PSM, Spring 2018: Final Exam

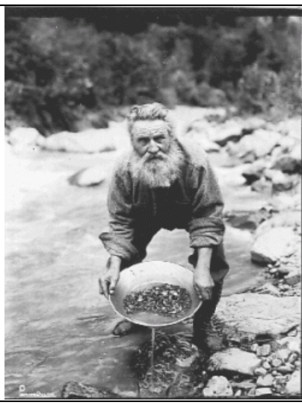
Your name:

How this works. *On the back side you find a list of 30 claims which are either true or false. To the left of each claim you find two boxes, one of which has a tiny* f *and the other a tiny* t *shown in it. If you think the claim is true, tick the box with the tiny* t*, and if you think the claim is false, tick the* f *box. In this way you can get a maximum score of 10 correct answers.*

How this is scored. *If you get everything right, you earn a 100% score. If you get 15 out of 30 right (which means random guessing), the score is 35%. (grade 5.0). Anything else is obtained by linear interpolation, and never less than 0%. This spells out to what you find in the following table:*

correct n	score
0 – 6	0
7	0,33
8	4,67
9	9,00
10	13,33
11	17,67
12	22,00
13	26,33
14	30,67
15	35,00
16	39,33
17	43,67
18	48,00
19	52,33
20	56,67
21	61,00
22	65,33
23	69,67
24	74,00
25	78,33
26	82,67
27	87,00
28	91,33
29	95,67
30	100,00

Answer box		Statements (claims marked with a "(!)" are a little more challenging than the rest)
T		1. Ω is the symbol standardly used to denote a set that is interpreted as a collection of "observation opportunities"
T		2. On a given universe Ω, infinitely many different RVs X_i: Ω → S_i can be defined.
F		3. If Y: Ω → [0, 1], there exists at least one $\omega \in \Omega$ such that $Y(\omega) = 0$.
F		4. If X, Y: Ω → ℝ are two real-valued RVs, then $(X \otimes Y)(\omega) = X(\omega) \cdot Y(\omega)$.
T		5. $(X_t(\omega))_{t\in\mathbb{R}}$ denotes a path of a stochastic process with continuous time.
F		6. Let $\mathcal{F}$ be a σ-field on S. Every subset $\mathcal{G} \subseteq \mathcal{F}$ which satisfies $\{\emptyset, S\} \subseteq \mathcal{G}$, is itself a σ-field.
F		7. If $\mathcal{F}$ is a σ-field on [0, 1) and $\mathcal{G}$ a σ-field on [1, 2], then $\mathcal{H} = \mathcal{F} \cup \mathcal{G} \cup \{[0, 2]\}$ is a σ-field on [0, 2].
F		8. (!) For every $n \in \mathbb{N}$, $n \geq 2$, there exists a set S and a σ-field $\mathcal{F}$ over S, such that the size of $\mathcal{F}$ is n.
T		9. $P(X \in A \cup B) = 0 \Rightarrow P(X \in A) = 0$
T		10. If $A \cap B = \emptyset$ and $P(Y \in C) > 0$, then $P(X \in A \cup B \mid Y \in C) = P(X \in A \mid Y \in C) + P(X \in B \mid Y \in C)$.
T		11. If X: Ω → $(S, \mathcal{F})$ is a RV, then $X^{-1}(S) = \Omega$.
T		12. (!) If X: Ω → $(S, \mathcal{F})$ is a RV, $(S', \mathcal{F}')$ a measurable space, $\varphi: S \to S'$ a $\mathcal{F}$-$\mathcal{F}'$-measurable function, and $A' \in \mathcal{F}'$, then $P(\varphi \circ X \in A') = P(X \in \varphi^{-1}(A'))$.
F		13. Let X_1, X_2 be two identically and independently distributed RVs X_1, X_2: Ω → ℝ, and $\omega \in \Omega$. Then $E[X_1] = ½ (X_1(\omega) + X_2(\omega))$.
F		14. For two real-valued RVs X, Y we have $\mathrm{Cov}(X, Y) \geq 0$.
T		15. (!) A fair coin is tossed repeatedly. Let $T(\omega)$ be the number of tosses until the first head comes up in a tossing trial ω. A gambler offers you a prize of $2^{T(\omega)}$ Euros upon the outcome of a tossing sequence ω if you pay him beforehand a charge of $2^{E[T]}$ Euros. Claim: your statistically expected prize is higher than this charge.
T		16. If $F(x)$ is the cdf of a real-valued RV X, then $F(x) \leq 1$ for all $x \in \mathbb{R}$.

T		17. $P(X \in A)$, $P_X(A)$, and $P(X^{-1}(A))$ all evaluate to the same number.
T		18. The covariance of two identically distributed, real-valued RVs X, Y is less than or equal to the variance of either of the two.
F		19. If X, Y are identically distributed real-valued RVs with values in the unit interval [0, 1], then for every subinterval $[a, b]$ one has $P(X \in [a,b]) > 0$ and $P(Y \in [a,b]) > 0 \Rightarrow P(X \in [a,b], Y \in [a,b]) > 0$
F		20. Two realizations of an infinite Markov chain $(X_i)_{i \in \mathbb{N}}$ can never be identical (stated in mathematical formalism: $\omega \neq \omega' \Rightarrow (X_i(\omega))_{i \in \mathbb{N}} \neq (X_i(\omega'))_{i \in \mathbb{N}}$).
T		21. Consider a k-state Markov chain $(X_i)_{i=0,1,\ldots}$ whose transition matrix is uniformly filled with $1/k$ entries. Then X_0 is independent of X_1.
T		22. Let p: $\mathbb{R}^n \to \mathbb{R}$ be a pdf. Then for all $A,B \subseteq \mathbb{R}^n$, where $A \cup B = \mathbb{R}^n$, one has $\int_{A \cap B} p(x)dx = \int_A p(x)dx + \int_B p(x)dx - 1$
F		23. Let X: $\Omega \to \mathbb{R}$ be a numerical RV. Then $\mathrm{Var}[X] \leq \mathrm{Var}[X^2]$.
T		24. Let X: $\Omega \to [0\ 1]$ be uniformly distributed, and Y: $[0\ 1] \to \{0, 1\}$ be defined by $Y(x) = 1$ if $x = ½$, else 0. Then $E[Y \circ X] = 0$.
F		25. Let $(X_n)_{n=1,2\ldots,k}$, where X_n: $\Omega \to$ {head, tail} describe the experiment of tossing a fair coin k times. Let T: $\Omega \to \{0, 1, \ldots, k\}$ be defined by $T(\omega) = i \Leftrightarrow X_1(\omega) =$ tail, …, $X_i(\omega) =$ tail, $X_{i+1}(\omega) =$ head; that is, T counts the number of an uninterrupted "tail" outcome sequence from the beginning to the first "head". Claim: T is binomially distributed.
F		26. Imagine you are a planespotter, monitoring landing aircraft at Frankfurt Rhein-Main airport. Let T be the RV that measures the time intervals between consecutive landings. Claim: T is (in good approximation) exponentially distributed.
F		27. Imagine you are panning for gold. Let $X \in \{0, 1\}$ be the "success" indicator, being 0 when there is not a single grain of gold found in a panning attempt, and being 1 if you find one or more grains. Claim: X is Poisson distributed. (Image source: thornews.com/2012/04/23/digging-for-gold-in-norway) 
T		28. The probability that an untrained monkey hitting a computer keyboard will type this very problem statement that I typed into this box is less than 1/2 to the power of the number of letters in this box.

F		29. Let $G \in$ {male, female, other} be the gender RV used in a customer profiling study. Let $\theta = (p_m, p_f, p_o)$ be the parameter vector made of the three probabilities characterizing the gender distribution in the customer population. Then the estimator t which always returns $t(x) = (0, 0, 1)$ is an unbiased estimator of θ.
T		30. Consider a coin tossing scenario with 3 tosses. Let q denote the probability of the coin coming up with "1". Then the maximum likelihood estimate of q given the data $x = (0, 0, 0)$ is $\hat{q} = 0$.