self-organizing approach. The high level states and Capacities restrict the organization of the system.

If enough information is available to construct a suitable task decomposition, it makes sense to use committed subpolicies. However, if it is not clear (for the designer) which kind of subtasks are needed, subpolicies need to start uncommitted.

Local Reward Functions

HASSLE uses local reward functions: subpolicies have their own rewards which are completely independent of the rewards the agent gets for solving the overall problem. This means that the behaviours are viewed independently from the overall task *unlike* for instance in HQ, MAXQ and HEXQ and can therefore be re-used fairly easy.

In this aspect HASSLE resembles RL-TOPs or Feudal-Learning or the way many *options* approaches. A behaviour is just rewarded for doing what it is ordered to do, not for contributing to the eventual (global) rewards or for completing the overall task.

Consider the situation in fig. 4.14 where the state space near the goal is depicted. It is clear that behaviours "A $\Rightarrow$ goal" and "B $\Rightarrow$ C" are very similar in this problem and could (even *should!*) be executed by the same subpolicy. This means that in both situations the agent should be rewarded if it

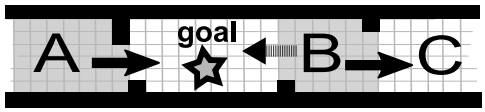


Figure 4.14: Subpolicies independent of global reward*: a navigation domain where behaviour "A ⇒ goal" and "B ⇒ C" are essentially the same. Behaviour "goal ⇐ B" is the opposite, but has a similar high level Q-value as "A ⇒ goal".*

executed the behaviour correctly, regardless of whether the behaviour actually brought the agent closer towards its overall goal.

Now suppose that the local (subpolicy) reward is dependent on the global reward. In that case the behaviours "A ⇒ goal" and "goal ⇐ B" would be rewarded with similar rewards (because in both cases the agent receives a reward for reaching the goal) and "B ⇒ C" would get a far lower (or zero or negative) reward because it moves the agent away from the goal.

But when training a certain behaviour, we are mainly interested in *that* behaviour, irrespective of whether it actually helps us in our overall task. The subpolicy only exists to execute transitions between high level states, and if that is what it should do, that is what is will be rewarded for, no matter the high level situation. So a dependence between the high level reward and the low level reward functions for the subpolicies is not desirable. This way of rewarding is exactly what the reward functions in HASSLE do (see section 4.1.1). A subpolicy will only be rewarded when it actually reaches the subgoal it was meant to reach.

Comparison – Final Remarks

It is interesting to note that HASSLE shares many features with both the Feudal-learning and the self-organizing approach, although HASSLE and Feudal-learning use look-up table, whereas the self-organizing approach is designed specifically for the use neural networks. The structure of the self-organizing approach is completely different than that of HASSLE or Feudal-learning.

Chapter 5

HABS — Self-Organizing Behaviours

From our analysis of HASSLE (section 4.2), we can conclude the following: the use of *state space abstractions* is useful. A focus on abstract states allows for a shift away from designing task decompositions by hand. Also, the use of *independent* reward functions for the subpolicies creates reusable behaviours.

Furthermore from the section on problems with HASSLE (section 4.3) we know that using subgoals explicitly as actions will not work, because it is the root of all sorts of problems. Instead we should aim for *behaviours that are defined relative to the abstract state space*, analogous to the primitive actions.

5.1 HABS — *Hierarchical Assignment of Behaviours by Self-organizing*

A new algorithm based on HASSLE cannot use absolute (high level) actions. Instead, when behaviours are defined in a relative way, there is a mapping between the behaviours (transitions between high level states) and the high level actions. It classifies each behaviour as belonging to one of the high level actions. This can be compared to how all primitive actions that move an agent Northward all map to the same primitive action *North*.

5.1.1 Short Circuiting HASSLE

A classification algorithm is used to map the transitions between high level states to characteristic behaviours. These behaviours are then added directly to the high level policy as (high level) actions. By this *short circuiting* of the HASSLE algorithm, both the Capacities and the use of *states (subgoals) as actions* can be avoided. The Q-values of the high level policy now directly determine which subpolicy is suited for which transition, because the Q-values give the value for a high level action (behaviour) in a high level state (see fig. 5.1).

The algorithm that results from short circuiting HASSLE is called HABS, which stands for *Hierarchical Assignment of Behaviours by Self-organizing* because instead of HASSLE (*Hierarchical Assignment of Subpolicies to Subgoals LEarning*) it does not use subgoals but organizes itself by dynamically assigning (classifying) behaviours to its uncommitted subpolicies.

Reverse Order

It is interesting to note that the relative nature of the HABS behaviours is accomplished by reversing the chain that links transitions between subgoals to subpolicies (see fig. 5.1). HABS starts with classifying many transitions to a small set of characteristic behaviours (*many-to-one*) and then associates each of these with a specific subpolicy (*one-to-one*). This reverse order is possible, if high level behaviours can be defined in a relative way (as is later explained). When that is the case, many transitions can be treated as roughly the same, and can be mapped to one group and one subpolicy.

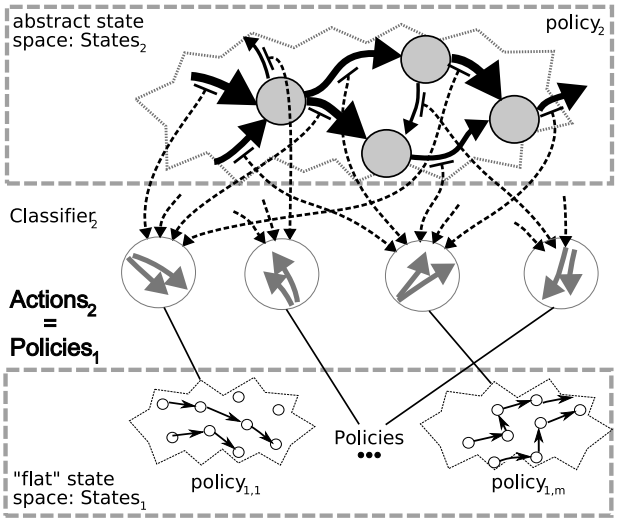


Figure 5.1: The hierarchical structure of* HABS: the large gray circles are the high level states. The black arrows represent the high level Q-values (the thicker they are, the higher the Q-value). The actions (behaviours) on the high level are classified to a small set of characteristic behaviours (dashed arrows leading towards the gray arrows). The classes are each associated with a particular subpolicy. Notations correspond to those in section 5.1.3.***

HASSLE on the other hand first treats each of the transitions as unique (*one-to-one*) and then relates each transition to all the subpolicies (*one-to-many*) by means of the Capacities (see fig. 5.2). The problems with HASSLE were all related to the number of high level actions, and their unique nature. But since HABS uses the characteristic behaviours, of which there are only a few, it does not suffer from the same problems.

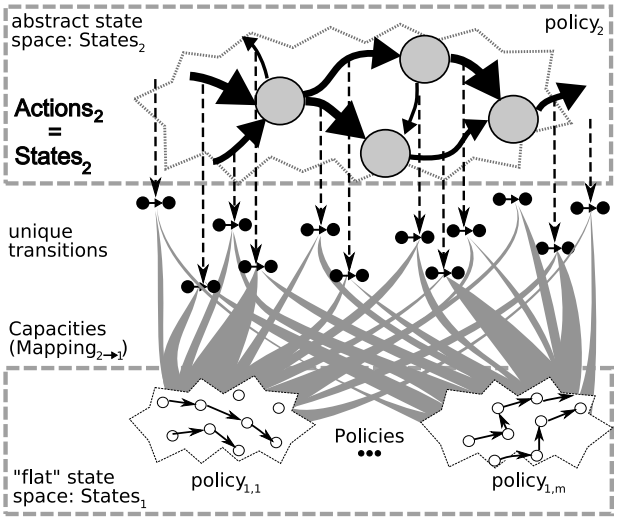


Figure 5.2: The hierarchical structure of* HASSLE: the large gray circles are the high level states (subgoals). The Q-values of the high level policy are indicated by the thickness of the black arrows between the subgoals. The ⊷ represent the (unique) transitions between subgoals. The Capacities ($Mapping_{2\rightarrow1}$) are represented by the thick gray triangles (thicker means higher capacity). Notations correspond to those in section 4.1.1. (Same as fig. 4.1, placed here for easy comparison with fig. 5.1).***

No More Explicit Goals

There is one *slight* problem with HABS as it is proposed here. By kicking out the *subgoals as actions*-idea, we have lost our easy way of training subpolicies. Without explicit (sub)goals we cannot train a

subpolicy for a subtask with the trivial heuristic that HASSLE uses: *the behaviour of the subpolicy is good, if the subgoal is reached*.[1]

Since there are no more goals, a new mechanism is needed for determining when a subpolicy should be rewarded (at least, if the classification is not given *a priori* by the designer). Self organizing is proposed as a solution, based on some extra restrictions on the state abstraction.

The idea is that a good representation of the abstract states will provide meaningful behaviours. Given such a state abstraction, it then becomes possible to classify and organize the occurring behaviours such that clusters of similar behaviours are identified. These clusters will represent behaviours that are *needed* to solve the problem and because the behaviours within a cluster are similar, it is possible to accomplish them with one (suitable) subpolicy which can be used directly as a high level.

HABS can start with uncommitted subpolicies because the classification is learned. The classification will provide the reward conditions for the subpolicies, but the classification is updated according to what the subpolicies do.

5.1.2 Comparing HABS to Other Approaches

In the previous chapter, several approaches were compared. Table 4.2 is reprinted here, with an extra column for HABS (table 5.1) to illustrate similarities between HABS and other approaches.

MAXQ and HEXQ focus on task decomposition (although they have some form of state abstraction) and therefore try to accomplish a structure like HABS by defining suitable pre and post conditions for their subtasks. They reduce the vast number of unique transitions to a manageable amount of subtasks by a suitable choice of conditions on each subtask.

	Options	MAXQ	HAM	HEXQ	HQ	Feudal	RL-TOPs	Self-org.	HASSLE	HABS
high level uses abstract states	−	+	−	+	−	+	+	±	+	+
abstract states composed of low level state features only		+		+		−	+	−	−	−
high level policy can (in principle) use neural network		±	±	−	−	+		+	−	+
focus on task decomposition		+	+	+	+	−	−	−	−	−
subpolicies used *directly* as actions by high level policy	+	+	+	+		−	+	+	−	+
subpolicies start uncommitted	±	−	−	−	±	−	−	+	+	+
subpolicy rewards independent of reward for overall task	±	−	−	−	−	+	+	+	+	+

Table 5.1: Comparison of different Hierarchical Approaches*: a "+" means that a feature is present, and "–" that it is not. The approaches are listed in the same order as they were described above. This is an extension of table 4.2. See there for more information.*

It is interesting to note, that this is somewhat similar to the structure *implicit* in RL-TOPs (see

[1] Note that this is not a problem for the primitive actions, even though they are also relative. This is because the primitive actions are *a priori* given and not learned, so there is no need of training or goals. Therefore if behaviours were predefined, this would not be a problem for the high level(s). This is somewhat similar to the way the Multi-Step actions (section 3.4.1) work: those options were also predefined instead of learned. This is however not suitable when the environment is not known well beforehand.

section 3.5.6). The RL-TOPs algorithm is based on the notion of a *teleo-operator* (called a TOP) which consists of a temporally extended behaviour (a Reinforcement Learning policy with a neural network as a function approximator) and a set of pre and post conditions. Although RL-TOPs still defines behaviours in terms of desired goals, it does so in a rather general way, by stating post conditions that the TOP should reach, not by stating specific (high level) states.

Both the TOPs and the subtasks in MAXQ or HEXQ are an attempt to define rather generic behaviours, that are not limited to certain specific sets of abstract states. Unlike the primitive actions, they still are somewhat goal directed (because of the post conditions they need to achieve) but this goal directed nature is what makes TOPs useful in planning[2].

Another possible parallel is between relative behaviours and Options. Some Options approaches allow for the inclusion of Options that do something "relative" like moving the agent through a small passage, instead of moving the agent to a certain goal. The Options approaches however do not use multiple layers or state space abstraction.

Feudal Learning as an *A Priori* Fixed Form of HABS

Feudal Learning is an interesting case because in its description Dayan et al. only note that each layer has a limited set of commands. As already noted in the comments in section 3.5.5, for each command the (sub)goal is already known *a priori* so this makes it somewhat similar to RL-TOPs or MAXQ and HEXQ with their pre and post conditions. However, the classification of the commands on each level is (in their example task) derived directly from the underlying geometry. HABS is built around the same principle, but unlike Feudal Learning it has no *a priori* defined behaviours. In a sense, Feudal Learning can be considered as a version of HABS where the classification is fixed from the beginning.

5.1.3 HABS Formalized

In this section the formal structure of HABS is presented. The same format as in section 4.1.1 is used to highlight the similarities and differences between HABS and HASSLE. The structure that is presented here, is in fact a framework. The details need to be filled in with heuristics, in order to get an algorithm.

HABS for n layers consists of the following (with $i \in \{1,\ldots,n\}$):

- sets of states $States_i = \{state_{i,1}, state_{i,2}, \ldots\}$:
  $States_1$ is the set of primitive states and $States_{i\geq 2}$ are sets of higher level states. Each of the $States_i$ can be continuous.[3]

- sets of actions $Actions_i = \{action_{i,1}, action_{i,2}, \ldots\}$:
  $Actions_1$ is the set of primitive actions, but $Actions_{i\geq 2} = Policies_{i-1}$ (i.e. actions are lower level subpolicies) so $action_{i,x} = policy_{i-1,x}$ for $i \geq 2$.

- sets of (sub)policies $Policies_i : \forall_k \left(policy_{i,k} \in Policies_i : States_i \times Actions_i \rightarrow \mathbb{R} \right)$:
  Standard Reinforcement Learning policies for each layer. The top layer has only one policy (so $Policies_n = \{policy_n\}$), but the other levels have m_i policies (so $Policies_i = \{policy_{i,1}, \ldots, policy_{i,m_i}\}$), each specializing in different subtasks. Only one (sub)policy is active at each level at each time step.

- *AgentInternals*: variables describing the observations and memory of the agent: $currentState_i \in States_i$, $currentAction_i \in Actions_i$, and indicators for the timeouts: $timeout_i$ and which policies are active on each layer ($policy_{i,active}$).

[2] RL-TOPs uses a planning tree on the high level, not Reinforcement Learning.

[3] For a hierarchy $|States_{i+1}| < |States_i|$ is needed. But if $States_i$ is continuous, $|States_{i+1}| < |States_i|$ needs to be understood in a different way. If two spaces are continuous, they both contain an infinite number of points, so comparing size that way is impossible. Instead, the statement $|States_{i+1}| < |States_i|$ should in that case be understood as indicating that the "flat" state space is viewed on a coarser level by the abstract state space. This could for instance be the case when the abstract states contain less variables or if it has the same number of variables, but averages values over a wide range.

- $n-1$ functions $Exec_{i\geq 2} : Actions_{i-1} \times Actions_{i-1} \times \ldots \times Actions_{i-1} \rightarrow \mathbb{R}^m$
  A means of getting from the actions that the agent executed with $policy_{i,active}$ to a suitable description $exec_i$. (Instead of defining it in terms of $Actions_{i-1}$ it could also be defined in terms of $Actions_1$. This does not matter, as long as a good representation for the executed behaviour on each level i can be defined.)

- $n-1$ classifiers $Classifier_{i\geq 2} : exec_i \rightarrow Policies_{i-1}$
  A behaviour $exec_i$ executed by a $layer_i$ policy is always classified as "belonging to" a $layer_{i-1}$ (sub)policy. This means that $Classifier_i(exec_i) = policy_{i-1,k}$ for a certain k. $Classifier_i$ classifies the actually executed behaviour $exec_i$ that results when the active policy $policy_{i,x}$ executes one of its actions $action_{i,active} \in Actions_i$. Since the actions for layers $i \geq 2$ are entire (sub)policies on the level below, $policy_{i,x}$ in fact executes $policy_{i-1,active}$. The classification defines sets of behaviours that are similar. (obviously there is no classifier for the lowest layer)

- stop conditions $Stop_i : AgentInternals \rightarrow \{terminate, continue\}$:
  Determine whether a (sub)policy should terminate when it has moved from S_i to S_i'.

$$Stop_{i<n} = \begin{cases} terminate & \text{If } timeout_i \vee distance(S_i, S_i') > \delta_i \\ continue & \text{otherwise} \end{cases}$$

$$Stop_n = \begin{cases} terminate & \text{If } timeout_n \vee \text{the agent reaches its goal} \\ continue & \text{otherwise} \end{cases}$$

- reward functions $Reward_i : AgentInternals \rightarrow \mathbb{R}$:
  The $Reward_{i<n}$ depend on how the higher level $Classifier_{i+1}$ classifies executed behaviour $exec$. If $Classifier_{i+1}(exec) = policy_{i+1,active}$ the agent needs to be rewarded. Other rewards/punishments are given for a wrong match and for timeout. During execution the policy receives 0 reward. Note that this is an internal reward, i.e. it is not received from the environment.

$$Reward_{i<n} = \begin{cases} 0 & \text{If } Stop_i = continue \\ 1 & \text{If } Stop_i = terminate \wedge Classifier_{i+1}(exec) = policy_{i,active} \\ \rho_{failed} & \text{If } Stop_i = terminate \\ & \quad \wedge\ Classifier_{i+1}(exec) = policy_{i,closestMatch} \\ & \quad \wedge\ closestMatch \neq active \\ \rho_{timeout} & \text{otherwise} \end{cases}$$

  For $Reward_n$ (top layer) the accumulated rewards that the environment gives the agent during execution of a subpolicy (possibly with nested subpolicies, etc), can be used because they are related to solving the overall problem[4].

Selecting Heuristics – Filling In The Details

In order to get a working algorithm, a Reinforcement Learning algorithm needs to be selected for all the policies.[5]. Several of these are described in section 2.3.

Also, a suitable abstraction of the state space is needed. Some general properties can be formulated (described in section 5.1.6) in general. But the selection of a certain abstract state space is related to which types of classification or distance measurement is used. A state abstraction on the assumption that high level actions can be approximated by *vectors* is described in section 5.2.

Furthermore classifiers are needed and the $distance(S_i, S_i')$ (i.e. the distance that the agent moved through state space from the start S of the subpolicy until the current situation S_i') needs a concrete measurement. Examples of these will be described in section 5.2.6.

[4] In sparse reward tasks this will amount to 0 when the goal is not reached, and $reward_{goal}$ when it is reached.

[5] Or even different Reinforcement Learning algorithms for different (sub)policies.

Algorithm 5: **HABS in pseudo code**: an example with two layers. **RLUPDATE (**...**)** (same as in algorithm 4) updates a policy with a Reinforcement Learning algorithm (see section 2.3). "NULL" indicates that a value is unknown.

HABS ::

while (TRUE) **do** // Policy_2 (high level) loop
  $accumReward_2 = 0$; // For high level reward
  agent is in abstract state $S \in States_2$;
  $policy_2$ selects (sub)policy $policy_{1,active}$; // HL-action = $\text{subpolicy}_{1,active}$
  while (TRUE) **do** // $(\text{Sub})\text{policy}_{1,active}$:(low level) loop
    agent is in state $s_t \in States_1$;
    $policy_{1,sel}$ selects primitive action $a_t \in Actions_1$;
    agent executes action a_t and receives reward $receivedReward_t$;
    $accumReward_2 \leftarrow accumReward_2 + receivedReward_t$; // Accumulate
    if $(Stop_1)$ **then BREAK**; // Terminate subpolicy
    else RLUPDATE ($policy_{1,active}, s_{t-1}, a_{t-1}, 0, s_t, a_t$**)**; // Sparse rewards
  end
  determine current abstract state $S^+ \in States_2$;
  if $(timeout_1)$ **then RLUPDATE (**$policy_{1,active}, s_{t-1}, a_{t-1}, \rho_{timeout}, s_t$, NULL**)**; // Timeout
  else
    determine behaviour *exec* that was executed by $policy_{1,active}$;
    calculate $Classifier_2(exec) = policy_{1,closestMatch}$;
    if $\left(policy_{1,closestMatch} = policy_{1,active}\right)$ **then** // Match:...
      RLUPDATE ($policy_{1,active}, s_{t-1}, a_{t-1}, 1, s_t$, NULL**)**; // ...reward
    else RLUPDATE ($policy_{1,active}, s_{t-1}, a_{t-1}, \rho_{failed}, s_t$, NULL**)**; // No match
  end
  update $Classifier_2$ with new data *exec*;
  RLUPDATE ($policy_2, S^-, oldSUB, oldAccumReward_2, S, policy_{i,closestMatch}$**)**;
  $S^- \leftarrow S$; // Save vars for...
  $S \leftarrow S^+$; // ...next iteration
  $oldAccumReward_2 \leftarrow accumReward_2$;
  $oldSUB \leftarrow policy_{1,closestMatch}$; // see section 5.1.4 for explanation
end

RLUPDATE (POLICY p**, STATE** s_{t-1}**, ACTION** a_{t-1}**, REWARD** r_t**, STATE** s_t**, ACTION** a_t**) ::**

switch $(favourite\ Reinforcement\ Learning\ algorithm)$ **do**
  case (*Q-Learning*) // See section 2.3.3
    update $Q(s_{t-1}, a_{t-1}) \leftarrow (1-\alpha)Q(s_{t-1}, a_{t-1}) + \alpha(r_t + \gamma \max_a Q(s_t, a))$;
  end
  case (*Advantage-Learning*) // See section 2.3.4
    update $A(s_{t-1}, a_{t-1}) \leftarrow$
    $(1-\alpha)A(s_{t-1}, a_{t-1}) + \alpha\left(\max_a A(s_{t-1}, a) + \frac{1}{k}\left(r_t + \gamma \max_{a'} A(s_t, a') - \max_a A(s_{t-1}, a)\right)\right)$;
  end
  ⋮
end

Control Flow

An example is given for two layers: the high level policy $policy_2$ runs until $Stop_2$ indicates termination. $policy_2$ selects a subpolicy $policy_{1,active}$ (its high level action) that it wants to execute when the agent is

in abstract state $S \in States_2$.

Control is passed to $policy_{1,active}$, which is executed until $Stop_1$ indicates termination. At each time step during execution the subpolicy $policy_{1,active}$ is updated with a 0 reward according to reward function $Reward_1$.

After $policy_{1,active}$ terminates its executed behaviour *exec* is determined with the function $Exec_2$. Then it is classified: $Classifier_2(exec) = policy_{1,closestMatch}$. If $closestMatch = active$ then $policy_{1,active}$ is rewarded because it was the closest match. If it is not the closest match because $policy_{1,active}$ executed a behaviour that fits better with some other $policy_{1,otherPolicy}$ then it would receive a punishment ρ_{failed}. If $policy_{1,active}$ terminated because of timeout it is also punished ($\rho_{timeout}$).

The higher level $policy_2$ is then updated according to $Reward_2$ (the accumulative reward) for selecting $policy_{1,active}$ in abstract state S. $Classifier_2$ is also updated.

The control flow (with two layers) is illustrated in algorithm 5 and the relation between components is depicted in fig. 5.1.

5.1.4 Replacing Desired with Actual Behaviour

HASSLE could use the actually reached subgoal instead of the desired subgoal. It just substitutes the high level action that it selected to execute, by the action that it did actually execute (see section 4.2.3). That way HASSLE can make more efficient use of its experience and correct errors due to not yet fully learned subpolicies or exploration (see section 4.2.3). If it ends up in subgoal X it will always use the knowledge that $(goto)\ X$ was the actually executed action.

HABS is very similar to HASSLE in this respect. After the active subpolicy has executed a high level action, the agent can observe which abstract action it has actually performed. Usually this will resemble what this subpolicy actually should do, but sometimes due to exploration it might have done something different. In fact, it will sometimes execute behaviour that "belongs" to another subpolicy (i.e. when $Classifier(exec) = policy_{1,actuallyExecuted}$ and $actuallyExecuted \neq selected$). In that case we can use this information to better train the high level policy. We actually know that the agent has performed behaviour *exec* so we can update it for $policy_{1,actuallyExecuted}$ instead of for selecting $policy_{1,active}$.

This is possible because the designer created the state abstraction and knows its mechanics. In that case an action can be deduced from the knowledge that the agent first was in abstract state A but now in B. HABS and HASSLE both use this principle to correct for errors (mainly in the beginning of learning).

5.1.5 Self Organization

If the classification is not given by the designer, it must be learned. HABS couples subpolicies to classifications, so this learning involves self organization on the part of subpolicies and classifications. This occurs because subpolicies are rewarded for executing behaviour that is classified as belonging to their own class, and on the other hand updating the classification towards recently executed behaviours (if they were successful, i.e. they matched).

During the learning phase, the agent can discover clusters of behaviours, and subpolicies are associated with these clusters. This is done by updating the classification with the behaviour $exec_{t \to t+\Delta t}$ that the active subpolicy $policy_{i,active}$ executed. If the recently executed behaviour $exec_{t \to t+\Delta t}$ is classified by $Classifier_{i+1}(exec_{t \to t+\Delta t})$ as belonging to the class that is associated with the subpolicy that executed $exec_{t \to t+\Delta t}$ or whether it is classified as belonging to the domain of another subpolicy. On the other hand the classification is used to measure their performance and reward the subpolicies accordingly.

Subpolicies are punished when they don't succeed in leaving a high level state (if they don't cover much distance if the state space is continuous) so they are *forced outward*. This prevents the subpolicies from specializing in *standing still* and becoming experts in *doing nothing at all*.

Burden Of Random Walk Exploration

The subpolicies (by means of a good classification) need to cover the occurring behaviours as good as possible, if they are to be useful as actions for the high level policy. But since there is no *a priori* knowledge about what behaviours are needed, the subpolicies start without meaningful behaviour and with randomly initialized characteristic behaviour. This is a crucial difference with other hierarchical Reinforcement Learning approaches, where the structure of desired behaviours (macro-actions, options, subtasks) is typically predefined.

This means that in the beginning (much) exploration is needed on both levels. The high level selects actions (subpolicies) to execute and the subpolicy (while exploring) might stumble upon new high level states. Since high level states are nearby they are reached early on in the learning phase by just randomly exploring. The overall goal on the other hand can only be reached by a long chain of primitive actions, so the expected duration before a random walk would reach the goal, is very high (because the time is quadratic in the distance).

This means that early during the learning phase, the agent is able to discover meaningful behaviours that provide transitions between abstract states. These experiences are (provided the state abstraction is good) clustered together, and the agent will be able to assign subpolicies to each of these newly discovered clusters. These subpolicies will specialize themselves, and will in turn be very useful in solving the overall problem because they execute useful behaviours (subtasks) that are purposeful. And as illustrated in section 3.1.3 and section 3.2.3, large non-random sequences of primitive actions allow the agent to explore further and faster by random walking.

In essence, the burden of random walking is during the learning phase shifted from the level of primitive actions to the level of behaviours. This principle can easily be extended to more than two layers. Once the agent has mastered (small) behaviours for the second layer, it will use these in random walking and then reach third layer abstract states. The random walking burden will be shifted to higher and higher layers.

5.1.6 State Abstraction Suitable for HABS

HASSLE uses abstract states in an absolute way, but as we illustrated in section 4.5 this is not an efficient way to define states, because each action becomes unique and unsuitable for generalization. Abstract states should be defined in a manner similar to the states in the original state space if we want abstract high level actions to be relative.

HABS uses its classification mechanism to go from many (absolute) transitions between high level states, to a few (relative) behaviours. If this classification were given *a priori* and if the designer just specified which high level transitions mapped to which behaviours, there would be no need to learn it, and neither would there be a reason to look for properties of suitable abstractions. However, if the problem is large, the abstract state space will also be large, and specifying the entire mapping might not be feasible.[6] So, we need a way to automatically discover this classification, and for that we need a suitable state abstraction.

The properties implicit behind the HASSLE abstract states, can also be used for HABS. HASSLE has the following properties (section 4.6):

(1) consistent mapping (to): states close together map to abstract states near each other (or the same abstract state)

(2) consistent mapping (from): neighbouring abstract states correspond to nearby regions in the original State Space

(3) a transition in abstract state space is a meaningful change in the original State Space

(4) abstract state space needs to be significantly smaller than the original state space

[6] Similar to why defining an *a priori* Filter is not feasible when the problem is too large (see section 4.4).

(5) there is a limited amount of groups of similar transitions between abstract states

All these properties still hold for HABS. The fifth HASSLE-property is related to the capacities and subpolicies: it assumes that there are only a limited amount of (substantially) different transitions. The Capacities map these transitions to different subpolicies, but the function of the Capacities is replaced by the identification of clusters in the Behaviour Space. The details of the fifth HABS-property will therefore depend on the specific heuristics that are used for the classification.

Circular?

It should be noted that the fifth property seem somewhat circular. We need some knowledge about the solution of the problem to derive a good state abstraction in order to solve the problem. We need to know whether a state abstraction is good (in terms of how many groups there are) but we will only know this for sure after we have mastered the problem and have actually experienced the occurring behaviours, and observed whether they are clustered appropriately.

This means that creating a good state abstraction is an iterative process where some intuition is needed for the designer (just as in the case where the designer needs to specify a task decomposition for an agent). The designer might try a state abstraction, observe the results, and perhaps tweak it a little and try it again.

Humans are experts in abstraction, and a state abstraction that seems reasonable to a human designer will often be according to the properties stated above. And besides that, subpolicies can use function approximators and are therefore flexible enough to handle some irregularity. After all, *quick and dirty* learning is desirable above painstakingly slow learning or no learning at all. So even if a hierarchy is not perfect, it will at least increase learning times and yield a good (though not perfect) solution in a reasonable time.

5.2 Heuristics For HABS — Filling in the Details

HABS is (as presented here) a framework. Depending on what choices are made, different algorithms are created. A choice needs to be made for a suitable state abstraction (and a means of classification) and for the termination criteria.

5.2.1 Action Space and Behaviour Space

For convenience, let's define an *Action Space* to be *the set of all possible difference vectors in that state space* (see fig. 5.3). The *difference vectors* are the vectors that result when the difference between the vector representations of arbitrary states is calculated. This means that the *Action Space* consists of all possible vectors that are confined within the dimensions of the state space (fig. 5.3).

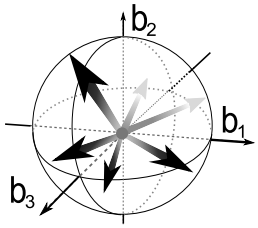


Figure 5.3: Behaviour Space*: the space of all possible difference vectors in a state space (with dimensions* b_1, b_2, b_3*). The arrows are examples of difference vectors between states.*

It is important to note that the primitive actions are also just vectors in an *Action Space*, they form a subset of the entire space. Moreover, the difference vectors that correspond to the primitive actions are the *only* vectors in the Action Space that are *real* in the sense that they do actually occur when solving

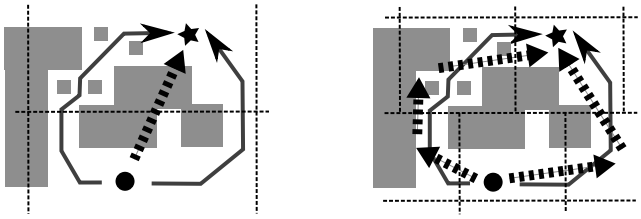
the problem. A transition between two states that are not connected by a primitive action, will not occur in the problem, and is only a theoretical possibility.

There is something else that can be said about the Action Space and the primitive actions. *If* the primitive actions are defined in a relative way (see section 4.5.1), then the primitive actions will all be cluttered together. All the instances of one type of primitive action (e.g. *"pickup"* or *"North"*) will map to the same (or nearly the same) difference vector in the Action Space because of their definition that was relative to the structure in the underlying state space.

We can also construct an Action Space for the Abstract State Space (like those that HASSLE and HABS use). The result is an *Abstract Action Space* or *Behaviour Space*. The actions in the Behaviour Space are entire behaviours consisting of sequences of primitive actions.

5.2.2 Assumption on Difference Vectors

The high level (abstract) actions are treated here as difference *vectors* between two abstract states. This is obviously a simplification.It is not hard to find a counter example against this assumption: suppose we have an obstacle and the agent needs to go around it (see fig. 5.4(a)). It can either go left (through a corridor) or right (open space). The difference vector is the same for both behaviours, because they end up in the same location. But the behaviour itself is very complex (moving through a corridor, avoiding obstacles, etc) and a subpolicy would have a hard time specializing on these widely different behaviours, even though the state abstraction suggests that they are in fact very similar.



(a) counter example (b) solution

Figure 5.4: Counter example*: the abstract state (indicated by dashed lines) are so large that two completely different behaviours (black arrows: left or right around the obstacle (gray area)) result in the same difference vector (dashed arrow).* ***Solution****: the abstract states are smaller, resulting in more – but less complex – behaviours.*

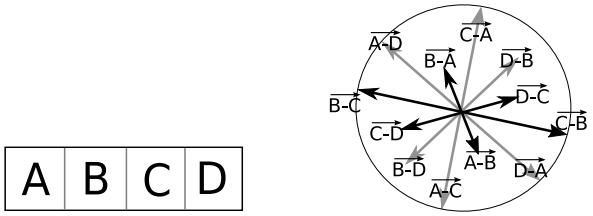
This counter example is the result of making the state abstraction too coarse. This is not what we want when we introduce hierarchies. One of the leading ideas was to obtain small reusable subtasks, but the task that the agent is confronted with here is itself a very large task.

Increasing the number of abstract states (i.e. making it more fine grained), could solve this problem (see fig. 5.4(b)). It results in more abstract states and behaviours, but these are less complex and more similar to each other. When abstract (high level) actions are roughly similar, this opens up possibilities for approximation. It now becomes possible for one subpolicy to specialize in moving through the corridor, and for another to do something else. Each of these behaviours is now different according to the abstraction.

5.2.3 HASSLE Behaviour Space

The Behaviour space that HASSLE uses, is very simple. Since the abstract state space has no structure and the abstract states are only *nominal*: they only have a "name" (or a number) as designation. In this case each difference $\overrightarrow{A-B}$ of two subgoals forms its own dimension, or in the case of $\overrightarrow{A-B}$ and $\overrightarrow{B-A}$ they share the same dimension, but are complete opposites!

For n subgoals there would just be $n \times (n-1)$ arrows (no transitions from A to A) pointing in $\frac{1}{2} \times n \times (n-1)$ different dimensions (because of the opposites). There are as many dimensions as there



(a) abstract state space (b) abstract action space

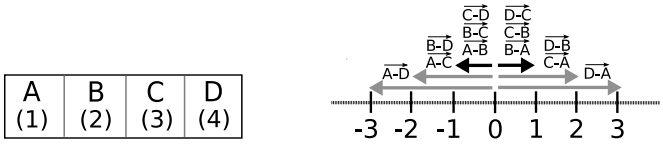
Figure 5.5: Abstract State Space for* HASSLE: a very simple abstract state space. Only A and B, B and C and C and D are connected.* ***Abstract Action Space****: because* HASSLE *subgoals are only nominal, there are* $4 \times (4-1)$ *different vectors for 4 subgoals, so there are* $\frac{1}{2} \times 4 \times (4-1) = 6$ *different dimensions, so each line depicts another dimension! The black arrows denote actually occurring transitions, the gray arrows are transitions that will not occur while solving the problem, because they are impossible transitions.*

are pairs of abstract states. Some of these arrows are actually occurring (high level) actions because they represent transitions between subgoals that are adjacent (see fig. 5.5).

5.2.4 Demands on the HABS Behaviour Space

The abstract state space that HASSLE uses, will not do for HABS. A space where each occurring transition has its own dimension leaves no room for defining the high level actions in a relative way and classifying them automatically – which was what HABS needs.

However, we are not restricted to nominal descriptions of the abstract states. These abstractions where derived from the underlying (original, flat) state space, and that state space has its own structure or geometry. In fact this structure is the reason that the primitive actions do not behave badly but can be defined in a way relative to this underlying structure (see section 4.5.1). Note that this is all under the assumption that the original state space is sufficiently structured, but this is not unreasonable, because many real-world problems have highly structured state spaces: usually Reinforcement Learning problem state spaces do not consist of states that are only nominally described.

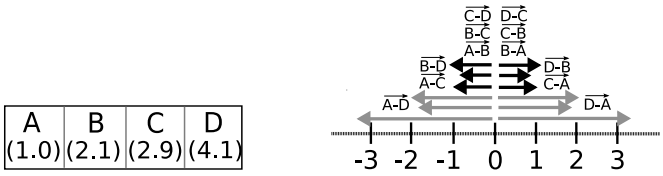


(a) abstract state space (b) abstract action space

Figure 5.6: Abstract State Space*: the same simple abstract state space as in fig. 5.5(a), but this time the abstract states have an internal structure that is reflected in their state description.* ***Abstract Action Space****: due to the internal structure of the abstract state space, the behaviour space is only one-dimensional. The black arrows denote actually occurring transitions, the grey arrows are transitions that will not occur while solving the problem (because they are impossible transitions). Many transitions (high level actions) map to the same difference vectors.*

If we make good use of the underlying structure when we create our abstract states, we can prevent the situation where each transition needs a new dimension because the states are all nominal. This is illustrated in fig. 5.6 where a very simple abstract state space and its accompanying abstract action space are depicted. The abstract states use the underlying geometry of the original state space in their descriptions and therefore many of the high level actions now map to the same difference vector.

This concept is not limited to abstractions that are perfectly regular and where the difference vectors are identical between many of the abstract states (as often is the case for the primitive actions between states). If high level actions are roughly the same (as in fig. 5.7) these difference vectors will still be close to each other even though they are not exactly the same. There are in fact several distinct groups



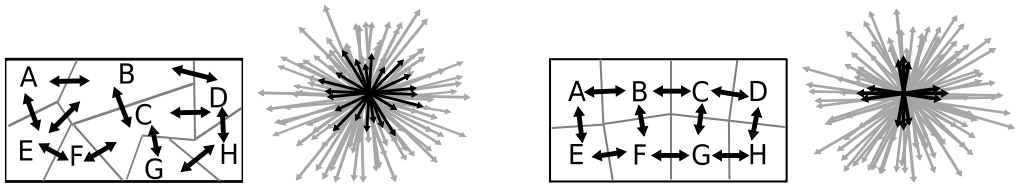
(a) abstract state space (b) abstract action space

Figure 5.7: Less regular Abstract State Space*: the same simple abstract state space as in fig. 5.5(a) and fig. 5.6(a), but now the state descriptions of the abstract states are less regular.* ***Abstract Action Space for the less regular abstract state space****: the abstract action space still has only one dimension. Many transitions (high level actions) are represented by roughly the same difference vectors.*

or clusters of difference vectors. This means that there is a good probability that one subpolicy would be able to execute all the behaviours in a cluster. We would only need as many subpolicies as there are clusters and that is also the number of actions that our high level policy would have.

Many Different Abstractions Are Possible

While it is true that HASSLE and HABS can work with the situation in fig. 5.8(a), it would cost a lot. There are so many different behaviours, that HASSLE and HABS would need many subpolicies (in fact, both would need the same number of subpolicies) and many of these subpolicies would only be good for executing a few transitions *or even only one*, because there are so many different transitions between high level states.



(a) state abstraction 1 (b) behaviour space 1 (c) state abstraction 2 (d) behaviour space 2

Figure 5.8: state abstraction 1 and behaviour space 1*: the behaviour space has two dimensions, but the actually occurring transitions between abstract states are scattered (black arrows) all over the behaviour space.* ***state abstraction 2 and behaviour space 2****: the behaviour space has two dimensions and the actually occurring transitions are clustered together around four vectors.*

Both algorithms can in principle deal with situations like fig. 5.8(a) as long as the dimension of the behaviour space is not too high, but it would need many subpolicies to cover all the needed behaviours. This is only a two dimensional example, but it is clear that many behaviours scattered all over a high dimensional behaviour space require way too many subpolicies to be practical.

The problem with the situation in fig. 5.8(a) is that all the (actually occurring) high level actions are scattered all over the behaviour space (fig. 5.8(b)). In this case it will be hard to use any generalization (or more layers), because each of the high level actions is different from the other, and if many of the high actions are still unique, we would still have an action explosion. HABS and HASSLE both need many similar transitions (property (5) in section 5.1.6). In fact, any hierarchical approach that wants to re-use subtasks, needs something like this fifth property.

But there are many ways to do state abstraction for a given state space. Suppose we would do it like in fig. 5.8(c), then we would obtain the behaviour space as seen in fig. 5.8(d). This does result in a desirable situation, because many high level actions are mapped to vectors that are near to each other and into four rather distinct groups.

Filling In The Fifth Property

In conclusion,we demand from our abstract state space that it gives us these useful clusters of high level actions, instead of behaviours scattered all over the place. This is illustrated in fig. 5.9.

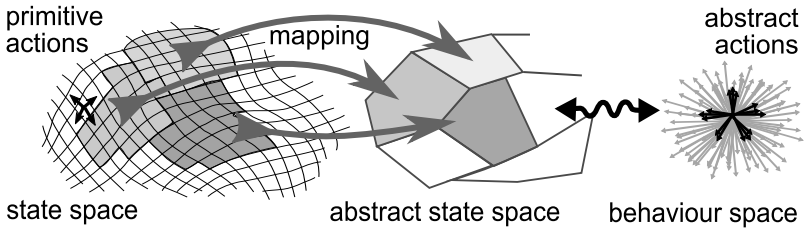


Figure 5.9: Abstract states as used in* HABS: the mapping from state space to an abstract representation preserves (some of the) underlying structure of the state space and ensures that the actually occurring high level actions are highly clustered (the black arrows). See fig. 4.3 (the* HASSLE *mapping) for comparison.***

In light of the analysis above about the need of an underlying structure in the abstract state space (in order to derive a good heuristic), we can now fill in the fifth property that we gave for HABS (see section 5.1.6). The fifth property can be made more precise in terms of the Behaviour Space and the assumption related to difference vectors (section 5.2.2) can be included:

(5'a) actually occurring transitions between abstract states need to be distributed *non-uniformly* in the Behaviour Space. They need to form a limited amount of distinct groups

(5'b) difference vectors that are similar should correspond to similar behaviours

In fact, the more non-uniformly the behaviours are distributed, the less distinct groups or clusters there are and the fewer subpolicies (i.e. high level actions) HABS needs. This means that a state abstraction is better if it has fewer clusters of behaviours.

5.2.5 Classification and Clustering

For the experiments described in this thesis, a very simple mechanism is used to do the classification. Since behaviours are treated as vectors (which is a simplification) it is reasonable to use a method that is suited to vectors, therefore an adaptive clustering algorithm was selected.

A cluster is assigned to each $subpolicy_i$. The cluster center $char_i$ can be considered the *characteristic behaviour* of that subpolicy. During learning, the cluster center is moved towards newly executed behaviour *if* this recent behaviour was already classified as belonging to this cluster. The update is done according to:

$$\overrightarrow{char}_{i,\,t+\Delta t} \leftarrow (1-\omega)\cdot\overrightarrow{char}_{i,\,t} + \omega\cdot\overrightarrow{exec}_{i,t\rightarrow t+\Delta t} \tag{5.1}$$

where $\overrightarrow{exec}_{i,t\rightarrow t+\Delta t}$ is the behaviour that the subpolicy executed starting at time t and ending at $t+\Delta t$ (calculated with $Exec_i$) and $\overrightarrow{char}_{i\,t}$ is the characteristic behaviour vector (i.e. cluster center) that was assigned to the subpolicy that just executed $\overrightarrow{exec}_{i,t\rightarrow t+\Delta t}$. The factor $\omega\,(0\leq\omega\leq 1)$ is learning rate determines how much the characteristic behaviour is moved towards the newly executed behaviour.

The definition of $Exec_i$ is related to the clustering. This function needs to calculate the executed behaviour $\overrightarrow{exec}_i$ and represent it in a suitable way. When the behaviours are treated as vectors, it is defined as the difference vector between the abstract states:

$$\overrightarrow{exec}_i = Exec(S,S') = \overrightarrow{S'_i - S_i} \tag{5.2}$$

Each subpolicy is assigned a cluster center and will specialize in that cluster, but these clusters will in turn provide the goal conditions for the subpolicy. If a subpolicy does something that is similar to "its"

cluster, then it will be rewarded, but at the same time the clustering is updated with this recently executed behaviour (equation 5.1). This way the subpolicies will tune in on clusters of behaviours that are actually occurring in the problem, they will self organize to cover the needed behaviours. If the clustering and the state abstraction are suitable, each characteristic behaviour (classification) will gravitate towards actual clusters (illustrated graphically in fig. 5.10).

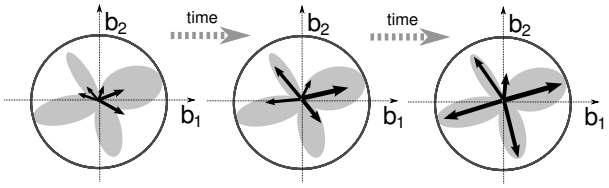
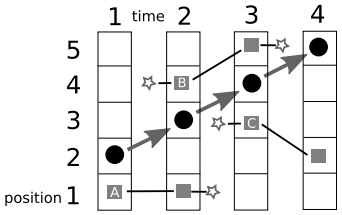


Figure 5.10: Changing characteristic behaviour vectors*. The gray areas depict pockets of behaviours that actually occur in the abstract state space. The arrows are the characteristic behaviours organizing themselves to cover the needed behaviours.*

Not All Features Are Related To Behaviour

In principle all elements of the behaviour vectors could be used. However, this is often not a good idea because many of the elements in the so-called *behaviour* vector, have nothing to do whatsoever with the behaviour of the agent. An object that moves by itself could create the illusion that its movement is actually part of the behaviour of the agent.

This can be illustrated by a simple example where an agent has only the two actions *Up* and *Down* and is situated in a grid world where objects move by themselves or appear and disappear autonomously (see fig. 5.11). The state (observation) of the agent consists of the following: $\langle position,\ object(+1),\ object(+2), object(+3),\ object(-1), object(-2),\ object(-3)\rangle$. The term $object(d)$ indicates whether or not there is an object present at distance d from the agent. So at the first time step, the observation is $< 2,\ 0,0,0,\ 1,0,0 >$ because the agent is at position 2, and there are no objects upwards (nothing at +1, +2 and +3), but there is one object (object A) one step downwards, so $object(-1) = 1$.



$$\overrightarrow{state_1} = \langle 2,\ 0,0,0,\ 1,0,0\rangle$$
$$\overrightarrow{state_2} = \langle 3,\ 1,0,0,\ 0,1,0\rangle$$
$$\overrightarrow{state_3} = \langle 4,\ 1,0,0,\ 1,0,0\rangle$$
$$\overrightarrow{state_4} = \langle 5,\ 0,0,0,\ 0,0,1\rangle$$

Figure 5.11: Autonomous objects*: this illustration shows four progressive time frames with an agent moving and some objects moving and (dis)appearing without involvement of the agent. The stars denote (spontaneous) creation or deletion of an object. The line are the "world lines" of the objects and the arrows depict the actions of the agent. To the right, the observations at different time steps are given. The state of the agent consists of the following:* $\langle position, object_{+1}, object_{+2}, object_{+3}, object_{-1}, object_{-2}, object_{-3}\rangle$*, where* $object_d$ *indicates whether or not there is an object present at distance* d *from the agent.*

If we would calculate the executed behaviour from time step t to $t+1$, using the difference between the vectors, we would arrive at:

$$\overrightarrow{behaviour_1} = \overrightarrow{state_2} - \overrightarrow{state_1} = \langle +1,\ +1,0,0,\ -1,+1,0\rangle$$
$$\overrightarrow{behaviour_2} = \overrightarrow{state_3} - \overrightarrow{state_2} = \langle +1,\ +1,0,0,\ +1,-1,0\rangle$$
$$\overrightarrow{behaviour_3} = \overrightarrow{state_4} - \overrightarrow{state_3} = \langle +1,\ -1,0,0,\ -1,0,+1\rangle$$

It is obvious that the first element of these vectors (+1) corresponds with the move upwards. However, some of the other elements are rather confusing. If the agent would try to discover clusters of similar behaviours in this action space, it would discover that $\overrightarrow{behaviour_1}$ and $\overrightarrow{behaviour_2}$ are similar for five out of seven elements, but that the $object_{-1}$ and $object_{-2}$ differ radically. On the other hand, for $\overrightarrow{behaviour_2}$ and $\overrightarrow{behaviour_3}$ the element denoting $object_{+1}$ is opposite, and so on.[7]

It would seem that elements in the environment that act on their own, are rather disruptive for the notion of an action space, because many of the features display contradictory values at different time steps. On the other hand, these disruptive elements can be identified easily. They are the features of the state (or state abstraction) that are not related to what the agent does, but to what is happening in the environment. These can be filtered out by the designer *a priori* (as is done for the experiments reported in this thesis, see section 6.4.2) or possibly even by the agent itself (see section 7.2.2 on future work with automatic detection of features).

5.2.6 Termination and Moving Significant Distances

The specific way in which the termination criteria are handled, depend on what kind of state abstraction is used. The definition for $Stop_{i<n}$ for HABS:

$$Stop_{i<n} = \begin{cases} terminate & \text{If } timeout_i \vee distance(S_i, S_i') > \delta_i \\ continue & \text{otherwise} \end{cases}$$

does not resemble that for HASSLE:

$$Stop_1 = \begin{cases} terminate & \text{If } timeout \vee (S \Rightarrow S' \wedge S, S' \in States_2 \wedge S \neq S') \\ continue & \text{otherwise} \end{cases}$$

Instead, the definition for the termination criteria for HABS is stated in terms of the (rather vague) $distance(S_i, S_i')$. This term indicates that the agent has moved a *significant distance through state space*. This leaves open how exactly we would determine the significance of a certain movement.[8]

This open formulation allows freedom on the part of the designer. A criterion similar to that of HASSLE could be used, but separation of the termination criteria from the abstract states, also allows the use of continuous high level states for determining subpolicy termination, or even a mixture of discrete conditions for termination *and* a continuous high level state for the high level policy.

Stopping Like HASSLE

If the state space abstraction $States_{i+1}$ is discrete, then the significant movement through state space *can* simply be equivalent to moving from one abstract state to another. The termination criterion for the subpolicies in the layer below ($discreteStop_i$) can then simply be:

$$discreteStop_i = \begin{cases} terminate & \text{If } timeout \vee (S_i \Rightarrow S_i' \wedge S_i, S'_i \in States_{i+1} \wedge S_i \neq S'_i) \\ continue & \text{otherwise} \end{cases}$$

This means that the active subpolicy terminates as soon as a new high level state is reached. This is exactly the same as in HASSLE. In effect the *significant distance* is implicit in the state abstraction: reaching a new high level state by definition means that the agent has moved a significant distance through the underlying "flat" state space.

Note that this way of defining significant movement, results in situations where the agent just steps from one side of the border to the other with one primitive action. This will always happen when state abstractions are discrete.

[7] The problem remains if we would use absolute coordinates for the positions of the objects instead of coordinates that are relative to the position of the agent. The fact remains, that the objects move with respect to each other, in whatever reference frame we take.

[8] The termination criteria could even be defined in a probabilistic manner, though not in with the definition presented here.

How To Stop In A Continuous (Abstract) State Space

The abstract state space can also have a continuous nature. In a continuous (abstract) state space the distance could be measured measured in terms of a difference vector between start and current location:

$$continuousStop_{i<n} = \begin{cases} terminate & \text{If } timeout \vee (distance(S,S') > \delta \ \wedge \ S,S' \in States_{n+1}) \\ continue & \text{otherwise} \end{cases}$$

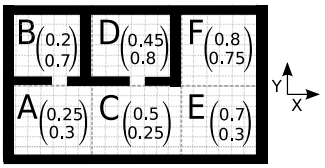
This *distance* could be the standard Euclidean distance $distance(S,S') = |\overrightarrow{S'-S}|$. However, some other measure of distance, more appropriate for that particular abstract state space, is also possible.

Hybrid Approach

The abstract state space could also be continuous, while the termination criterion is still discrete (or vice versa). The high level policy uses the continuous abstract state (for instance because it has a neural network as function approximator) and at the same time the active subpolicies are terminated when a new (discrete) area, similar to that used by HASSLE, is reached.

5.3 A Simple Example

To get an idea of how HABS works, a simple example (fig. 5.12) will be presented. This is the same setup as in the HASSLE example, section 4.1.3. Only now the states have a better (more informative) description: coordinates derived from the structure of the state space.



$$\begin{aligned} states_2 &= \{room\,A_{(0.25;0.3)}, room\,B_{(0.2;0.7)}, \ldots, room\,F_{(0.8;0.75)}\} \\ actions_2 &= \{subpolicy_1, subpolicy_2, \ldots, subpolicy_7\} = Policies_1 \\ Actions_1 &= \{North, East, South, West\} \\ States_1 &= \{all\ grid\ cells\} \end{aligned}$$

Figure 5.12: The Example Grid World

Heuristics — State Abstraction, Classification and Termination

The coordinates provide the the first heuristic (see section 5.2). With the coordinates of the underlying state space, the high level abstract states can be given coordinates (for instance with a clustering algorithm). These coordinates will become the descriptions of the abstract states. The abstract states can now be used to define behaviours and termination criteria. A subpolicy always terminates if it reaches a new abstract state (and of course when it reaches its timeout).

The behaviour space is just a two-dimensional space (see fig. 5.13(a)). The executed behaviour is defined as

$$\overrightarrow{exec} = Exec(S,S') = \overrightarrow{S'-S} = (x_2 - x_1; y_2 - y_1)$$

and the actually occurring behaviours are just the vectors belonging to the differences between adjacent high level states.

There are seven subpolicies, and each of them gets assigned a (initially completely random) vector $\overrightarrow{char}_i$. These vectors are the characteristic behaviour vectors of the subpolicies. These vectors will be updated according to equation 5.1:

$$\overrightarrow{char}_{t+\Delta t} \leftarrow (1-\omega) \cdot \overrightarrow{char}_t + \omega \cdot \overrightarrow{exec}_{t \to t+\Delta t}$$

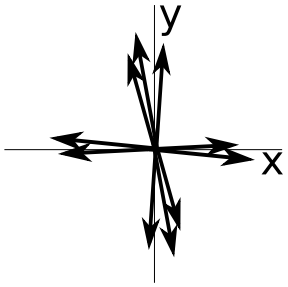
The characteristic vectors, together with a means of determining distance constitute $Classifier_2$. The distance between vectors in the behaviour space are measured with the Euclidean distance function:

$$||\vec{w} - \vec{v}|| = \sqrt{\sum_i (w_i - v_i)^2}$$

Learning

If the agent finds itself in room (abstract state) A, it can select one of the high level actions $subpolicy_1$, $subpolicy_2$, ..., $subpolicy_6$. Suppose it selects $subpolicy_6$. Perhaps the agent is going to explore (selecting a random high level action) or perhaps $Q(room\,A, subpolicy_6)$ has a higher value than any other high level action in that state.

The agent hands over execution to $subpolicy_6$. The subpolicy will execute some primitive actions (i.e. from $Actions_1$) and after some time either reach a new high level state, or it times out[9]. During execution, it will (perhaps) receive some rewards, which are summed up (*summedReward*) and will be used for the high level after termination.



(a) behaviour space

	cluster center	distance to *exec* $(0.25; -0.05)$
$subpolicy_1$	(-0.039 ;-0.321)	0.396
$subpolicy_2$	(0.109 ;-0.549)	0.519
$subpolicy_3$	(0.057 ; 0.727)	0.801
$subpolicy_4$	(-0.454 ;-0.117)	0.707
$subpolicy_5$	(-0.007 ; 0.001)	0.262
$subpolicy_6$	(0.361 ; 0.051)	0.150
$subpolicy_7$	(0.013 ;-0.002)	0.242

(b) characteristic behaviour vectors

Figure 5.13: Example Behaviour space *for fig. 5.12* ***characteristic behaviour vectors****: a snapshot of the cluster centers, and the Euclidean distance to the executed behaviour exec* $= (0.25; -0.05)$.

Rewarding the Subpolicy and Policy

After the subpolicy terminates, the agent can observe what high level state it is in, and can determine what high level action it has executed (as explained in section 5.1.4). Suppose our agent actually wandered into room C, then its actually executed behaviour $\overrightarrow{exec}$ can be calculated:

$$\overrightarrow{exec} = \vec{C} - \vec{A} = (0.5; 0.25) - (0.25; 0.3) = (0.25; -0.05)$$

The next step is to classify $\overrightarrow{exec}$ with $Classifier_2$. The classifier checks the distance between $\overrightarrow{exec}$ and all the cluster vectors (see table 5.13(b)). The winning cluster *is the cluster belonging to subpolicy 6*, so $Classifier_2(\overrightarrow{exec}) = subpolicy_6$. This means that $subpolicy_6$ has executed a behaviour that "belongs" to it, and needs to be rewarded for that. It receives a reward of 1 for its last (low level) action. During execution of the subpolicy, all the primitive actions get a zero reward.

Suppose that it was not $subpolicy_6$ that the agent had selected, but $subpolicy_7$, but the agent still ended up in room C. In that case the winning cluster was not the cluster that executed the behaviour. In that case it receives ρ_{failed} as a punishment. The other alternative is, that the active subpolicy failed to get out of the high level state it started in, and times out. If a timeout occurs it is punished with $\rho_{timeout}$. In these cases the clustering is not adjusted.[10]

[9] If no timeout is used, a (bad) subpolicy could go on literally forever by just staying inside the high level state (*looping*).

[10] A more complicated classifier could move $char_i$ away from the behaviour executed by $subpolicy_i$ (*exec*) if it was not the winner.

On the high level, the agent will update $policy_2$ (equation 2.18, Q-learning):

$$Q(room\,A, subpolicy_6) \leftarrow (1-\alpha)\cdot Q(room\,A, subpolicy_6) + \gamma \cdot summedReward$$

Adjusting the Clustering

Finally, the clustering $Classifier_2$ needs to be adjusted (assuming that $Classifier_2(\overrightarrow{exec}) = subpolicy_6$):

$$\begin{aligned} \overrightarrow{char}_6 &\leftarrow (1-\omega)\cdot \overrightarrow{char}_6 + \omega \cdot \overrightarrow{exec} \\ &= (1-0.03)\cdot(0.361;0.051) + 0.03\cdot(0.25;-0.05) = (0.358;0.048) \end{aligned}$$

After all this, the high level policy can select a new subpolicy to execute.

Chapter 6

Experiments

Several experiments were done to test both the fix for HASSLE (the filter proposed in section 4.4) and the new algorithm HABS. The first part of the experiments deals with HASSLE-filters. The second with experiments where HABS uses a tabular Q-values function on its high level (and is compared to unaugmented HASSLE). The third (and most extensive) part consists of experiments where HABS uses a neural network on its high level and is compared to a flat learner.

The experiments are partly spatial, but also contain non-spatial elements. This is done in order to show that HABS and HASSLE can handle problems whose state space has elements which are not spatial (but still have some inherent geometry or structure) like objects which can be picked up and dropped and which can either be or not be in the agent's possession.

6.1 The Environment

6.1.1 Grid Worlds

All experiments were done in grid worlds (like the one in fig. 6.1) where the agent needed to retrieve one or several (similar) objects and drop them at a designated location. Some of the cells were marked as *walls* (i.e. impenetrable) and other cells as *drop zones*.

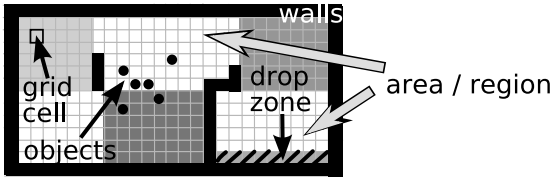


Figure 6.1: Grid World*: an example of a grid world. The small squares are the (grid) cells, the large (gray scaled) areas are the (high level) regions and the shaded cells represent "drop zones". Black cells are walls (i.e. inaccessible). Black dots represent objects.*

High Level Regions

The cells in the grid worlds are clustered into areas or regions. These regions are used for the high level states (see below). They represent a spatial abstraction on the grid world. The regions form natural divisions of the state space, for example rooms or corridors (see fig. 6.1).

Objects

There were one or more objects present in the environment (see fig. 6.1). These objects were distributed randomly over the free cells (i.e. not on walls and not on drop zones). There could only be one object

in each grid cell, but the agent could carry many objects at the same time (up to its cargo capacity). The objects carried by the agent were not considered as being in a grid cell.

The agent in all experiments *always* received a reward of 1 when it dropped an object at a *drop zone*.[1] In some of the experiments additional "pickup rewards" were also applied in order to help the learners in solving the task: the agent received a small "pickup reward" (0.1) for picking up the object, and a corresponding small punishment (-0.1) for dropping it (regardless of the location).

6.1.2 The Agent

For the experiments with HASSLE and HABS, the agent was given two layers (therefore the subscripts are $_{ll}$ and $_{HL}$ instead of the less descriptive $_1$ and $_2$). On the top layer there is always only one policy and the low level always consisted of several (sub)policies. This number varied depending on the complexity of the task. Both the policy and the subpolicies use *Advantage Learning*, (see section 2.3.4) in these experiments. The "flat" learners that were used for comparison, also used this Reinforcement Learning algorithm. Advantage Learning was selected instead of the more common Q-learning, because it works better with neural networks due to its scaling factor K.

Low Level Motor Controls

The agent was given the primitive actions *North*, *East*, *South*, *West*[2], *Pickup* and *Drop* (similar to Dietterich's well-known taxi task [27]). These actions were executed perfectly: when the agent selected one of the actions, it executed exactly that action. Actions (and sensors) without noise are used because we are (for now) not interested in how well a Hierarchical Reinforcement Learning agent performs with noisy motor controls or sensors. Given the fact that all layers in the hierarchy use standard Reinforcement Learning techniques, the tolerance for noise in motor controls or sensors will probably be similar to that of flat learners with the same techniques.

Low Level Sensors

The low level sensors (the low level states) are (in principle) the same in all experiments for HASSLE and HABS. The lower level $\overrightarrow{state}_{ll}$ results from observation. It is a vector composed of the following:

$$\overrightarrow{state}_{ll} = \overrightarrow{radar}_{ll\,objects} +\!\!+ \overrightarrow{radar}_{ll\,dropZones} +\!\!+ \overrightarrow{radar}_{ll\,walls} +\!\!+ \overrightarrow{position} +\!\!+ \overrightarrow{cargo} \qquad (6.1)$$

where $\vec{a} +\!\!+ \vec{b}$ denotes concatenation of vectors $\vec{a}$ and $\vec{b}$.

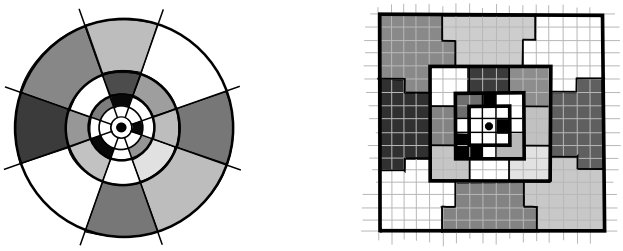
Part of this observation vector ($\overrightarrow{radar}_{ll\,objects}$, $\overrightarrow{radar}_{ll\,dropZones}$ and $\overrightarrow{radar}_{ll\,walls}$) was generated by a sensor grid. The area around the agent was divided into concentric circles – although perhaps "concentric squares" would be a better term because the grid world uses squares as cells (see fig. 6.2(a) and (b)). The sensor grid was also divided into eight arcs.

Areas between circles that are further away from the agent are larger because the space between the circles increases. This allows for a fine grained observation near the agent but at the same time prevents an overflow of information. Areas that are further away are not observed in detail but less fine grained. For the grid world, this sensor grid was implemented by replacing the circles with squares, as can be seen in fig. 6.2(b).

The division in rings and arcs creates areas (in multiples of eight) and each of the areas corresponds with an element in the observation vector. The *average* density of the observed entity in the area is the value that is used in the observation vector (i.e. the grayscales in fig. 6.2). Each of the observable entities

[1] Simpler experiments were done where the agent only needed to reach a certain location. These experiments served only to develop and debug the algorithms and are not shown here, except for one example in the section on HASSLE filters.

[2] Some experiments were also done with the actions *Move*, *Rotate-left*, *Rotate-right*, *Pickup*, *Drop* but these didn't provide different results, other than that all times were somewhat longer, because each turn costs the agent another time step with these motor controls.



(a) circular sensors (b) square sensors

Figure 6.2: Circular Sensors*: concentric circles around the agent (black dot) and a division in eight arcs. The further away from the agent, the larger the space between the circles.* ***square sensors****: an adaptation of the circular sensors to a rectangular grid. Each area yields one value for the observation vector. The colours represent the values of each area.* ***Zooming in on the sensor grid****: the average density of the observed quantity in each of the 24 areas (8 per ring) is a value in the observation vector (a gray area in fig. 6.2(b)). In this example the vector resulting from this observation would be:* $\langle \frac{1}{28}, \frac{2}{24}, \frac{3}{28}, \frac{4}{24}, \ldots, \frac{3}{8}, 0, 0, \frac{1}{6}, \ldots, 1, \frac{2}{3}, 0, \frac{1}{3}, 1, \frac{2}{3}, 0, 0, 0, 1, 1, 0, 1, 0, 0, 0, 0 \rangle$.

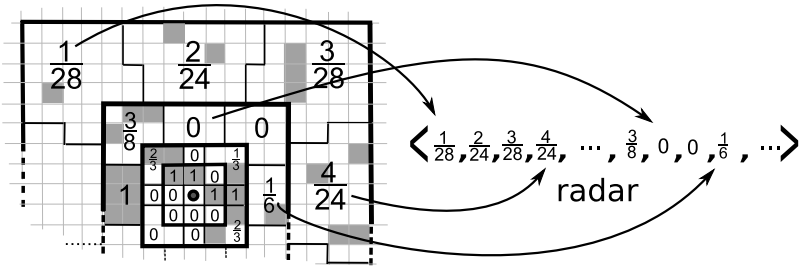


Figure 6.3: Zooming in on the sensor grid*: the average density of the observed quantity in each of the 24 areas (8 per ring) is a value in the observation vector (a gray area in fig. 6.2(b)). In this example the vector resulting from this observation would be:* $\langle \frac{1}{28}, \frac{2}{24}, \frac{3}{28}, \frac{4}{24}, \ldots, \frac{3}{8}, 0, 0, \frac{1}{6}, \ldots, 1, \frac{2}{3}, 0, \frac{1}{3}, 1, \frac{2}{3}, 0, 0, 0, 1, 1, 0, 1, 0, 0, 0, 0 \rangle$.

(i.e. *walls*, *objects* and *drop zones* has its own sensor grid. An example of a calculation of part of the observation vector is illustrated in fig. 6.3. For experiments involving only one object, the area that the object was in simply yielded 1 instead of the average. For experiments with more objects, the average was also used for $\overrightarrow{radar}_{ll\ objects}$.

The position[3] of the agent was added as two scaled values between 0 and 1 (scaled to the width and height of the grid world) to $\overrightarrow{state}_{ll}$, resulting in the vector $\overrightarrow{position} = (x_{scaled}, y_{scaled})$.

The cargo part $\overrightarrow{cargo}$ equals 1 if the agent is carrying the object, and $\overrightarrow{cargo} = (0)$ otherwise.

The Subpolicies

HABS and HASSLE both use the same kind of subpolicies. Each of these subpolicies consists of a set of neural networks. Each of the primitive actions has its own neural network, so for the low level each subpolicy has six networks, and each of the networks calculates $Q(s,a_i)$ for action a_i for one of the subpolicies.[4]

The low level structure is the same for all HASSLE and HABS experiments (although details like the

[3]Initial experiments without the position were also carried out but the results did not differ much from those with information about the position, so they are not described here.

[4]The alternative is to give each subpolicy only one large network with as many outputs as there are primitive actions. If there is no hidden layer, these two alternatives amount to exactly the same structure. If there is a hidden layer, there is a difference: the hidden layer is used by each of the actions in the latter case, but not in the former. Initial experiments showed that separate networks results in significantly higher performance, probably because the actions cannot interfere: they don't share the hidden layer.

number of hidden neurons or learning rates differ).

6.1.3 Some Remarks on Boxplots

Many of the graphs presented in this chapter use box plots, which are an easy way of indicating the spread of a data set. The box indicates the middle 50 percent of the data (all data between the first ($x_{.25}$) and third ($x_{.75}$) quartile). The thick line indicates the median.

For a normal box plot, all values that are lower than $x_{.25} - 1.5 \cdot (x_{.75} - x_{.25})$ or higher than $x_{.75} + 1.5 \cdot (x_{.75} - x_{.25})$ are considered "*outliers*". The "*whiskers*" (i.e. the lines) would indicate the data point closest to (but inside) these boundaries. The outliers themselves would be represented by dots. However, since the graphs in this thesis often compare multiple plots, a full blown box plot would become to complicated, obfuscating the graph. Therefore in this thesis the bars above and below the box indicate the maximum and minimum value in the data set.[5]

Some of the box plots in this thesis are not using a data set from one experiment run several times with the same settings. *Instead, several experiments with different settings were run several times, each yielding an average performance for that particular setting.* These averages are then used as the data set for a box plot, resulting in a plot that shows the spread of averages over different settings.

This way of showing the data was selected because it gives a better idea of how robust the algorithms are and how they perform for various different settings. A wide spread means that different settings yield very different performances and indicates that the algorithm in question needs much fine tuning. Even if in the best case it can perform very good, it could still be outperformed on average by another algorithm that has far better performance over a wider range of settings. It is also easier to tune the latter algorithm because it performs good on a wider range of settings.

6.2 Augmenting HASSLE — Filtering

As explained in section 4.4, HASSLE can be augmented by some sort of filtering mechanism to improve learning times. A very simple learning[6] filter was implemented to illustrate this principle (section 4.4.2).

The learning filter consists of a filter table (similar to the high level Q-values table) that registers the amount of success that the agent has had in executing a certain action in a certain state. The success rate obviously changes during the learning process, so some sort of moving average or window is needed. This is implemented by updating the *successRate* value in the direction of 1 when a certain action resulted in reaching a new state, and to 0 when the agent didn't leave its current high level state:

$$successRate(state, action) \leftarrow 0.9 \cdot successRate(state, action) + 0.1 \cdot newSuccess(state, action) \quad (6.2)$$

where 0.1 functions as a learning rate and $newSuccess(state, action) = 1$ when a new high level state is reached and 0 otherwise. This is similar to a Reinforcement Learning update, but without discount.[7]

When the *successRate* is above a threshold for a certain action, a constant[8] value σ is added to the Q-value of that action:

$$\sigma(state, action, successRate) = \begin{cases} \sigma & \text{If } successRate(state, action) > 0.05 \\ 0 & \text{If } otherwise \end{cases} \quad (6.3)$$

The combined value $Q(state, action) + \sigma$ is used for action selection instead of $Q(state, action)$, using

[5] Experiments with the same settings were repeated at least eight times, for some experiments more.

[6] Creating an *a priori* filter is not interesting enough to investigate further, since it would only involve selecting by hand which high level actions are available at which time

[7] It might be interesting though to expand this filtering mechanism to include discounting.

[8] Using $\sigma(state, action, successRate) = successRate(state, action)$ or some other (monotonously) increasing function would favour the actions that are easiest. Using a constant gives equal boost to each action that the filter allows.

the Boltzmann selection mechanism (described in section 2.4.2):

$$P_{BoltzFilter}(s,a_i) = \frac{e^{(Q(s,a_i)+\sigma(s,a_i,successRate))/\tau}}{\sum_{a' \in actions} e^{(Q(s,a')+\sigma(s,a',successRate))/\tau}} \tag{6.4}$$

where $P_{BoltzFilter}(s,a_i,successRate)$ gives the probability of selecting a_i in s using the filter mechanism, s is the current state, a_i is the action under consideration and τ is the temperature.

6.2.1 Tabular Representation of the High Level Q-Values

The high level states for tasks where the high level has a tabular representation of the Q-function, are rather straightforward. As explained above, the cells in the grid world are clustered. These regions are used for the high level state.

The tabular representation was used for the experiments with only one object, so it suffices to use one boolean that indicates whether the agent is carrying the object or not. The high level state (i.e. sensor) is the tuple of the high level region the agent is in, the high level region the object is in, and a boolean for possession of the object:

$$\overrightarrow{state}_{HL,\text{HASSLE}} = \left(region_{agent}, region_{object}, \overrightarrow{cargo}\right) \tag{6.5}$$

Obviously $region_{object} = region_{agent}$ if the agent is carrying the object. This means that nearly half[9] of the possible combinations, like $(region_A, region_B, 1)$ will not occur in reality. It is impossible for the object to be in $region_B$ and the agent to be in $region_A$ and possess the object at the same time.

This does not pose a huge problem for HASSLE, although it means it has nearly twice as many high level actions as would be possible if the illegal possibilities were filtered out. Furthermore it is not a large factor. Leaving the illegal possibilities available as high level actions, illustrates that HASSLE can cope with them.

Capacities

HASSLE has its Capacities mechanism to match high level actions to subpolicies. For this, some parameters need to be assigned values. This was done during initial testing. The Capacities mechanism was of the following form (eq. 4.2):

$$\begin{aligned} C_{i,act}(start,goal) &\leftarrow C_{i,act}(start,goal) + \Delta C_{i,act}(start,goal) \\ \text{with} \quad \Delta C_{i,act}(start,goal) &= \begin{cases} \alpha_C^r \cdot \left(\gamma_C^{\Delta t} - C_{i,act}(start,goal)\right) & \text{success} \\ \alpha_C^f \cdot (0 - C_{i,act}(start,goal)) & \text{failure} \end{cases} \end{aligned}$$

where α_C^r and α_C^f are learning rates, and γ_C is a discount, measuring performance. HASSLE worked well with $\alpha_C^r = \langle 0.03, 0.003 \rangle$, $\alpha_C^f = \langle 1, 0.1 \rangle \cdot \alpha_C^r$ and $\gamma_C = 0.99$ (all yielding the same performance) so these values were used for all HASSLE experiments. The Capacities also need a selection mechanism. As in [1] and [2], Boltzmann selection was used, with $\tau = 0.03$ (but other values yielded similar results).

Subpolicies

HASSLE uses subpolicies with linear neural networks. The learning rate α_{ll} for each of the low level networks was set to 0.01 and for the high level to 0.05. The high level uses a tabular representation. Both layers use a simple ε-greedy selection.

[9] Nearly half of the possible combinations will not occur, because if the agent is not carrying the object, all combinations are possible (half of the total number). But if the agent is carrying the object, only $(region_{agent}, region_{agent}, 1)$ is possible, but this is only a very small part of all the possible combinations of regions.

6.2.2 Experiment 1 – Moving to a Fixed Location

Two tasks were created to investigate the performance of the simple learning filter. The first consists of an environment with several rooms (fig. 6.4, with same dimensions as the environment used to test HASSLE in [1] and [2], but with slightly different layout). Each of the rooms was considered a high level state, and the corridors were divided into areas of similar size. If the agent reaches the target, it receives a non-zero reward and the episode ends. If the agent fails to reach the target in the allotted time of 1000 (low level) steps, the episode is also terminated.[10] At the beginning of each episode, the agent is placed randomly on one of the cells (target excluded) of the grid world. The calculated optimum is an average of 54 steps. This is calculated by selecting 10^6 random starting locations for the agent and for each calculate the shortest path to the target.

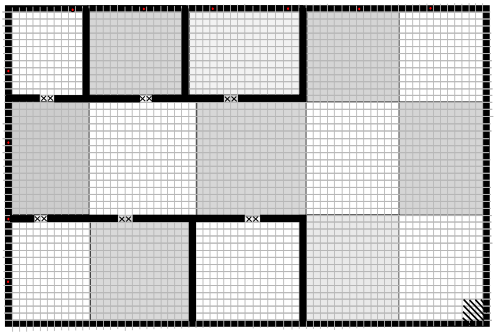
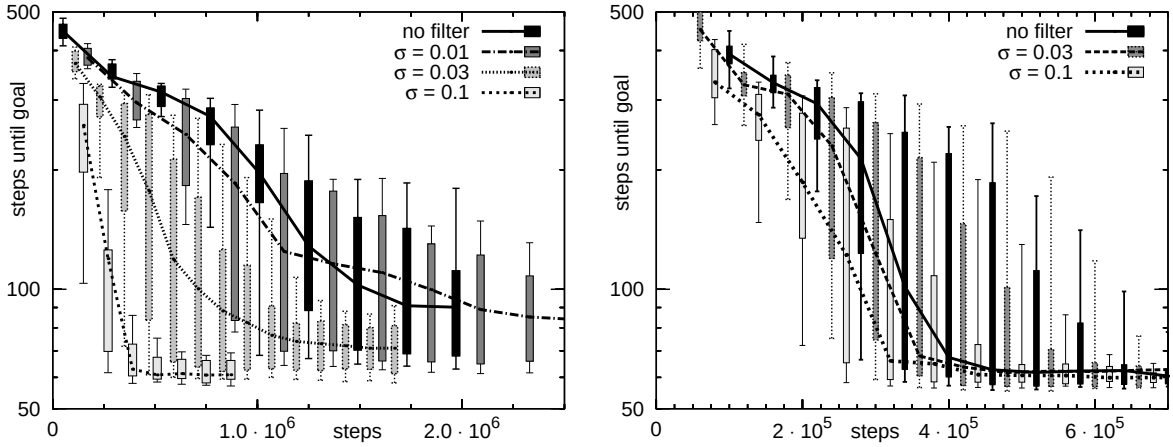


Figure 6.4: environment for experiment 1*: a grid world similar to that presented in [1] and [2], consisting of* 68×48 *cells and 15 high level states. The shaded area represents the target.*



(a) $\gamma_{HL} = 0.95$ (b) $\gamma_{HL} = 0.99$

Figure 6.5: performance of filters in experiment 1*:* HASSLE *without filtering is compared with* HASSLE *augmented with filters (for various settings of the filter constant* σ*).* ***(a)*** $\gamma_{HL} = 0.95$ *and* ***(b)*** $\gamma_{HL} = 0.99$*. The graphs show boxplots over different settings (*$\{10, 15\}$ *subpolicies,* $\tau_{ll} = \{0.0167, 0.04\}$ *and* $\tau_{HL} = \{0.0167, 0.033\}$*.* $\alpha_{ll} = 0.003$*,* $\alpha_{HL} = 0.03$*,* $\gamma_{ll} = 0.95$*,* $\gamma_{HL} = 0.99$*,* $K_{ll} = 0.3$ *and* $K_{HL} = 0.3$*. Higher values for the temperatures (*$\tau_{ll} = 0.10$ *or* $\tau_{HL} = 0.10$*) for either of the levels resulted in bad performance for both filtered and unfiltered* HASSLE*) The averages of the different settings are the data for the boxplot, resulting in plots that show the robustness of the algorithms.*

The graphs shown in fig. 6.5(a) and fig. 6.5(b) illustrate the performance of HASSLE[11] with and

[10] In fact, the agent was allowed to finish the current high level action (i.e. until timeout or success), meaning that at most $1000 + timeout_{ll}$ steps were taken.

[11] HABS is also able to solve this problem in roughly the same time as HASSLE, i.e. in $2.5 \cdot 10^5$ time steps, reaching conver-

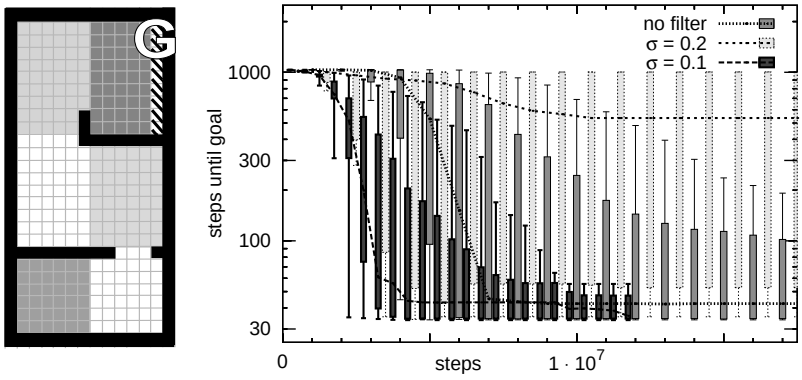
without filter. For this experiment, two different high and low level Boltzmann temperatures and two different numbers of subpolicies were tried (resulting in 8 different combinations of settings). Several runs were done for each of these settings and averages were calculated. These averages are then displayed in the form of a box plot.

When a discount of 0.95 is used for this task, the unaugmented HASSLE is unable to even reach the optimum value. The filtered version has no problems at all for this low discount. In fact, given a good value of the constant σ the filtered version can solve this task in under half a million low level steps very consistently for all settings (which is the same as unaugmented HASSLE with $\gamma_{HL} = 0.99$). Higher σ values (not shown here) will deteriorate the learning process, whereas lower values signify less influence of the filter. A value of $\sigma = 0.01$ for this experiment amounts to the same as not using a filter at all. HASSLE can also reach nearly the same performance without filtering (indicated by the bottom of the boxplots) but only for some of the settings. Other settings result in average performances that are far from optimal. Filtering ($\sigma = 0.1$) can reach values near 57 steps and on average reaches 61 steps. HASSLE without filtering can still reach performance below 70 for some settings, but other settings perform as bad as 180 steps on average.

The graph depicting the results for a high level discount $\gamma_{HL} = 0.99$ shows that HASSLE is able to catch up better for higher discounts. On average it performs nearly as well, *but* its spread is wider than when using a strong filter ($\sigma = 0.1$). Unfiltered HASSLE is able to reach a similar highest performance as the filter, around 57 or 58 steps, and the average performance is also similar, around 60 or 61 steps.

6.2.3 Experiment 2 – Retrieving an Object

A second experiment was done to see how the simple filter would perform on a task with more high level states, because that is where the filtering should benefit the most. An object is introduced at a random location in the environment shown in fig. 6.6(a), and the agent needs to retrieve this object and drop it at one of the cells marked as a 'drop location' ("G"). As explained in section 6.2.1, the high level state of the agent consists of the area the agent is in, together with the area the object is in and a boolean indicating whether the agent is carrying the object. This means that there are $7 \times 7 \times 2 = 98$ high level states and therefore 98 high level actions, *most of which* are not reachable from many of the other high level states, because the agent cannot for example go from $< 1, 5, false >$ to $< 1, 3, true >$ in one high level step.



(a) s-curve (b) results

Figure 6.6: experiment 2*: an object needs to be retrieved in an s-curved grid world.* ***performance of filters in experiment 2****:* HASSLE *without filtering is compared with* HASSLE *augmented with filters (for two settings of the filtering constant σ). The graphs show boxplots over several different settings (in order to show robustness). The graphs show boxplots over different settings: $\{10, 15\}$ subpolicies, $\gamma_{ll} = 0.95$, $\gamma_{HL} = \{0.95, 0.99\}$, $\tau_{ll} = \{0.02, 0.04\}$, $\tau_{HL} = \{0.02, 0.04\}$, $\alpha_{ll} = 0.003$, $\alpha_{HL} = 0.03$, $K_{ll} = 0.3$ and $K_{HL} = 0.3$.*

gence just below 60.

The same effects as in experiment 1 can be observed in experiment 2 (see fig. 6.6(b). Unaugmented HASSLE is able to reach a near optimal value, but clearly not for all tested settings. Using the filter (with $\sigma = 0.1$ makes the algorithm twice as fast (on average) and reaches a near optimal performance of 34 (optimal ≈ 33.7) for some settings, and an average of 55 for the worst case settings. HASSLE without filtering reaches similar values for its best case settings, but does far worse for other settings (100 or even 200 steps on average) and takes far longer time. Setting the filter constant σ too high ($\sigma = 0.2$) results in failure for some of the runs, that is why on average it performs worse than unaugmented HASSLE and HASSLE with more moderate filtering.

6.2.4 On the Usefulness of Filtering

Experiment 1 and 2 show that (some kind of) filtering can be useful. A filter biases exploration (and thereby learning) towards high level actions that can actually be executed by the agent. HASSLE augmented with a filter can reach the same (or better) performance in a shorter time – if the filter is properly tuned. With a good filter HASSLE is able to perform good under a wider range of parameters.

The filtering proposed here can only work in conjunction with Boltzmann selection however, which means that it is also dependent on the weakness of Boltzmann selection: it will be slow when there are many actions because the computationally heavy function e^z needs to be calculated for all actions.[12]

It is not hard to give a good guess for the value of σ because the effectiveness of this constant is related to the size of the problem and the Q-values that are expected. If σ is far greater than the Q-values, the value $e^{(Q(s,a_i)+\sigma)/\tau}$ will be nearly the same (i.e. $e^{(\sigma)/\tau}$) for each of the actions a_i. This would result in a uniform probability distribution *over all reachable actions* for the Boltzmann selection. If σ is too low then the difference between $e^{(0+\sigma)/\tau}$ (a reachable state, but no Q-value) and $e^{0/\tau}$ (no reachable state) will be too small to make a difference, again resulting in a nearly uniform probability distribution, this time over all actions, both reachable and unreachable, which is simply the situation that HASSLE also runs into without the filtering and before learning meaningful Q-values.

6.3 Comparing HASSLE, HABS and the Flat Learner

HASSLE (in its unaugmented form, without any filters) is compared to HABS on two large tasks to demonstrate the limits of the HASSLE architecture. A "maze" and a "big maze" environment were created and the task was to collect the object (placed at a random location at the start of every new episode of 1000 time steps) and drop it at the drop area.

When the agent dropped the object at one of the correct locations it received a reward of 1. Two alternatives were tried for the maze: one where this is the only reward that the agent ever gets, and another where the agent also receives the small "pickup rewards" for picking up an object.

6.3.1 The Learners

HABS and HASSLE both use subpolicies with linear neural networks. HABS uses the same $\overrightarrow{state}_{ll}$ as HASSLE (equation 6.1). Both layers of both algorithms use a simple ε-greedy selection.[13]. HABS and HASSLE use a tabular representation for their high level policies.

[12] When the Q-values are approximated with – for instance – a neural network, calculating them becomes more time consuming than the Boltzmann selection. But HASSLE works with a tabular high level policy, so its Q-values are simply stored in a look-up table (which is many times faster than a neural network) and in that case the computation e^z is relatively heavy.

[13] It is difficult to solve this task with HASSLE using Boltzmann selection (the experiments used for tuning HASSLE with Boltzmann selection are not reported here). This is probably due to the fact that there are so many high level actions. Boltzmann selection (see equation 2.21) has random selection (uniform distribution) as one of its limits (for high τ, the other is ε-greedy selection for low τ). With many actions, a lower selection temperature τ is needed, creating a higher selection pressure. This has its own problems however, which are mainly computational: Iff τ is low, then the value of $e^{Q(s,a')/\tau}$ near the target (where $Q(s,a') \approx 1$) will sky-rocket and easily become larger than what the programming language can compute with the standard data types.

A few heuristics need to be selected for HASSLE and HABS. They both need a suitable state space abstraction. Furthermore, HABS needs criteria for classification (section 5.2.5) and for termination (section 5.2.6). Despite the fact that all these heuristics depend on each other, they will be described separately. It should be noted however, that a choice for a particular state abstraction is partly driven by considerations about what kind of classification it could yield, and vice versa (see section 5.1.6).

HABS Heuristics — Abstract State Space

HASSLE can use the vector $\overrightarrow{state}_{HL,\text{HASSLE}}$ defined earlier (equation 6.5) in section 6.2.1. HABS needs a suitable representation of the state abstraction, because it needs structure to be able to discover behaviour *vectors*A nominal description of the high level state in terms of regions contains not enough information. Therefore the position of the regions is used.[14] This results in the vector:

$$\overrightarrow{state}_{HL,\text{HABS}} = \left(X_{agent}, Y_{agent}\, X_{object}, Y_{object}, \beta \cdot \overrightarrow{cargo}\right) \tag{6.6}$$

where β is a scaling factor, which is needed to give the vector element for the cargo roughly the same size as the elements for the position to ensure that a change in cargo (picking up or dropping the object) creates a difference between vectors that is of the same order as the difference created by moving from one region to another.[15] A value of $\beta = 0.1$ worked well, but this obviously depends on the size of the grid world and on the distances between abstract states[16].

HABS Heuristics — Classification

The vector $\overrightarrow{state}_{HL,\text{HABS}}$ was used as a measure for the executed behaviour when the agent moved from one high level state to another. The behaviour space therefore consists of the dimensions contained in $\overrightarrow{state}_{HL,\text{HABS}}$. The characteristic behaviour vectors are allowed to self organize in this space. A simple vector clustering algorithm (eq. 5.1, as described in section 5.2.5) was used, that moves a cluster center ($\overrightarrow{char}$) towards the recently executed (and observed) behaviour $\overrightarrow{exec}_{t\rightarrow t+\Delta t}$:

$$\overrightarrow{char}_{t+\Delta t} \leftarrow (1-\omega)\cdot\overrightarrow{char}_t \;+\; \omega\cdot\overrightarrow{exec}_{t\rightarrow t+\Delta t} \tag{6.7}$$

The characteristic behaviour vectors were always initialized with small random values.[17]

HABS Heuristics — Termination Criteria

The termination criteria for HABS were very simple. Entering a new high level region or increasing or decreasing the amount of objects in cargo by one, indicated the termination of a subpolicy. This amounts to stopping criteria equal to those of HASSLE (see section 5.2.6).

Flat Learner

For comparison, the tasks were also solved using a standard Reinforcement Learning agent . This "flat" agent used a tabular representation of the Q-function. The location (x- and y-coordinates) were used as state (because they are unique). The tabular representation allows a rather high learning rate. The flat learner used advantage learning (k = 0.3, same as for HASSLE and HABS) and uses Boltzmann selection.

[14] This position is the average (scaled) position of all cells contained in the region and is denoted (X,Y) where X and Y are scaled to a value $\in [0,\ 1]$ according to the dimensions of the grid world.

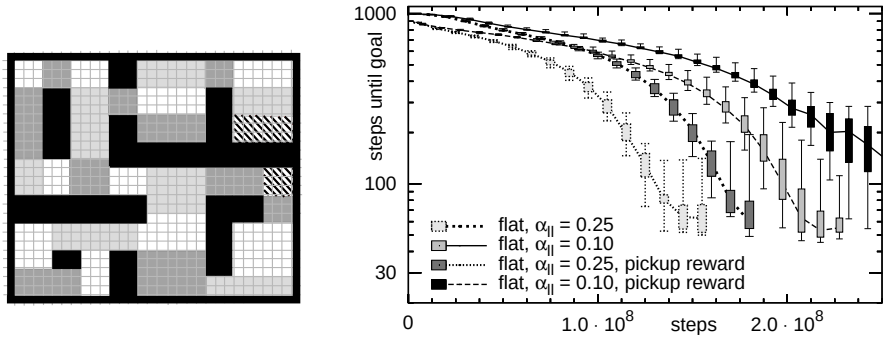
[15] Some sort of scaling will always be needed when different types of dimensions are compared. If comparing dimensions is impossible, the classification by vectors cannot be done in the simple way it is done here. In that case, a behaviour might for instance consist of multiple vectors, each representing differences in parts of the behaviour space that *are* comparable. See section 7.2.1 (Future Work)

[16] When a continuous high level state space is used, the scaling could depend on the distance used in the termination criteria, see section 5.2.6.

[17] Some tests were done with initializing each vector with the first behaviour that the accompanying subpolicy actually executed. This made no difference however, because none of the subpolicies is actually capable of performing reasonable behaviours in the beginning.

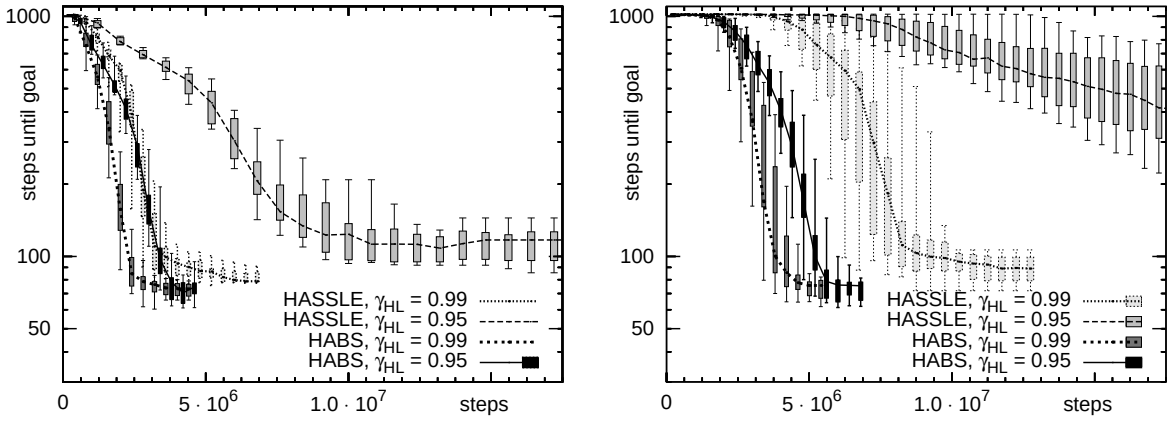
6.3.2 The Maze

The Maze (fig. 6.7(a)) has 29 areas, resulting in $29 \times 29 \times 2 \approx 1.7 \times 10^3$ different high level states (*area that the agent is in* × *area that the object is in* × *whether or not the agent carries the object*). The flat learner is able to solve the task in somewhere over $1 \cdot 10^8$ time steps and reaches a best result of around 50 steps (see fig. 6.7(b)). The calculated optimum for this problem is 40.2.



(a) maze (b) results flat learner

Figure 6.7: (a) maze*: The maze is* 31×28 *cells, has 29 areas, resulting in* $29 \times 29 \times 2 \approx 1.7 \times 10^3$ *abstract states.* ***(b) flat learner results****: with and without the "pickup reward" and for* $\alpha_{flat} = \{0.25, 0.10\}$*.* $K_{flat} = 0.3$*,* $\tau_{flat} = 0.02$ *(same results for* $\tau_{flat} = 0.01$ *and* $\tau_{flat} = 0.04$*) and* $\gamma_{flat} = 0.99$



(a) pickup reward (b) no pickup reward

Figure 6.8: maze experiment*: box plots over averages of runs for* HASSLE *and* HABS*.* ***(a) pickup reward****: The agent receives an additional reward of 0.1 when it picks up the object, and a reward of -0.1 when it drops the object.* ***(b) no pickup reward****: there is no additional reward.* ***settings****:* HASSLE *and* HASSLE *have 10 subpolicies,* $e_{ll} = \{0.15, 0.1\}$*,* $e_{HL} = \{0.1, 0.05\}$*,* $\alpha_{ll} = 0.01$*,* $\alpha_{HL} = 0.05$*,* $\gamma_{ll} = 0.95$*,* $\gamma_{HL} = 0.99$*,* $K_{ll} = 0.3$ *and* $K_{HL} = 0.3$*,* $\alpha^r_C = \{0.03, 0.003\}$ *and* $\alpha^f_C = \{1, 0.1\} \times \alpha^r_C$ *(*α^r_C *and* α^f_C *are learning rates for the Capacities mechanism, see section 4.1.2).* HABS *uses* $e_{ll} = \{0.15, 0.1\}$*,* $e_{HL} = \{0.1, 0.05\}$*, and* $\omega = \{0.001, 0.01, 0.1\}$ *(*ω *is the learning rate for the clustering). The agent receives a reward of 1 for dropping the object at the correct location.*

HABS and HASSLE can both solve this task. HASSLE is however much more sensitive to the high level discount γ_{HL} than HABS, which makes is harder to fine tune (see fig. 6.8(a) and 6.8(b)). Furthermore, HABS is able to reach a slightly better performance, especially when the task gets more complicated (i.e. without the "pickup reward" that splits the large task into two smaller tasks).

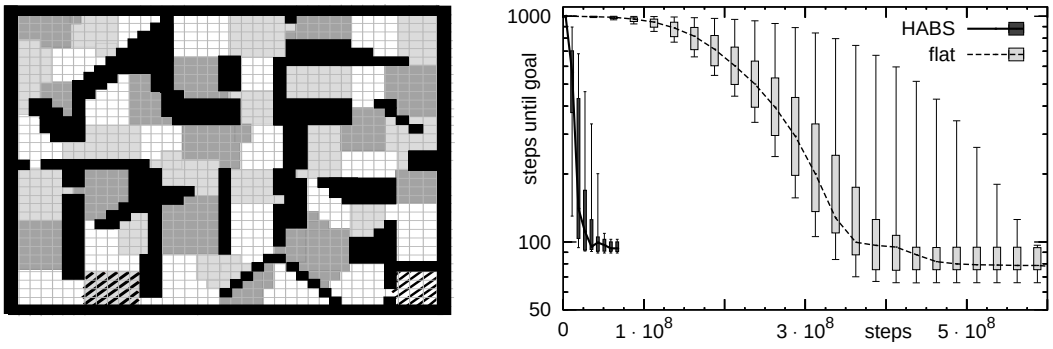
The most striking difference is that HABS has no real problems when the task increases in size (no more "pickup rewards"), whereas the number of steps that HASSLE needs to reach convergence,

increases much more. The time that HABS needs until convergence roughly doubles but HASSLE need three times as much low level steps at its best performance (for $\gamma_{HL} = 0.99$).

6.3.3 The Big Maze

A second maze was created with more high level states and a more complicated layout. This "big maze" (fig. 6.9(a)) has 50 areas, resulting in $50 \times 50 \times 2 \approx 5.0 \cdot 10^3$ high level abstract states. The agent only received a non-zero reward when the object was correctly dropped.

All experiments done with HASSLE on this large environment, simply *fail* due to lack of memory (on a computer with 500 Megabyte of RAM). The "big maze" clearly marks the limits of HASSLE. The smaller maze, with some $1.7 \cdot 10^3$ high level states, was still manageable, but an environment with three times as many abstract states pushes the envelope too far.



(a) big maze (b) results

***Figure 6.9: (a) big maze**: 36×39 cells, 50 regions. **(b) results**: performances for HABS and the flat learner for a range of parameters. HABS: 25 subpolicies, $\gamma_{ll} = 0.95$ $\gamma_{HL} = \{0.95, 0.99\}$, $\varepsilon_{ll} = 0.85$, $\varepsilon_{HL} = \{0.01, 0.05\}$, , $\alpha_{ll} = 0.01$, $\alpha_{HL} = 0.03$, $K_{ll} = 0.3$, $K_{HL} = 0.3$, $\rho_{failed} = \{1, 0.3, 0, -0.3\}$ Flat: $\varepsilon_{flat} = \{0.7, 0.85, 0.95\}$, $\alpha_{flat} = 0.2$, $\gamma_{flat} = \{0.9, 0.95, 0.99\}$ and advantage $K_{flat} = 0.3$. (Higher discount γ_{flat} results in slower learning.)*

Performance

HABS can solve the "big maze" task consistently in under well $5 \cdot 10^7$ time steps, whereas the flat learner takes $4 \sim 6 \cdot 10^8$ time steps. The flat learner is able to reach a higher performance of around 70 but HABS only reaches 90. The optimum for this task is 59.6. If accuracy is of the essence and there is enough time, the "flat learner" is preferable. HABS is however able to solve this problem in a *quick-and-dirty* way an order of magnitude faster and with far lower memory requirements.

Memory Requirements

The failure of HASSLE, due to lack of memory, is not surprising, because five thousand high level states amount to $(5.0 \cdot 10^3)^2 \approx 2.5 \cdot 10^7$ Q-values that need to be stored (i.e. for each of the state-action-pairs), and on top of that, the same amount *for each subpolicy* because of the Capacities tables.[18] This means that HASSLE would need to store some 2.5 to $5.0 \cdot 10^8$ values if 10 or 20 subpolicies are used. Coded in (standard *Java*) *doubles* of 32 bits, this would mean some 2 to 4 Gigabyte of memory. Obviously some memory can be saved by only storing high level states that are actually possible within the environment. This roughly halves the amount of memory needed.[19]

[18] The few kilobytes needed for storing the neural networks, can safely be ignored.

[19] A hash table was used to accomplish this: only storing a high level state and all accompanying high level actions when the agent was in that particular high level state.

Furthermore, memory can be saved by using 4 *byte* instead of 8 *byte* (Java-*double*) values, or even a 2 *byte* coding (resulting in coarser representation, but since the Q-values are limited to a relatively small range, this does not pose a problem: it only needs to approximate $[-10, 10]$ (to be on the safe side), not all of $\mathbb{R}$.). With all these approaches combined, the amount of memory was still too much for the available computer.

Obviously the precise boundary is dependent on the amount of RAM memory that a computer has (and also partly on how efficient the algorithm is programmed and the memory is used) but the square Q-values tables (as many high level actions as there are states) and the accompanying Capacities-tables (with the same dimensions) are an integral part of HASSLE, and will therefore always cause this kind of trouble[20]. Large amounts of abstract states clearly present problems for HASSLE because of their enormous memory requirements.

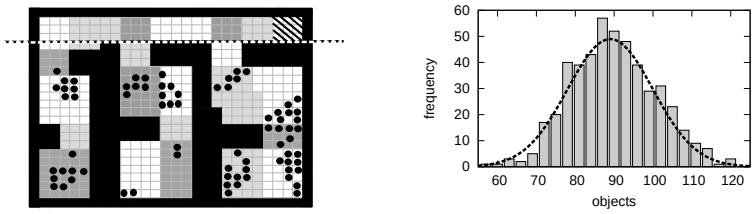
The memory requirements of the "flat" learner are less extreme. It doesn't have the high level subgoals and neither does it have the Capacities and subpolicies, so it can describe the problem in $2 \times (39 \times 36)^2 \approx 4 \cdot 10^6$ states ($position_{agent} \times position_{object} \times hasObject$), and therefore some $2 \cdot 10^7$ Q-values (one for each of the six primitive actions in each state). This requires something just under $2 \cdot 10^2$ Megabytes (when 8 byte Java-*doubles* are used).

HABS needs to store far fewer Q-values than either HASSLE or the "flat" learner. It has the same number of high level states, but does not use these states as high level actions, but uses its subpolicies. And since there are only a few subpolicies, HABS does not have a square Q-values table but only needs to store $n \times 5.0 \cdot 10^3$ Q-values (where n is the number of subpolicies, for each $Q(state, subpolicy_i)$-pair), together with the few Kilobytes for the subpolicy neural networks. This amounts to some $10^2 \sim 10^3$ Kilobytes, orders of magnitude less than HASSLE or the flat learner.

6.4 The Cleaner Task — Description

One of the reasons that HABS was designed, was that the structure of HASSLE is unsuitable for use of neural networks on the high level. The following task was designed to illustrate the ability to use neural networks for the high level policy.

An environment is created (see fig. 6.10(a)) that contains randomly scattered objects that need to be picked up and dropped at the drop zone. The agent is allowed to carry as many as 10 objects in its cargo bay. The agent has 1000 time steps to return as many as the objects as possible to the drop area, after which the episode was terminated and a new one was started with the agent and the objects at random locations. All the graphs depict the average number of objects that was dropped at the drop zone per episode.



(a) cleaning environment (b) object distribution

Figure 6.10: snapshot of the cleaner environment*: the black-and-white dotted line denotes the boundary for the objects. No object was placed randomly above this boundary at the beginning of the episode.* ***distribution of objects****: objects are placed at random at the beginning of each episode, with an average of 89 objects and a standard deviation of 11 (normal distribution). A histogram over 486 runs is shown.*

[20] If a filter were used, yet *another* huge table would be added to this!

6.4.1 Object Placement

The number of objects is determined by the (normal) distribution shown in fig. 6.10(b) with an average of 89 and a standard deviation of 11. At the beginning of each episode the objects are scattered in 20 patches or clusters (5×5 cells, the centers of which are selected randomly), but these patches may overlap and not all the cells in a patch are filled with an object (see fig. 6.10(a) for a snapshot). No objects were placed above the dotted line in fig. 6.10(a) to make the task harder.

Objects In The Cargo Bay

The agent uses neural networks as function approximators both on the lower level and on the higher level and its cargo can be up to 10 objects (its *capacity*). Simply giving the number of objects in cargo $\in (0,1,\ldots,10)$ or scaled ($\in \left(\frac{0}{10},\frac{1}{10},\ldots,\frac{10}{10}\right)$) would give the neural network not enough information. Differentiating between 6 or 7, or even between or something or nothing in cargo, is very hard for a function approximator.

The current cargo of the agent was therefore represented in a more complicated manner to give the neural networks a better opportunity to discriminate between values. The cargo part ($\overrightarrow{cargoBay}$) of the observation vector consists of the following elements:

$$\overrightarrow{cargoBay} = \left(\frac{cargo}{capacity},\ 1 - \frac{cargo}{capacity},\ cargo \geq_? 2,\ cargo \geq_? 4,\ cargo \geq_? 6,\ cargo \geq_? 8 \right) \tag{6.8}$$

where $cargo \geq_? n$ is defined as:

$$cargo \geq_? n = \begin{cases} +1 & \text{If } cargo \geq n \\ -1 & \text{If } cargo < n \end{cases}$$

The vector $\overrightarrow{cargoBay}$ is substituted for the simpler $\overrightarrow{cargo}$ used in equation 6.1, resulting in:

$$\overrightarrow{state}'_{ll,\text{HABS}} = \overrightarrow{radar}_{ll\,objects} +\!\!+ \overrightarrow{radar}_{ll\,dropZones} +\!\!+ \overrightarrow{radar}_{ll\,walls} +\!\!+ \overrightarrow{position} +\!\!+ \overrightarrow{cargoBay} \tag{6.9}$$

6.4.2 High Level States Used with Function Approximation of the Q-Values

For experiments where multiple objects need to be collected, a tabular representation of the Q-Values is not suitable.[21] This also means that another representation for the high level states is needed, because $\overrightarrow{state}_{HL,\text{HABS}}$ as presented above (in section 6.2.1), gives too little information for a neural network. The problem is complicated further because there is an *a priori* unknown (but high) number of objects present in the environment, and the state representation needs to be able to represent all these varying numbers of objects and locations accurately.

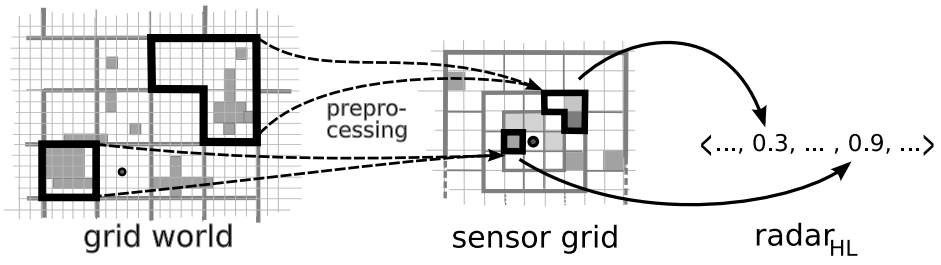


Figure 6.11: High Level Sensor Grid*: The grid world is viewed on a coarser level by the high level sensor grid $radar_{HL}$. Blocks of many grid cells (in this case 5×5 cells) are averaged and with the resulting values the same calculation as in fig. 6.3 (the low level sensor grid, $radar_{ll}$) is done.*

[21] In fact, that is exactly the reason why such an experiment was selected: to demonstrate that HABS can use function approximators for its high level, unlike HASSLE, and therefore solve tasks that HASSLE can't because of its tabular nature.

This is a problem that is very similar to how the low level needs to represent the locations of all the walls, objects and drop zones. In fact, the high level sensors $\overrightarrow{radar_{HL,}}$ can be created similar to those of the low level. But now each square in fig. 6.3 represents many grid cells (for instance blocks of 5×5 cells), instead of only one grid cell. This process is depicted in fig. 6.11. The resulting $\overrightarrow{radar_{HL,objects}}$, $\overrightarrow{radar_{HL,dropZones}}$ and $\overrightarrow{radar_{HL,walls}}$ are then used to create $\overrightarrow{state'_{HL,\text{HABS}}}$ (similar to $\overrightarrow{state_{ll}}$):

$$\overrightarrow{state'_{HL,\text{HABS}}} = \overrightarrow{radar_{HL,objects}} +\!+ \overrightarrow{radar_{HL,dropZones}} +\!+ \overrightarrow{radar_{HL,walls}} +\!+ \overrightarrow{position} +\!+ \overrightarrow{cargoBay} \quad (6.10)$$

Preprocessing the grid world in this manner means that the smallest areas in the high level *sensor grid* (i.e. those consisting of only one square in the sensor grid in fig. 6.11) now represent an average over a block of many grid cells in the grid world.

Reduced Radar Range

In most experiments, the high level sensor grid consisted of only two or three rings. This was done because the fourth ring is so far away from the agent, that most of the time it actually observed cells *outside* the grid world! In those cases the radar detecting the presence/absence of walls by default registered walls (i.e. "outside" is inaccessible) and no objects or drop zones.[22]

HABS Heuristics — Classification and Filtering Out Irrelevant Features

For classification, the vector described in equation 6.6 and clustering described in equation 6.7 are used:

$$\overrightarrow{state}_{HL,\text{HABS}} = \left(X_{agent}, Y_{agent}\, X_{object}, Y_{object}, 0.1 \cdot cargo\right)$$

and

$$\overrightarrow{char}_{t+\Delta t} \leftarrow (1-\omega) \cdot \overrightarrow{char}_t \;+\; \omega \cdot \overrightarrow{exec}_{t \rightarrow t+\Delta t}$$

This vector worked well for the experiments with the tabular high level policy as well as the Cleaner task with the function approximator as high level policy.

For the experiments reported in this thesis, the relevant features are determined *a priori* by the designer. The features for the high level abstraction are the cargo and position of the agent and of the object. For determining *behaviours*, the position of the object is actually irrelevant. This position is only relevant for the agent when it needs to *select* what kind of behaviour it will execute, but the area the object is in, is not related at all to what kind of behaviour the agent *executes*. It is needed in the abstract state, but is it not needed at all for classification of behaviour.

This irrelevant feature could have been excluded *a priori* in the classification phase[23] but this was not done. Leaving the irrelevant feature(s) in the vector, and using them in the classification (even though their influence can only be disruptive or neutral at best) shows that HABS can deal with some irrelevant features.

HABS Heuristics — Termination Criteria

The termination criteria for HABS for the function approximator tasks, are the same as for the tabular case. Entering a new high level region or increasing or decreasing the amount of objects in cargo by one, indicated the termination of a subpolicy.

[22] If the preprocessing takes blocks of 5×5 cells, the fourth ring ranges from 22 cells from the agent, up to 42 cells away ($22\ cells = 2\ (from\ the\ center\ area\ to\ the\ first\ ring) + 5\ (first\ ring) + 5\ (second\ ring) + 10\ (third\ ring)$ and $42 = 22\ (until\ third\ ring) + 20\ (fourth\ ring)$, whereas the grid worlds were between 20 and 40 cells wide.
During initial experiments with the range of the high level radar, it was discovered that adding or removing the fourth ring did not make any difference (apart from saving a lot of running time due to smaller observation vectors and therefore smaller neural networks!). Even removing the third ring (only using two rings with a range of only 12 cells), resulted in rather good results, although somewhat lower than with three rings.

[23] Excluding irrelevant features can be done by assigning *zero* as a scaling factor for the irrelevant elements of the vector.

This is an example of the hybrid case mentioned in section 5.2.6. The abstract state space is treated as continuous by the function approximator (the neural network), although technically the low and high level radar observations are only discrete (though *astronomical* in size: each of the roughly 100 (for the low level) or 80 (for the high level) values in the observation vector delivered by the radar, can have many values ranging between 0 and 1).[24]

6.4.3 Forcing HABS to Explore

Since HABS works with neural networks on both levels in the Cleaner task, there were some concerns about whether Boltzmann selection would do the trick. Neural networks are sometimes unstable with Reinforcement Learning (leading to early suboptimal convergence or even to all weights fading to zero exploding to infinity) and on top of that HABS uses subpolicies that are untrained at the start.

The high level uses subpolicies as its actions, so if a subpolicy is not yet fully trained, it might be that it fails too often on a certain task, making the Q-values for that subpolicy rather low. Another subpolicy that does something different, but with higher Q-values, would then be preferred by the high level policy.

Suppose that it is optimal for the agent to completely fill its cargo bay, but that it is still not that good in locating and picking up objects, meaning that it wastes a lot of time searching and eventually the 1000 time steps are over and the episode stops. Even though it did collect some objects in its cargo bay, it didn't return them to the drop zone and therefore didn't receive any rewards. On the other hand, if it just picked up one object, and then raced for the drop zone and repeated this process over and over it *definitely would* receive some rewards, albeit only small ones.

This (in fact inferior) course of action gets the highest Q-values because one subpolicy (locating and picking up objects) often fails initially. There is a risk that after a while – once the pickup-subpolicy is fully learned – the Q-value of selecting this subpolicy *again* once the agent has one object in cargo, is so small compared to the Q-value for subpolicies that will bring the agent to the drop zone, that the agent will never (or only after a very long time) discover that it is more beneficial to go on collecting objects until its cargo bay is nearly full, and only *then* return to the drop zone. Instead it would get stuck in suboptimal behaviour.

Combining Boltzmann and ε-Greedy Selection

To avoid the situation described above, it seemed prudent to select a random action with ε probability and use Boltzmann selection only with $(1-\varepsilon)$ probability ($\varepsilon = 0.01$ in all experiments). This leads to what could be called Boltzmann-ε-selection:

$$a_{selected} = \begin{cases} \textit{from the distribution } P_{Boltz} & \text{with probability } (1-\varepsilon) \\ \\ \textit{random} & \text{with probability } \varepsilon \end{cases} \tag{6.11}$$

where P_{Boltz} indicates the Boltzmann selection that was defined in equation 2.21 and *random* denotes that an action is selected with uniform probability from the set of actions. So just as in ε-greedy selection, sometimes actions are selected completely at random, but with (large) probability $(1-\varepsilon)$, the normal Boltzmann selection is applied. This means that the probability of selecting a_i is:

$$P_{Boltz-\varepsilon}(s,a_i) = (1-\varepsilon)\frac{e^{Q(s,a_i)/\tau}}{\sum_{a' \in actions} e^{Q(s,a')/\tau}} + \frac{\varepsilon}{||actions||} \tag{6.12}$$

where s is the current state, a_i is the action under consideration, $P_{Boltz}(s,a_i)$ gives the probability of selecting a_i in s, $||actions||$ stands for the number of actions and τ is the temperature.

This is a safeguard against premature convergence to a clearly suboptimal behaviour. Even if the probability of selecting a subpolicy according to the Boltzmann selection mechanism is (very close to)

[24] Some initial trials were conducted with using a continuous measure for the distance as a stopping criterion. However, there was no time to investigate this option any further. It will be described in more detail in "Future Work" (section 7.2).

zero, it will still be selected once in a while by the random selection (on average about 1 in a 1000 times, because the cleaner uses 10 subpolicies and $\varepsilon = 0.01$).

6.4.4 Expected Performance on the Cleaner Task

This task is too difficult to solve optimally with a simple calculation. Unlike the previous experiments, no exact value can therefore be given for the optimum value. A human is able to collect all objects within the allotted time[25], but a human probably uses visual cues and planning unavailable to the learners. For comparison, a flat learner was created that also needed to solve the task.

HABS uses two sensors ("radars"), one for its low level and one for its high level, and the flat learner needs the same information as HABS, in order to make it a fair trial. But the flat learner by definition only has one layer, so all this information is combined into one vector which is used as its observation. Its state $\overrightarrow{state}_{flat}$ is therefore defined as:

$$\overrightarrow{state}_{flat} = \overrightarrow{state}'_{ll,\text{HABS}} \;+\!+\; \overrightarrow{state}'_{HL,\text{HABS}} \tag{6.13}$$

Tuning the Flat Learner

The "pickup reward" divides the task into several smaller tasks (*find and pickup objects, find target and drop objects, find and pickup objects, find target and drop objects, …*). This helps the flat learner to solve the task in a reasonable time and yields a good measure of what performance to expect.

The flat Reinforcement Learner that needs to solve this task, was tuned in order to find out what kind of performance is typical for this task. Extensive testing (see fig. A.2 in Appendix A.1) shows that the performance of the flat learner depends highly on several variables. Most striking are the influence of the ε and γ parameters.

The discount γ_{flat} needs to be around $0.95 \sim 0.97$, otherwise the agent cannot learn the task. The ε_{flat} parameter (determining the amount of "greediness" of the selection) has a large influence on how long the agent takes to learn the task. A low value of ε_{flat} vastly increases the learning time, but the policy that is eventually learned is better, because random – suboptimal – actions are selected with smaller frequency.[26]

The flat learner can solve the task reasonably well[27] in some $5 \cdot 10^6$ steps, returning an average of around $60 \sim 65$ objects. The best performance (around 70) is only reached after $4 \cdot 10^7$ steps, but the agent can not do this consistently: some runs fail while others reach this high performance. On average there are 89 objects in the environment so the flat learner is able to collect and return just over three quarters of the objects in the allotted time on an average run.[28].

6.4.5 Running Time

HABS and the flat learner both make extensive calculations (a large part of which are the forward- and backpropagation parts of all the neural networks). Table 6.1 gives an impression of the time that the flat learner and HABS need with various configurations.

[25] Episodes with a longer duration of 3000 steps, show that both the flat learner and HABS can increase there performance to around $70 \sim 80$ on the task without "pickup rewards".

[26] It is of course possible to adjust the value of ε_{flat} during learning, starting with a high (quick-and-dirty) value, and gradually increasing the accuracy by decreasing ε_{flat}. The result in terms of time needed to convergence would be somewhere in between that of the quick and the accurate values for ε_{flat}. The performance would be roughly equal to that of the accurate ε_{flat} value. This would require another parameter and additional tuning.

[27] The flat leaner could solve the task a little faster with Boltzmann selection and "pickup rewards", but was slower with Boltzmann selection when no "pickup rewards" were used. See section A.1.3 in the Appendix for a short discussion.

[28] HABS was also tested on this simple task. It is able to achieve nearly the same (suboptimal) performance that most of the flat learners reach (just under 60, not shown here) but in only $2 \sim 3 \cdot 10^6$ so it is slightly faster. No extensive tuning was done for HABS in this simple cleanup task. In fact, the parameters were selected based on the results of the harder task without the pickup rewards.

	actions/ms
flat learner (5 hidden neurons)	12.5
flat learner (15 hidden neurons)	5.7
flat learner (25 hidden neurons)	3.8
HABS (2 hidden neurons (ll), 5 hidden neurons (HL), 10 subpolicies)	13.4
HABS (2 hidden neurons (ll), 15 hidden neurons (HL), 10 subpolicies)	11.6
HABS (5 hidden neurons (ll), 5 hidden neurons (HL), 10 subpolicies)	10.3
HABS (5 hidden neurons (ll), 15 hidden neurons (HL), 10 subpolicies)	9.7

***Table 6.1: Running times**: the average speeds (in primitive actions per second) for various configurations. Calculated on a Pentium IV with 2.4 GHz and 500 MB RAM.*

The important factor for HABS is the number of low level neurons. The subpolicy neural networks are used at every time step, so at every time step there is a forward propagation for all six primitive actions, and a back propagation for one of these six actions. The high level policy only uses its neural network once every few (possibly only once every $timeout_{ll}$ steps in the beginning)

Since the flat learner uses its neural networks every time step, but it also needs it for the kind of large scale learning that HABS does with its *high level policy*, it has a large disadvantage. Even more so, because it has only one state vector as input, which is the conjunction of both the low level state and high level state of HABS (otherwise HABS would receive information that the flat learner does not, and the trial would not be fair). It therefore has a larger input vector to process at every time step, than HABS does, if HABS has the same number of hidden neurons for its subpolicies as the flat learner has for its policy. HABS on the other hand has an extra overhead because of the clustering and other calculations related to its layered structure, and of course the high level policy with its neural network.

The difference is most notable with many hidden neurons. If the flat learner uses 15 neurons, and HABS uses the same amount for its high level (even though it only needs 5 to solve the problem), it can still be twice as fast because it only needs a few (2 already proved enough) hidden units for its subpolicies.

6.5 The Cleaner Task

For the real Cleaner task the agent received no no extra help in the form of "pickup rewards". As can be seen in fig. A.4, this task is rather tough for the flat learner.

6.5.1 The Flat Learner

Without the extra help of the "pickup rewards", the best flat learners need some $2 \cdot 10^7$ steps (compared to only $3 \sim 5 \cdot 10^6$ steps with the "pickup rewards"!). Furthermore, the variance in convergence time is larger, so some learners take far longer to reach good performance (or don't reach any significant performance at all). Results of these experiments can be found in Appendix A.1.4.

Two settings are interesting: 15 hidden neurons, $\gamma_{flat} = 0.97$, $\varepsilon_{flat} = 0.1$ and $\alpha_{flat} = \{0.01, 0.02\}$ (see fig. 6.12). The first ($\alpha_{flat} = 0.01$) provides a high performance (just above 60) about half the time[29]. The other alternative is to go for a "quick and dirty" solution with a higher learning rate ($\alpha_{flat} = 0.02$). This allows the flat learner to reach values between 50 and 55 fairly consistently.

[29]The boxplot visualisation is somewhat misleading here, because roughly half of the runs reach performance around 60 and the other half stay near 0. The boxplot "box" therefore only contains one or two runs. Nearly all other runs are either in the top or at the bottom. That is why the lines denoting the lowest and highest quarter of the data are so small.

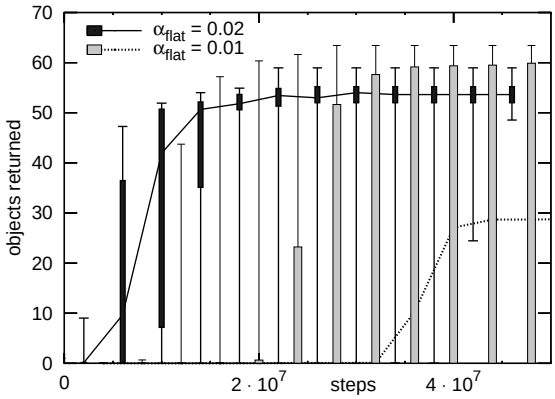


Figure 6.12: flat learner*: no "pickup rewards". 15 hidden neurons,* $\alpha_{flat} = 0.02$*,* $\varepsilon_{flat} = 0.1$ *and* $\gamma_{flat} = 0.97$*. For* $\alpha_{flat} = 0.005$ *there was zero performance.*

6.5.2 HABS does Some Cleaning Up

Even though it is also interesting to see how HASSLE and HABS relate to each other on tabular problems, the focus of HABS is on the use of neural networks on the high level. HABS was therefore most extensively tested on the Cleaner task. The most interesting results are presented here.

The High Level Policy

The performance of the high level is virtually independent of the number of hidden neurons (fig. 6.13(a)). A lower number of neurons does degrade performance slightly, but even with only one neuron in the hidden layer, HABS is still able to solve the task (although about half the time it only reaches a lower performance of 30 or 40 objects retrieved). This indicates that the structure of the problem as viewed on the high level, is fairly uncomplicated. One or two hidden neurons are sufficient[30].

Just as with the flat learner, a learning rate α_{HL} of around 0.01 proved best (see fig. 6.13(b)). The flat learner uses its neural network both for large scale and small scale learning, but for HABS these functions are done by the subpolicies and the high level policy respectively.

Interdependencies

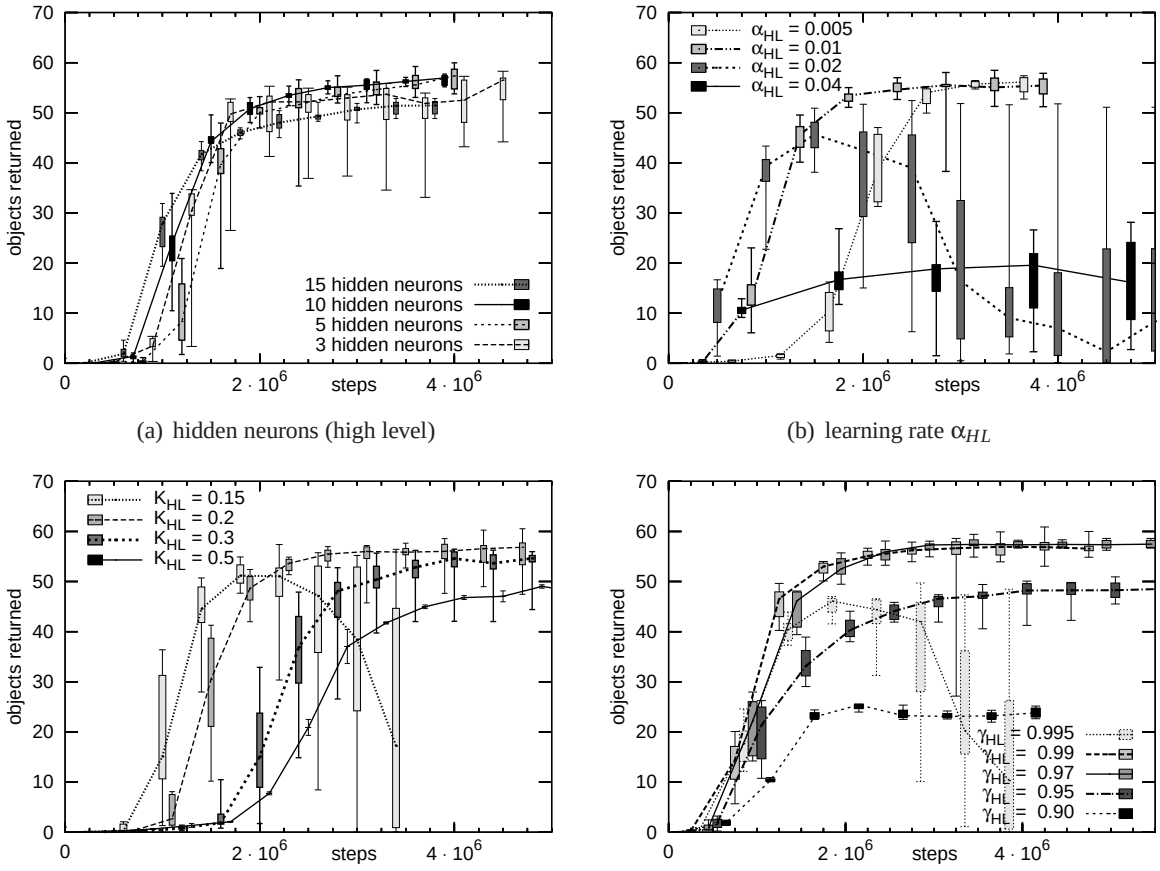
Fig. 6.13(d)) gives an overview of behaviour of learners with different γ_{HL} (for $\tau_{HL} = 0.025$). It is plausible the range of good values for γ_{HL} is dependent on the selection mechanism. The discount determines the Q-values, and the selection mechanism is highly dependent on *differences between* these values.[31]. For this reason the discount was investigated for other values of the Boltzmann temperature τ (see fig. 6.14). It turns out that lower values of τ_{HL} are more stable and give better performance for all tested γ_{HL} values. For $\gamma_{HL} = 0.99$ all tested values of τ_{HL} give good performance, but if γ_{HL} is lowered, then the learner with the highest τ_{HL} is the first to drop. If γ_{HL} is lowered further to $\gamma_{HL} = 0.95$ then the performance of the learners with lower τ_{HL} values also start to degrade.

As can be seen in fig. 6.14, the performance of HABS for other τ_{HL} is roughly the same. The discount is of great influence on the performance of the learner. A value that is too high or too low leads to low performance. Values around $\gamma_{HL} = 0.97 \sim 0.99$ yield good results.

The same kind of dependency was suspected of the discount and the Advantage scaling factor K_{HL} because this factor scales the Q-values and therefore may also have an impact on the action selection.

[30] One or two neurons for each of the networks: there are as many networks as there are high level actions a_i, because each network only calculates $Q(s, a_i)$ for one action.

[31] Parameters like the learning rate α_{HL} and the number of hidden neurons are presumably not related to the discount: the speed of adjustment of the weights of the neural network is independent of the *differences* between the different Q-values.



(a) hidden neurons (high level)

(b) learning rate α_{HL}

(c) Advantage K_{HL}

(d) discount γ_{HL}

Figure 6.13: **HABS** **– *Settings related to the high level policy****: No "pickup rewards". **Defaults**:* $\tau_{ll} = 0.05$, $\tau_{HL} = 0.025$, $K_{ll} = 0.3$, $K_{HL} = 0.3$, $\gamma_{ll} = 0.95$, $\gamma_{HL} = 0.99$, $\alpha_{ll} = 0.01$, $\alpha_{HL} = 0.01$, *2 hidden neurons (low level), 5 hidden neurons (high level), 10 subpolicies, subpolicy-timeout* $= 20$, $\rho_{failed} = 0$, $\rho_{timeout} = -1$, $\omega = 0.03$. ***(a) hidden neurons (high level)****: 5 hidden neurons (low level).* ***(b) learning rate*** α_{HL}*:* $K_{HL} = 0.2$*, 7 subpolicies.* ***(c) Advantage*** K_{HL}*: (defaults).* ***(d) discount*** γ_{HL}*:* $\rho_{failed} = 0.3$, $K_{HL} = 0.2$.

The results of several tests with different values for these parameters are depicted in fig. 6.15. There seems to be no significant dependency between the τ_{HL} and the Advantage K_{HL} because the learner behaves roughly the same for varying K_{HL} with each of the tested temperatures.

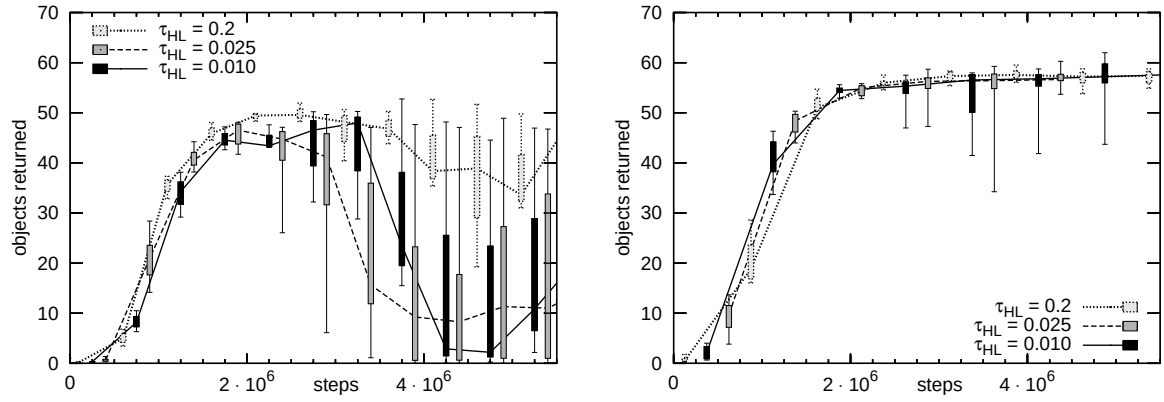
The effects of different K_{HL} values at $\tau_{HL} = 0.025$ is depicted in fig. 6.13(c). Smaller K_{HL} values give a boost in convergence time, but are less stable. Higher values of K_{HL} give worse and/or slower performance.[32]

The Subpolicies

The low level subpolicies are able to do their subtasks with as few as two neurons in the hidden layer (see fig. 6.16(a)). There is no significant difference in performance between two or five neurons. This was to be expected: the subtasks are designed to be *simple* behaviours. A value of $\alpha_{ll} = 0.01$ for the subpolicy learning rates was found adequate in initial testing. This learning rate was used consistently in all HABS Cleaner experiments. No results regarding performance with varying low level learning rates α_{ll} are presented here.

The selection temperature for the low level policies (fig. 6.16(b)) show a smaller window of good parameters: $\tau \in [0.025, 0.05]$ yields a good performance. But since the behaviour of the subpolicies is

[32] Initial tests confirmed that $K = 0.2 \sim 0.3$ was also suitable for the low level subpolicies (K_{ll}) and for the flat learner (K_{flat}).



(a) Boltzmann τ_{HL} at $\gamma_{HL} = 0.995$

(b) Boltzmann τ_{HL} at $\gamma_{HL} = 0.99$

(c) Boltzmann τ_{HL} at $\gamma_{HL} = 0.97$

(d) Boltzmann τ_{HL} at $\gamma_{HL} = 0.95$

Figure 6.14: (a – d) **HABS** ***– Varying Boltzmann temperature*** τ_{HL} ***for*** $\gamma_{HL} = \{0.995, 0.99, 0.97, 0.95\}$*: No "pickup rewards".* $\tau_{ll} = 0.05$*,* $\tau_{HL} = 0.025$*,* $\gamma_{ll} = 0.95$*,* $\gamma_{HL} = 0.99$*,* $\alpha_{ll} = 0.01$*,* $\alpha_{HL} = 0.01$*, 2 hidden neurons (low level), 5 hidden neurons (high level), 10 subpolicies, subpolicy-timeout* $= 20$*,* $\rho_{failed} = 0$*,* $\rho_{timeout} = -1$*,* $\omega = 0.03$*.*

rather similar in the different tasks, good values in one task (like one of the maze tasks) turn out to be good values in another task, and little real tuning is needed.

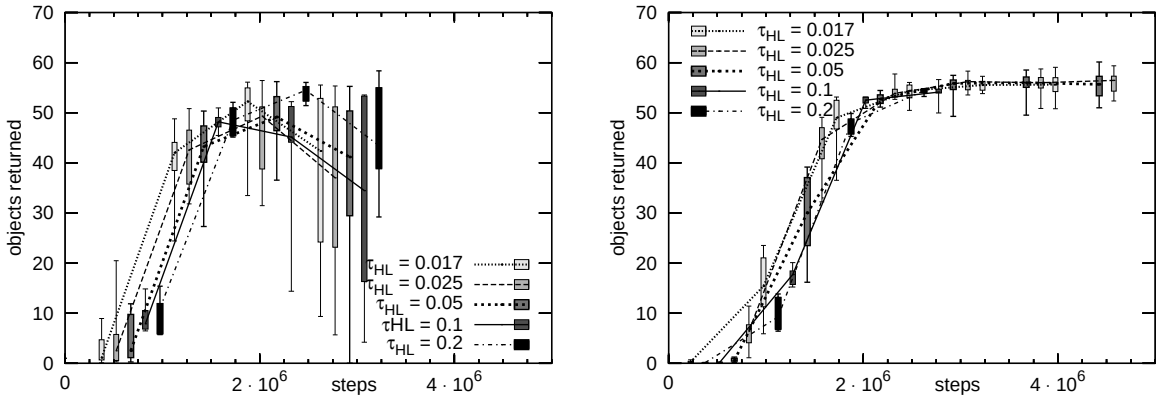
The number of subpolicies that HABS uses to solve a task fig. 6.16(c)), does not influence the performance it reaches, unless of course there are so few subpolicies that not all behaviours can adequately be covered. This is the case when only five subpolicies are used. It is easy to imagine that there are six distinct behaviours in the cleanup task: four behaviours for moving (roughly) in the cardinal directions, one for locating and picking up objects, and one for dropping them.

Tests with five subpolicies show that the agent manages to retrieve as many objects as with seven or more subpolicies *for some runs, but not for others*. This is remarkable. Apparently HABS is sometimes able to use one subpolicy to execute two kinds of behaviours. One of the runs with 5 subpolicies was investigated, and the following characteristic vectors were observed after $3 \cdot 10^6$ steps (i.e. near convergence):

$$char_1 = (-0.0795, +0.0022, -0.0362) \quad char_2 = (+0.1265, -0.0036, -0.0000)$$
$$char_3 = (-0.0141, -0.1696, \ 0.0000) \quad char_4 = (-0.0000, +0.0000, +0.1000)$$

The agent used $char_1$ for dropping the objects at the drop zone, but also for moving to the left.

Tests with four subpolicies the agent is unable collect any significant amount of objects. The performance does occasionally reach 1 or 2 because the agent by accident drops an object correctly, but nothing more.



(a) Boltzmann τ_{HL} for $K_{HL} = 0.15$

(b) Boltzmann τ_{HL} for $K_{HL} = 0.2$

(c) Boltzmann τ_{HL} for $K_{HL} = 0.3$

(d) Boltzmann τ_{HL} for $K_{HL} = 0.5$

Figure 6.15: (a – d)* HABS *– Varying Boltzmann temperature τ_{HL} for $K_{HL} = \{0.2, 0.3, 0.5\}$: No "pickup rewards". $\tau_{ll} = 0.05$, $\tau_{HL} = 0.025$, $K_{HL} = 0.3$, $\gamma_{ll} = 0.95$, $\gamma_{HL} = 0.99$, $\alpha_{ll} = 0.01$, $\alpha_{HL} = 0.01$, 2 hidden neurons (low level), 5 hidden neurons (high level), 10 subpolicies, subpolicy-timeout = 20, $\rho_{failed} = 0$, $\rho_{timeout} = -1$, $\omega = 0.03$.

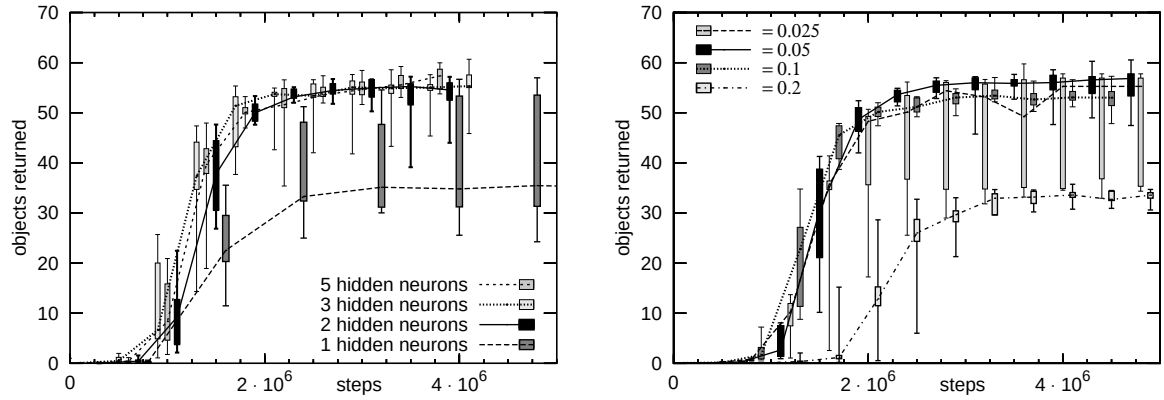
Clustering and Self Organizing

The parameters related to the clustering and self organizing show some surprises. These parameters are the clustering learning rate ω and the rewards (or punishments) for timeout ($\rho_{timeout}$) and for executing a behaviour that is not clustered as belonging to the current subpolicy (ρ_{failed}).

It was suspected that punishing ($\rho_{failed} < 0$) slightly would work best, but instead HABS reaches better performance if failure (but not timeout) is rewarded slightly ($\rho_{failed} = 0.3$), as shown in fig. 6.16(e). Even if the subpolicy did something that actually needs to be clustered in another cluster (i.e. belongs to another subpolicy), it still interferes more to punish this (wrong) behaviour than to reward it a little bit. Doing something wrong is apparently (in this environment) more similar to doing it right than to doing nothing at all (timeout).

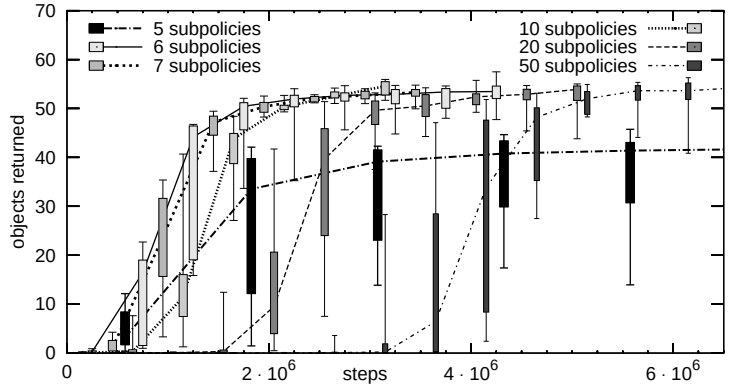
Punishing a subpolicy for doing nothing at all ($\rho_{timeout} < 0$) proved useful (see fig. 6.16(f)). A negative return of $\rho_{timeout} = -1$ or $\rho_{timeout} = -2$ results in faster convergence and slightly higher performance.

The clustering learning rate ω behaved rather good (fig. 6.16(d)) for a very wide range of values, only $\omega = 0.001$ proved too low, though not on all runs. Apparently even with this very small ω value, the learner is still able to cluster the data most of the time. This is probably because it executes so many iterations. If HABS takes $3 \cdot 10^6$ steps to reach convergence and if a subpolicy on average terminates every 10 steps and there are 10 subpolicies, then each of the clusters has had some 10^4 adjustments of its cluster center.

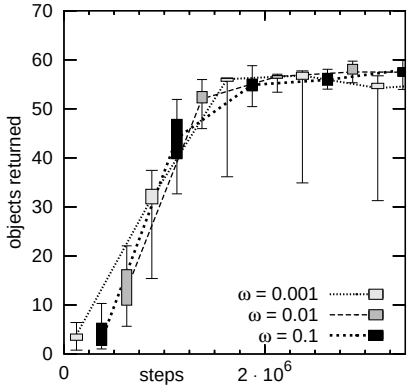


(a) hidden neurons (low level)

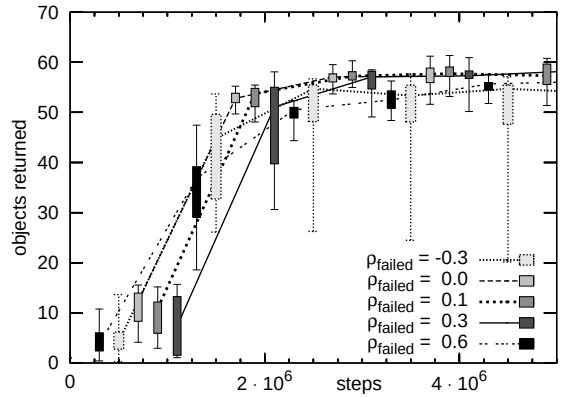
(b) Boltzmann τ_{ll}



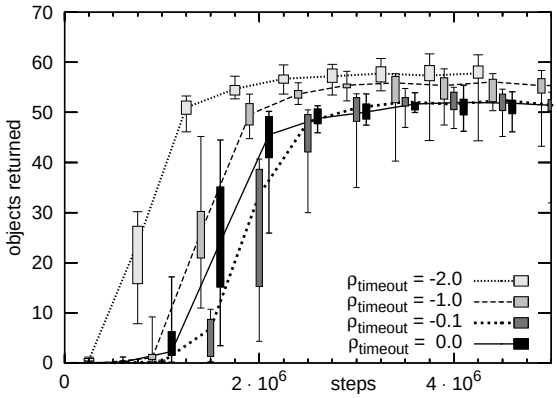
(c) number of subpolicies



(d) clustering learning rate ω



(e) fail-reward ρ_{failed}



(f) timeout-reward $\rho_{timeout}$

Figure 6.16: **HABS – *Settings related to subpolicies and clustering***: *No "pickup rewards".* ***Defaults:*** $\tau_{ll} = 0.05$, $\tau_{HL} = 0.025$, $K_{ll} = 0.3$, $K_{HL} = 0.2$, $\gamma_{ll} = 0.95$, $\gamma_{HL} = 0.99$, $\alpha_{ll} = 0.01$, $\alpha_{HL} = 0.01$, *2 hidden neurons (low level), 5 hidden neurons (high level), 10 subpolicies, subpolicy-timeout* $= 20$, $\rho_{failed} = 0$, $\rho_{timeout} = -2$, $\omega = 0.03$. ***(a) hidden neurons (low level):*** *(defaults),* ***(b) Boltzmann*** τ_{ll}***:*** $\tau_{HL} = 0.05$, $\rho_{timeout} = -1$. ***(c) number of subpolicies:*** $\tau_{ll} = 0.10$, $\rho_{timeout} = -1$. ***(d) clustering learning rate*** ω***:*** $\rho_{failed} = 0.3$. ***(e) fail-reward*** ρ_{failed}***:*** *(defaults).* ***(f) timeout-reward*** $\rho_{timeout}$***:*** *(defaults).*

Robustness

As can be seen from the data presented here, HABS is fairly easy to tune for this hard problem.[33] HABS is able to reach convergence in just over $2 \cdot 10^6$ time steps, and it can reach average performances

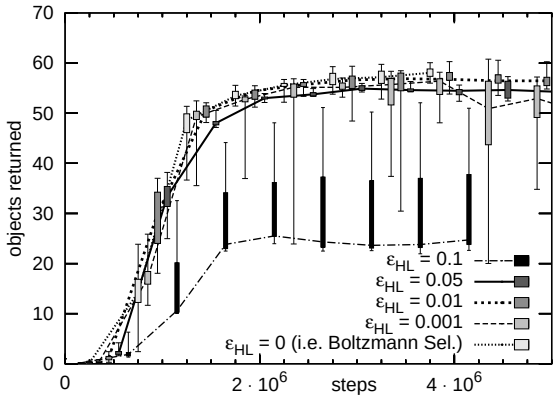
[33] That is, using Boltzmann-selection. Using ε-greedy selection proved harder: with ε-greedy the performance was inferior with same settings of K_{HL}, γ_{HL}, α_{HL} numbers of hidden neurons, subpolicies and settings for HABS clustering!

of between 55 and 60 objects returned per episode. This is as good a performance as the flat learner is able to reach, but HABS does it far more consistently and for a wide range of parameters. The flat learner needs an order of magnitude more time (over $2 \cdot 10^7$ steps) to reach convergence, and the variance in the moment of convergence is far greater. This is related to the random walking distance for both algorithms: the flat learner has a very long chain of actions, but HABS a far shorter one because once its subpolicies start acting non-randomly, it only needs to random walk on the high level. Shorter distances result in less variance in the time when convergence is reached.

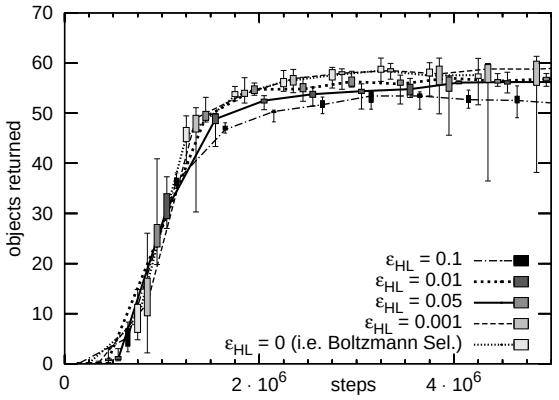
6.5.3 Forcing HABS to Explore, Some Tests

As explained in section 6.4.3, HABS was forced to explore with probability $\varepsilon_{HL} = 0.01$. Initial tests suggested this value, so it was used as a precaution in all above experiments.

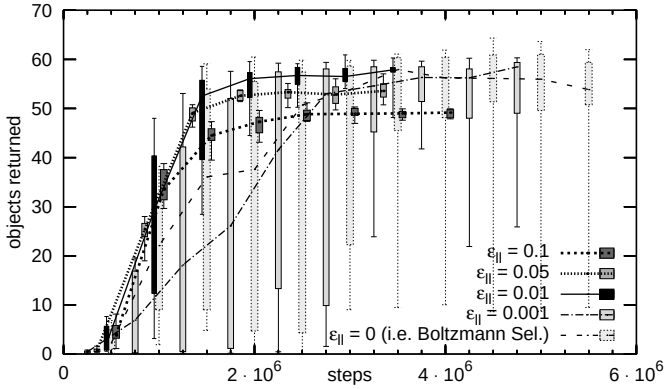
Using Boltzmann-ε selection on the high level seems to make no real difference for convergence time and reached performance for most of the runs. Significantly higher ε_{HL} obviously lead to lower performance, because if $\varepsilon_{HL} = 0.05$, in 5% of the time, some subpolicy is selected at random. For a lower Boltzmann τ this led to increasingly worse performance: for $\tau = 0.1$ and $\varepsilon = 0.1$ the performance is still above 50 (fig. 6.17(b)), but for $\tau_{HL} = 0.025$ and $\varepsilon = 0.1$ it has dropped to an average of about 30 (fig. 6.17(b)) with only some of the runs reaching 50.



(a) Varying ε_{HL} at Boltzmann $\tau_{HL} = 0.025$



(b) Varying ε_{HL} at Boltzmann $\tau_{HL} = 0.10$



(c) Varying ε_{ll}

Figure 6.17: Boltzmann-ε selection: *no "pickup rewards".* ***defaults:*** $K_{ll} = 0.3$, $K_{HL} = 0.2$, $\gamma_{ll} = 0.95$, $\gamma_{HL} = 0.99$, $\alpha_{ll} = 0.01$, $\alpha_{HL} = 0.01$, *2 hidden neurons (low level), 5 hidden neurons (high level), 10 subpolicies, subpolicy-timeout* $= 20$, $\rho_{failed} = 0.3$, $\rho_{timeout} = -1$, $\omega = 0.03$. ***(a) Varying*** ε_{HL} ***at Boltzmann*** $\tau_{HL} =$ ***0.025:*** $\tau_{ll} = 0.05$. ***(b) Varying*** ε_{HL} ***at Boltzmann*** $\tau_{HL} =$ ***0.0.10:*** $\tau_{ll} = 0.05$. ***(c) Varying*** ε_{ll}***:*** $\tau_{HL} = 0.025$.

However, when ε_{ll} was varied (fig. 6.17(c)), the agent was observed to take a lot more time for some

runs with very low ε_{ll} (if $\varepsilon_{ll} = 0$ this amounts to regular Boltzmann selection, not shown here but same results as with $\varepsilon_{ll} = 0.001$), but otherwise performed the same as with $\varepsilon_{ll} = 0.01$. Increasing ε_{ll} had the same effect as with increasing ε_{HL}: lower performance because more random actions are selected.

It seems that a little precaution was good, especially for the low level. Boltzmann-ε selection with $\varepsilon_{ll} = 0.01$ performs nearly the same as Boltzmann selection (Boltzmann-ε selection with $\varepsilon_{ll} = 0$) but is less variance in the time when convergence is reached.

Combining the best of Boltzmann selection (selection proportionate to Q-values) and ε-greedy selection (forcing exploration with probability ε) is – at least for HABS – slightly more stable than either of the two selection mechanisms (see Appendix A.2 for tests with HABS and ε-greedy selection).

Chapter 7

Conclusion and Future Work

7.1 Conclusions

In this thesis some of the problems were analyzed that arise when Reinforcement Learning is used in combination with hierarchies. This knowledge was used in designing a new Hierarchical Reinforcement Learning algorithm called HABS. The existing algorithm HASSLE by Bram Bakker and Jürgen Schmidhuber [1, 2] was used as a starting point for this new algorithm.

State Space Abstraction and Task Decompositions

It is clear that Hierarchical Reinforcement Learning can benefit from using state space abstraction, where each higher layer has a coarser view of the state space. State space abstraction allows the designer to use his (perhaps only intuitive) grasp of a problem, to create coarser representations of the problem. Furthermore, the designer is not forced to solve the problem himself: there is no need to create a *task decomposition*. The agent can learn which behaviours it should execute in what abstract state.

Behaviours Should Be "Relative", Not "Absolute"

HASSLE uses high level states as its high level actions. In section 4.5 it was shown that one of the mayor problems with high level actions is that they are (often) defined in an absolute sense (unlike primitive actions), referring to fixed locations in the (abstract) state space.

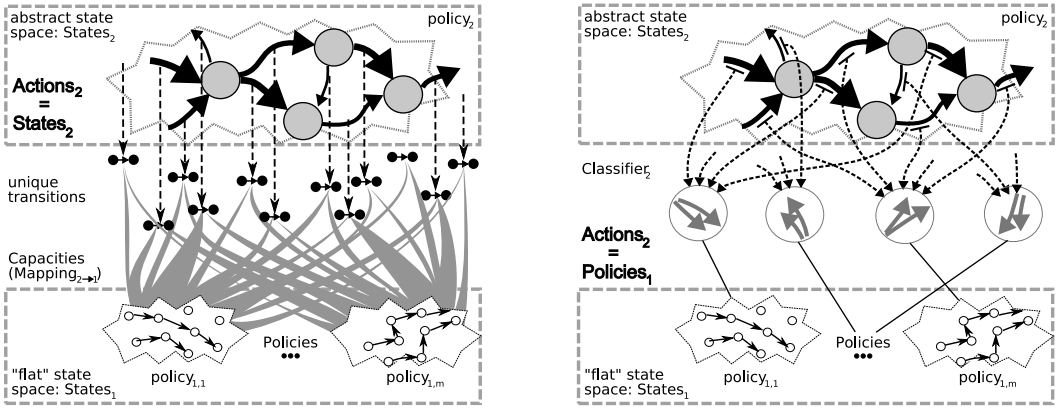
First of all, no (or virtually no) generalization is possible when high level actions are defined in an absolute way (see section 4.3.1). A high level action that is good in one particular abstract state, is useless in nearly all other abstract states, because it is defined in terms of specific abstract states. This is unlike the primitive actions or for instance the *teleo-operator* (TOP) in RL-TOPs. Primitive actions and the TOPs in RL-TOPs are only defined in terms "relative" to the starting point, formulated in terms of *doing* something, not in *reaching states*.

When generalization is impossible, it is not useful to use more than two layers (see section 4.3.3). When a third layer is introduced, we run into the problem that the middle layer should consist of sub-policies that are applicable everywhere.

The third problem is specific to HASSLE (or structures like it), because it stems from the use of high level states *as high level actions*, so other algorithms that use similar constructions will also be hampered by this effect. If such a construction is used, there occurs an *action explosion*: the amount of high level actions grows with the number of high level states (see section 4.3.2). This is something unusual and certainly something undesired in Reinforcement Learning. A larger problem has more states and therefore more high level abstract states, and therefore requires more time to learn. But in HASSLE the extra burden is that a larger problem also implies a larger number of high level actions to be explored, increasing the learning time and the memory requirements even more.

A Solution - Do it in Reverse

In this thesis a solution was proposed that can most easily and intuitively be illustrated by two pictures (fig. 7.1), showing the structure of HASSLE and HABS. Compared to HASSLE, HABS does its mapping from high to low level in reverse order. Instead of treating all the transitions between abstract states as unique high level actions and then mapping each of these to a limited set of subpolicies, it first maps many transitions to more generic high level actions or behaviours and then associates each of these generic classes of behaviours to one subpolicy.



(a) HASSLE (b) HABS

Figure 7.1: **HASSLE***: same as fig. 4.1* **HABS***: same as fig. 5.1*

Obviously this is not the only way in which a structure like HABS could be created: other types of mapping and/or classification could be used instead, or the classification could be fixed *a priori* (resulting in an algorithm somewhat like Feudal Learning) or the mapping between classes of behaviour and subpolicies could be flexible. In this thesis a simple version of HABS was presented. The high level behaviours were represented as vectors and the classification was done using a simple clustering algorithm on these vectors.

7.1.1 Results

Augmenting HASSLE with Filters

HASSLE was tested with and without filters on two experiments. It turns out that, given the right filtering, the augmented form of HASSLE is more robust for a range of parameters.

Comparing HABS with HASSLE

HABS was compared to HASSLE, to see whether HABS is a good solution to the problems that were identified with HASSLE. In order to see the destructive effect of the action explosion, experiments were done in larger environments to see whether HASSLE would indeed be hampered by its many high level actions, and to see whether HABS would be any good as an alternative. In one experiment HASSLE slowed down, in the other it needed orders of magnitude more memory than was actually available (resulting in a crash on every run). The experiments also showed that HABS was able to perform this difficult task without any problems.

Comparing HABS with "Flat" Learners - Tabular Case

HABS also outperforms "flat" Reinforcement Learning in speed an order of magnitude in the same maze problem. This is not surprising since Hierarchical Reinforcement Learning algorithms are designed especially with that goal in mind. These experiments only confirm that the time that HABS needs for its

self organizing and classification, is insignificant in comparison with the time saved by using a hierarchy. This means that behaviours need not be fixed *a priori* but can be identified during execution.

It should be noted that it is difficult to compare "flat" tabular learners with HABS because HABS always has function approximators as its subpolicies, even when it uses a tabular representation for its high level. This is unavoidable, because HABS treats its subpolicies directly as high level actions, and if these subpolicies would be tabular, they would not be able to generalize, which would in effect result in a situation where HABS needs to learn *each* of the individual high level behaviours that would normally be generalized over by a function approximator. Comparison between HABS or HASSLE and a flat learner is therefore always unfair.

Comparing HABS with "Flat" Learners - Function Approximator Case

HABS was created to be suitable for the use of function approximators, such as neural networks, to approximate the value functions at all levels in its hierarchy. This means that it can boldly go where HASSLE cannot. To illustrate this, an experiment (with an abstract state space too large for tabular representation of the Q-values) was designed where the agent was rewarded for collecting (and correctly returning) objects scattered all over the world.

The "flat" learner was now equipped with a comparable neural network (with hidden layer). HABS was both faster and more reliable. It was faster – both in time per primitive action (because of the smaller networks) and in steps until convergence – and also had a wide range of settings that allowed it to perform very good.

In contrast, the "flat" learner was very hard to tune, and even after much tuning and searching through many parameters, it still often performed very badly. This is probably due to the fact that the distance that needs to be travelled through state space to accomplish the goal, is very large. This means that random walking takes an enormous time which allows the neural network to fade out before the agent has had time to discover something meaningful. HABS on the other hand doesn't run into this problem because its random walking distances are smaller, both on the high and the low level, due to the hierarchical structure.

Suboptimality

The suboptimality of HABS and HASSLE when compared to the "flat" learner was to be expected. It is probably because the characteristic behaviours are too simple (see future work for more on this), or the abstract state space does not allow optimal policies.

In fact, this is a common characteristic of hierarchical approaches: optimal behaviour given the constraints of the hierarchy may be near optimal, but slightly suboptimal given the space of all policies. However, often this is a price worth paying for far more efficient learning in general, and the ability to learn in cases where Reinforcement Learning without hierarchies is completely infeasible because of the distances involved and the random walking it implies.

7.2 Future Work

Several approaches for future work on HABS are given. They could be classified in several ways. Some of the suggestions deal directly with better representations for the characteristic behaviours (curves instead of vectors or decomposing the behaviour space into smaller subspaces (7.2.1) and automatically detecting features (7.2.2)) whereas the other two (continuous termination criteria (7.2.3) and extending the hierarchy to more layers (7.2.4)) do not. On the other hand, the first topic (curves and decomposition) could be considered heuristics that help the designer, whereas the remaining three are solutions to problems that arise when the tasks become larger (and more continuous).

7.2.1 Representation of Behaviours: Curves and Decomposition

It should be expected that practical problems arise when HABS is extended to larger problems. Introducing a the third layer (see below) will necessarily imply a very coarse abstraction, which means that a simple vector representation for the behaviours might not be suitable any more. Even for only two layers, this situation already could occur.

This issue was already dealt with briefly in section 5.2.2. It was noted there, that high level actions that result in the same difference vector, should in fact be comparable. But the more abstract the problem is represented, the greater the risk that this assumption will not hold when *vectors* are used.

Vectors, Adieu!

The first option is to look at other representations for the path between two locations in the abstract state space. Another function could be used to represent the path that the agent must take from one location to another. Instead of a characteristic behaviour *vector*, something like a characteristic behaviour *curve* (see fig. 7.2) or *spline* or something else entirely, could be tried.

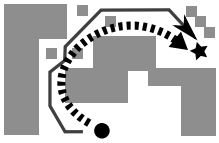


Figure 7.2: Bezier curve*: the executed behaviour between start (dot) and end (star) is represented by a simple bezier curve (dashed arrow), instead of a vector. The curve is a better high level description of the behaviour, than a vector from start to end.*

Decomposing the Characteristic Behaviour

But suppose we have a problem where we *know* that some of the abstract high level dimensions are completely unrelated to some of the other dimensions. In this case it would be difficult to use one vector as a representation for the characteristic behaviour. In fact, we know that some of the dimensions are incomparable. In fact, it would be counter intuitive to try to squeeze all these values into one vector using many different scaling factors for all the values and to try to use this many-dimensional space to self-organize these vectors in.[1]

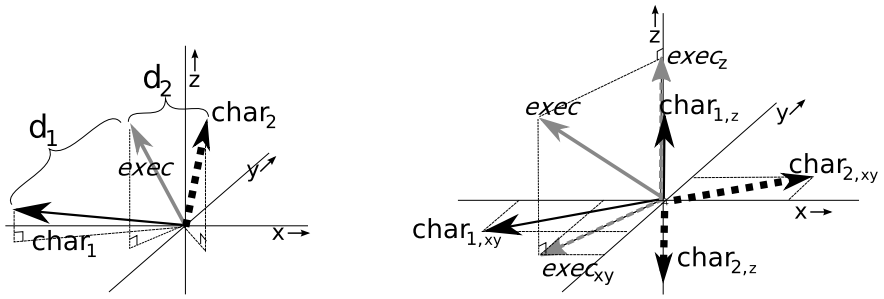
As an example, suppose that we have three (abstract) dimensions, x,y and z. The usual approach would be, to compare the executed behaviour *exec* with all the characteristic behaviours (associated with the subpolicies) in this three-dimensional behaviour space. As illustrated in fig. 7.3(a) this could result in strange classifications. The *exec* vector is closer to v_2 than v_1 ($d_2 < d_1$) but the executed behaviour with regard to the x and y dimensions was almost opposite to what v_2 does in those dimensions. In fact, v_1 is much more similar if only x and y were considered. Unfortunately it is simply not the closest vector when all dimensions are considered.

The alternative would be to decompose the behaviour space and create smaller spaces consisting of comparable dimensions. This approach would make sense when the dimensions in the (abstract) state space are very different from each other and cannot easily be compared by introducing a scaling factor (as is done in section 6.2.1). In our example the dimensions x and y are related and can be compared[2] but one dimension (z) is incomparable to the other two. In this case it would be useful to split the behaviour space into two smaller subspaces (behaviour subspaces).

The first subspace is generated dimensions x and y, and the second space is formed by dimension z. This means that a characteristic behaviour now consists of *two* vectors, one in the xy-space, the other in

[1] In fact, this "counter intuitive" but simpler approach was taken in the experiments conducted here. This illustrates that (useful) *a priori* knowledge is not in principle needed in simple situations.

[2] The dimensions x and y might for instance both be spatial, or both related to (different kinds of) objects in possession. Dimension z measures something completely different, e.g. fuel-consumption or energy.



(a) simple behaviour space (b) separating dimensions

Figure 7.3: (a) Simple behaviour space*: all dimensions in the behaviour space are taken together and used for one behaviour vector (exec). The characteristic behaviour vectors (black arrows) can assume any value.* ***(b) behaviour space with separated dimensions****: the behaviour space is separated into two parts (xy and z). Each characteristic behaviour consists of two vectors: one in the xy-space and one in the z-space (the first characteristic behaviour consists of the two solid black arrows, the second has dashed arrows). The executed behaviour exec (gray arrow) is decomposed and projected onto the two spaces, and its projections $exec_{xy}$ and $exec_z$ are used for classification.*

the space generated by *z*. When the agent has executed a certain behaviour (*exec*), this behaviour needs to be decomposed into its projections on all the spaces (as illustrated in fig. 7.3(b)). The resulting $exec_{xy}$ and $exec_z$ can then be used for classification.

This classification needs to combine the distance from $exec_{xy}$ to $char_i, xy$ and $exec_z$ to $char_i, z$. One alternative is, to use something quadratic: $distance^2(char_i, exec) = distance^2_{xy}(char_{i,xy}, exec_{xy}) + distance^2_z(char_{i,z}, exec_z)$. This would lead to the normal Euclidean distance, but the difference with regarding all dimensions as one space, is that with separate vectors, *there are separate processes of self-organization*: each of the vectors is independent of the others. If all the values are taken together in one vector, the different (but in principle incomparable) dimensions influence each other, because they are used in determining which subpolicy is the best match for an executed behaviour. If the behaviour space is separated into several subspaces, and if the characteristic vectors are constrained to those subspaces then unwanted influence can be avoided. The downside is that we would need as many subpolicies as there are combinations of vectors from all Behaviour subspaces.

7.2.2 Automatic Detection of Relevant Features

In many cases where it is not *a priori* clear which of the features of a state space abstraction are relevant for behaviour *but* on the other hand it is known that there could be features that are less relevant then others. When this happens it is not possible to exclude irrelevant features *a priori* (as was done in the experiments presented here, see section 5.2.5 and 6.4.2).

Some kind of automatic detection of the relevant features is suggested here. Features that are correlated to behaviour also have a statistical property that can be exploited for automatic detection:[3] the prediction error between *exec* and the characteristic behaviour that is the closest match, will be small.

If these features cannot be appropriately described by a few clusters, the average error between the features and the corresponding values in the characteristic behaviours will always by high. This suggests some kind of error minimalization algorithm. The expected difference (scaled to for instance its standard deviation) for feature *i* between the nearest cluster $cluster_k$ and the executed action *exec* will be large for irrelevant features but small for features related to behaviour. Each of these features contributes to the overall error $E_{exec,cluster_k}$. Each feature *i* is assigned a weight (similar to how each input in a neural network is assigned a weight). This weight increases whenever its contribution to the error $E_{exec,cluster_k}$ is low, and decreases when its contribution is high.[4] This relation can for instance be ensured

[3] Perhaps this property is even used implicitly by the designer when relevant features are identified *a priori* .

[4] This error probably needs to be related somehow to the standard deviation in the feature in question, to avoid problems

by demanding that the sum of all weights remains constant ($\sum weight_i = 1$). Perhaps each characteristic behaviour will have a separate set of weights or one set could be shared between all subpolicies.

Using weights to automatically scale each feature will result in a process where *on average* the features that are related to the behaviour will end up with high weights. This is because on average they are more properly clustered then the irrelevant ones, and therefore on average have a lower error. Maximizing the weights of relevant features and minimizing those of the irrelevant ones, will ensure the lowest average error between executed behaviour and the characteristic behaviour vectors. This should happen intertwined with the self organization of the characteristic behaviour vectors. In the beginning some exploration in the weight-space is unavoidable, because it will take some time before the subpolicies will start to behave non-randomly. From that moment on, a better set of weights will lead to better classification, and better classification will lead to more consistent training of the subpolicies, and so on, presumably.

7.2.3 Continuous Termination Criteria

In section 5.2.6 it was stated that HABS can in principle use continuous termination criteria:

$$continuousStop_{i<n} = \begin{cases} terminate & \text{If } timeout \vee (distance(S,S') > \delta \ \wedge \ S,S' \in States_{n+1}) \\ continue & \text{otherwise} \end{cases}$$

However, in the experiments presented in this thesis, only discrete criteria were investigated due to a lack of time. Using a continuous termination criterion would imply that the agent agent after each lower level step needs to compare its higher level state and measure the difference between its current state and the state it was in when it started its current (sub)policy.

As a distance measure for two vectors in the behaviour space, the Euclidean distancecould be used. But in situations where it is not known what the relevant features are, using this metric would mean that some completely irrelevant feature could change a large amount and bring the distance over the threshold δ even though no *significant* change would have been achieved by the agent.

When automatic detection of features – as proposed above – is used, it makes sense to also use the feature weights in the definition of the distance. The length of the *weighted* difference vector

$$distance(S,S') = \sqrt{\sum weight_i \cdot (S'_i - S_i)}$$

would presumably be a better metric than just the length of the difference vector.

It would make sense to use automatic feature detection and a weighted (continuous) termination criterion. If the relevant features are not clear, and cannot be *a priori* be excluded, they need to be detected. But if the relevant features are not clear, they cannot be used for simple (possibly discrete) termination criteria and not even for something as convenient as $distance(S,S') = |\overrightarrow{S' - S}|$. This would mean an extra complication because good sets of feature weights would result in good termination criteria and the other way around. But bad weights would result in wrong (premature or late) termination of subpolicies and therefore probably degrade the capacity of the agent to identify and classify meaningful behaviours.

7.2.4 Three or More Layers

It would also be useful to study performance of HABS on larger problems, where three or more layers (together with function approximators) are needed. The groundwork for HABS has already been done because HABS is shown to behave well when neural networks are used for the second level. This means that there is – in principle – no problem with extending the structure and adding new layers with more abstraction on top of it. Because of the fact that the policies on the second layer can be regarded as relative actions, applicable anywhere in the problem, means that HABS can also use these second layer policies as subpolicies in a third layer, and so on.

with features that always remain zero.

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Appendix A

Cleaner Task

A.1 The Flat Learner (with "Pickup Rewards")

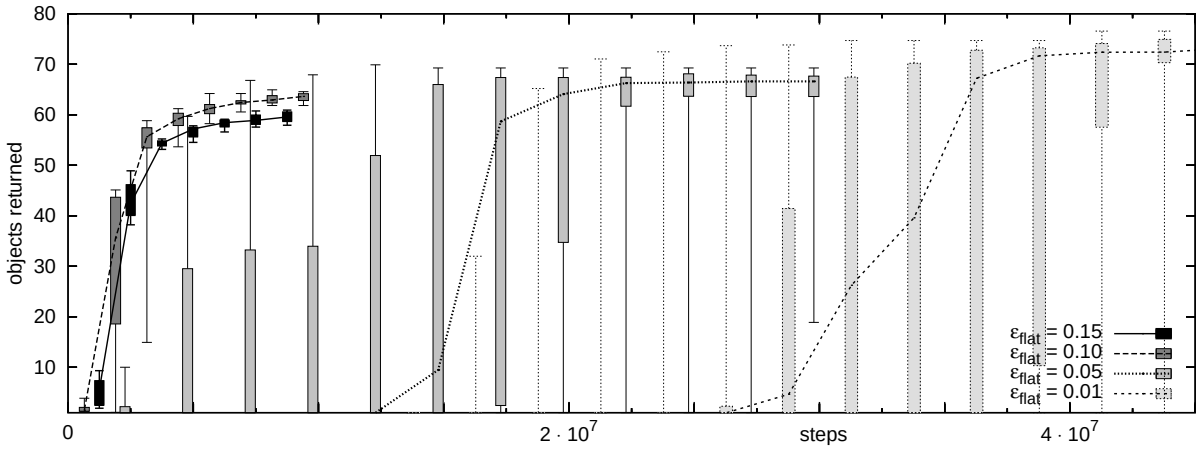


Figure A.1: flat learner – ε-greedy selection*: results for the flat learner when the "pickup rewards" are used. The total reward accumulated per episode (1000 time steps) is displayed.* $\alpha_{flat} = 0.01$, $\gamma_{flat} = 0.97$ *and 15 hidden neurons.*

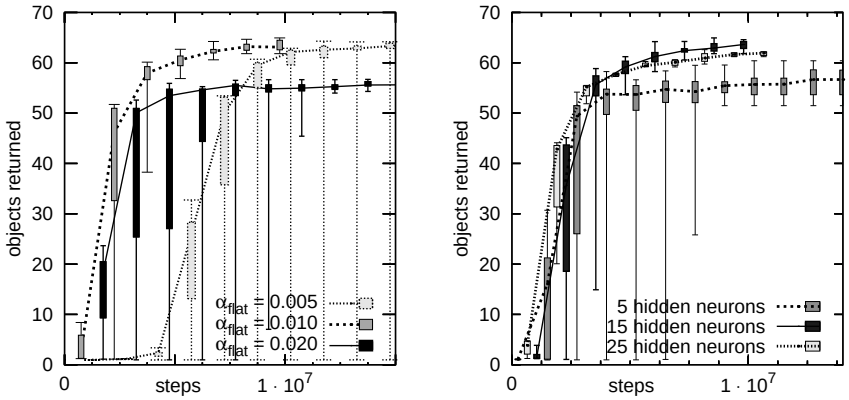
The flat Reinforcement Learning agent was tuned on the simpler Cleaner task with the "pickup rewards". This gives good indication of what performance to expect in the harder task without the small rewards.

Parameters – with ε-Greedy Selection

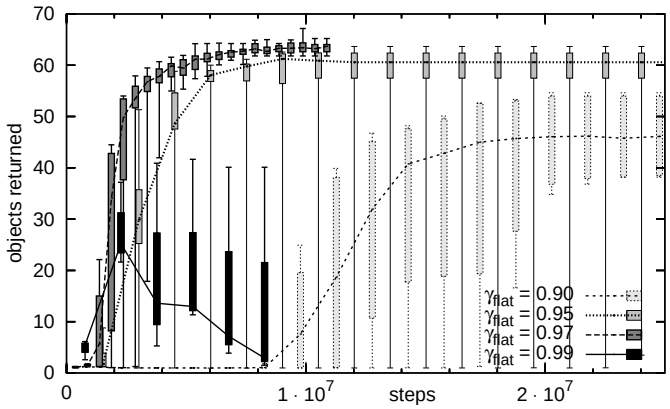
If ε is low ($\varepsilon_{flat} = 0.01$, fig. A.1) the flat learner is able to reach a very high result (over 70 objects returned on average). However, it takes over $3 \cdot 10^7$ time steps to do this on average, and some runs don't show any progress even after $5 \cdot 10^7$ time steps! Higher values ($\varepsilon_{flat} = 0.1$) yield lower results (around 65 objects returned) *but* are able to learn this 10 times as fast.

The discount parameter γ_{flat} is also of significant influence. When γ_{flat} is too high ($\gamma_{flat} = 0.99$, fig. A.2(c)) the learner is unable to reach a good performance. This is probably because a (too) high value of γ_{flat} makes discriminating between actions with nearly equal Q-values too hard (see section 3.1.4). A low value of $\gamma_{flat} = 0.9$ also leads to bad performance (because the reward signal vanishes too fast).

The flat learner was able to solve the task with 5 (good) or 15 hidden neurons (best) per neural network (a network for each of the 6 primitive actions). Increasing the number of neurons to 25 leads to a lower performance, probably due to overfitting. This is shown in fig. A.2(b). More hidden neurons mean more calculations, so smaller networks are desirable from a computational viewpoint.



(a) learning rate α_{flat} (b) hidden neurons



(c) discount γ_{flat}

Figure A.2: flat learner – "pickup rewards" – ε-greedy selection*: results for the flat learner when the "pickup rewards" are used. The total reward accumulated per episode (1000 time steps) is displayed. (The graphs have the same scale for easy comparison.)* ***Defaults:*** $\varepsilon_{flat} = 0.1$, $\alpha_{flat} = 0.01$, $\gamma_{flat} = 0.97$ *and 15 hidden neurons.* ***(a) learning rate*** α_{flat}*: (defaults).* ***(b) hidden neurons****: (defaults).* ***(c) discount*** γ_{flat}*:* $\varepsilon_{flat} = 0.05$*.*

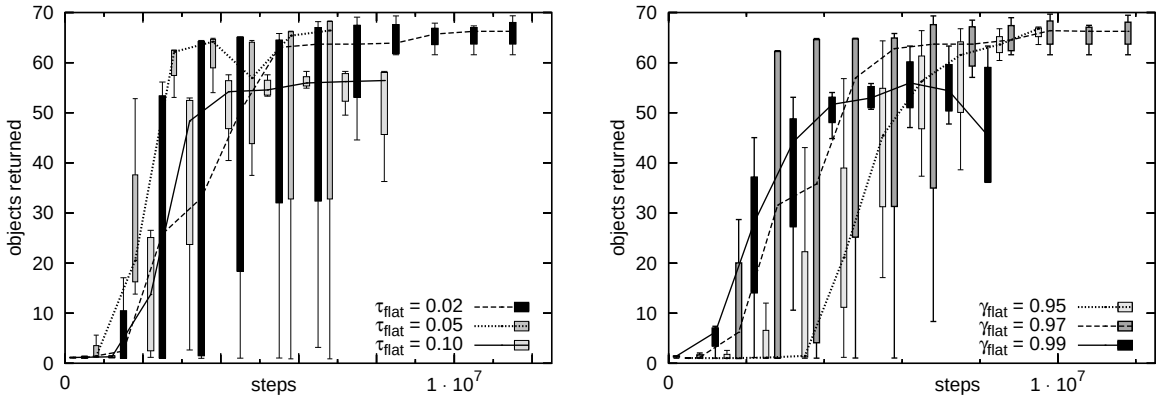
The learning rate α_{flat} (fig. A.2(a)) cannot be pushed too high, because a high value lowers performance. This is probably due to the fact that the adaptions that the network makes in its weights, are too large: the network steps over the optimal value. A low value is possible, but increases the time until convergence (which is undesirable).

A.1.1 Parameters – with Boltzmann Selection

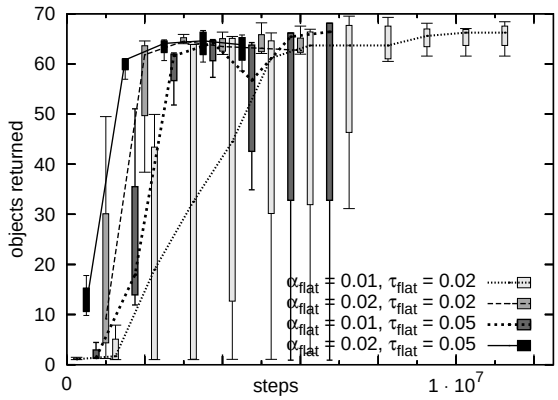
fig. A.3 The flat learner in combination with Boltzmann selection was also briefly investigated. The behaviour with respect to the discount γ_{flat} is the same as with the ε selection (a peak at around $\gamma_{flat} = 0.97$, see fig. A.3(b)).

The relation between the selection and the performance is different from when ε-greedy selection is used. For ε-greedy selection a sharper selection (lower ε_{flat}) meant higher performance *but* far longer time until that performance was reached. With the Boltzmann selection this does not seem the case (see fig. A.3(a)). Different temperatures do yield different performances (higher τ_{flat} result in lower performance) but there does not seem to be a real difference in the time until that convergence is reached (at least for the values that were investigated).

The learning rate α_{flat} behaves stranger (fig. A.3(c)). The higher value $\alpha_{flat} = 0.02$ that does *not* work for the ε-greedy case, does perform good here. However there seems to be a dependence between



(a) Boltzmann τ_{flat} (b) discount γ_{flat}



(c) learning rate α

Figure A.3: flat learner – "pickup rewards" – Boltzmann selection*: results for the flat learner when the "pickup rewards" are used. The total reward accumulated per episode (1000 time steps) is displayed. (The graphs have the same scale for easy comparison.)* ***Defaults:*** *$\tau_{flat} = 0.02$, $\alpha_{flat} = 0.01$, $\gamma_{flat} = 0.97$ and 5 hidden neurons.* ***(a) Boltzmann*** *τ_{flat}: (defaults).* ***(b) discount*** *γ_{flat}: (defaults).* ***(c) learning rate*** *α_{flat}* ***for various Boltzmann*** *τ_{flat}* ***values****: (defaults)*

temperature τ_{flat} and learning rate α_{flat}. When the learning rate is lowered from $\alpha_{flat} = 0.02$ to 0.01, the time until convergence for the learner with a low value of $\tau = 0.02$ increases, but not for the higher value of τ.

A.1.2 Performance

The performance of the flat learner is highly dependent on the selection parameter ε_{flat} and the discount γ_{flat} if ε-greedy selection is used. Values around $\varepsilon_{flat} = 0.1$ and $\gamma_{flat} = \{0.95, 0.97\}$ yield good performance. Lower ε_{flat} values result in the best performance (around 70 objects retrieved), but also in far longer times to convergence (up to an order of magnitude longer!).

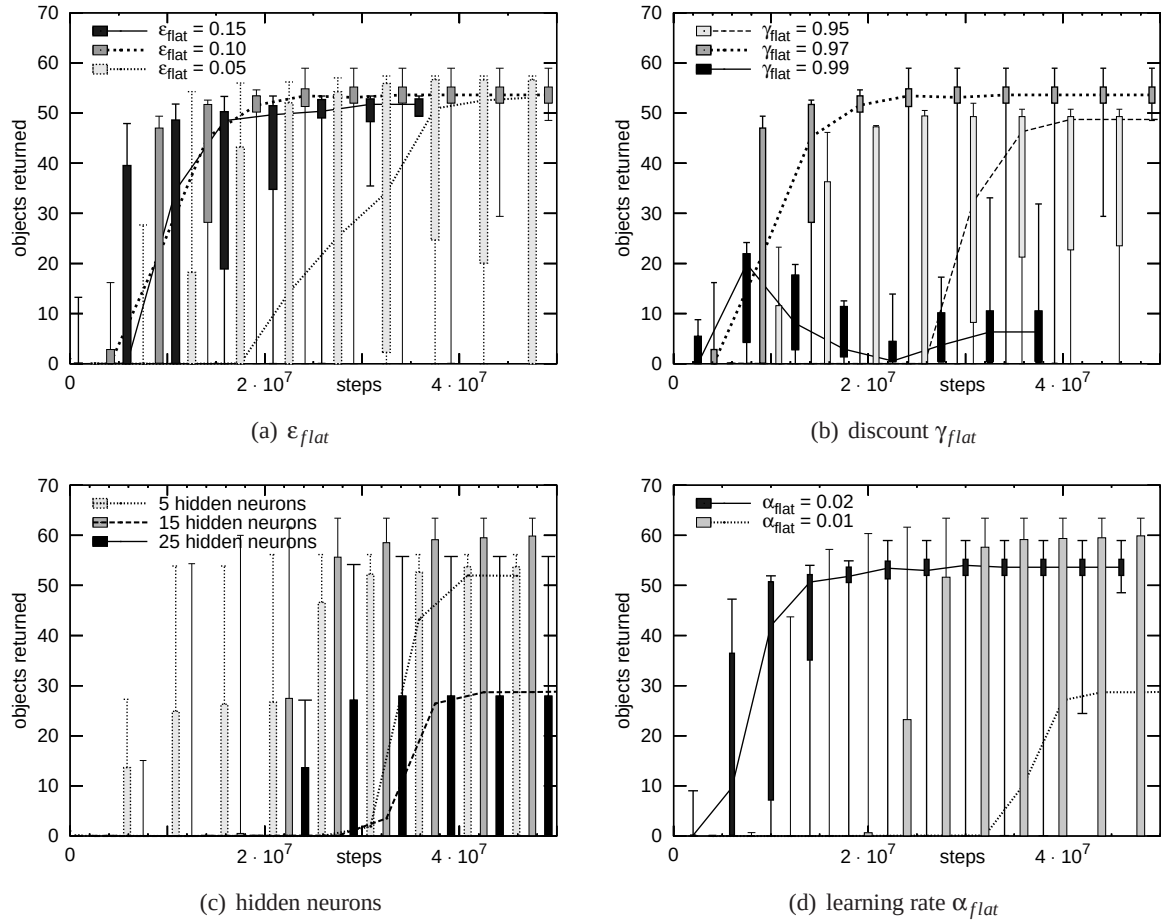
If the flat learner is used in conjunction with Boltzmann-selection, it is able to reach the same performance, but is does so faster (convergence in $2 \sim 3 \cdot 10^6$ steps).

A.1.3 Boltzmann Versus ε-Greedy Selection

It is observed that the flat learner on average seems to learn slightly faster (but no better performance) with Boltzmann selection than with ε-greedy selection in the Cleaner task *with* the "pickup rewards". However, for the Cleaner task *without* these extra rewards, it took *longer* to reach convergence for Boltzmann selection (not shown here), but it reached slightly higher performance of around $60 \sim 65$.

A.1.4 Cleaner Task (without "Pickup Rewards")

In fig. A.4 the results for the flat learner without pickup rewards are displayed. The flat learner needs $2 \sim 4 \cdot 10^7$ steps to arrive at a performance of around $50 \sim 60$.



(a) ε_{flat} (b) discount γ_{flat}

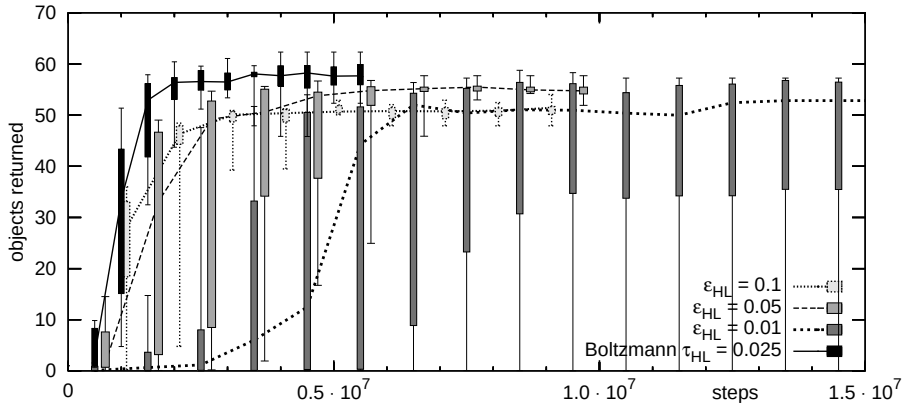
(c) hidden neurons (d) learning rate α_{flat}

Figure A.4: flat learner*: no "pickup rewards".* ***Defaults****: 15 hidden neurons,* $\alpha_{flat} = 0.02$*,* $\varepsilon_{flat} = 0.1$ *and* $\gamma_{flat} = 0.97$ ***(a)*** ε***-greedy****: (defaults).* ***(b) discount*** γ_{flat}*: (defaults).* ***(c) hidden neurons****:* $\alpha_{flat} = 0.01$*.* ***(d) learning rate*** α_{flat}*: for* $\alpha_{flat} = 0.005$ *there was zero performance.*

A.2 HABS

A.2.1 Asymmetries

The flat learner learned better with Boltzmann selection when "pickup rewards" but better with ε-greedy selection without the extra help (Appendix A.1.3). Something similar was observed for HABS, but *the other way around* (see fig. A.5)! Using Boltzmann selection yielded slightly less variance in the time that convergence was reached (when no "pickup rewards" were used). For ε-greedy selection, it sometimes happened that HABS did not learn anything at all in a reasonable time (in comparison with HABS together with Boltzmann selection), even though all settings other than the selection mechanism were the same. Boltzmann selection proved better for HABS, and was therefore used to solve the Cleaner task (without the "pickup rewards") in section 6.5.



(a) ε-greedy

Figure A.5: **HABS — ε*-greedy versus Boltzmann selection***: (no "pickup rewards") for* $\varepsilon_{ll} = \{0.90, 0.95, 0.99\}$ *and* $\varepsilon_{HL} = 0.99$ *(for ε-greedy selection),* $\tau_{ll} = 0.05$ *and* $\tau_{HL} = 0.025$ *(for Boltzmann selection),* $K_{ll} = 0.3$*,* $K_{HL} = 0.2$*,* $\gamma_{ll} = 0.95$*,* $\gamma_{HL} = 0.99$*,* $\alpha_{ll} = 0.01$*,* $\alpha_{HL} = 0.01$*, 2 hidden neurons (low level), 5 hidden neurons (high level), 10 subpolicies, subpolicy-timeout* $= 20$*,* $\rho_{failed} = 0.3$*,* $\rho_{timeout} = -2$*,* $\omega = 0.03$*.*

For Reasons Unknown ...

The reason for these asymmetries is unknown, but they might be related to the number of actions that both algorithms have. The flat learner has only six primtive actions, but HABS uses 10 (or even 20) subpolicies. With so many (high level) actions, it will happen more often that two Q-values have nearly the same value. In that case ε-greedy selection will with probability $(1-\varepsilon)$ select the maximum action, but Boltzmann selection will assign roughly the same probability to the winner and the runner-up. This might help the *comeback kid* to really make a comeback: the action with the second highest Q-values has a greater chance of being selected.

Boltzmann selection is therefore more helpful to actions that (eventually) are the best, but (currently) have a slightly lower Q-value: they have a better chance of catching up and becoming the highest Q-value. With fewer actions this effect becomes less and less of a problem because the probability that these actions are selected at random, is higher.

HABS has (high level) actions that first need to be learned. This means that there is a good chance that subpolicies will run into exactly this problem: they might still be inadequate for a certain behaviour, but as their performance improves, the Q-value of selecting this subpolicy as high level action, becomes better. The flat learner does not have this extra complication, because the primitive actions don't change.