

## Machine Learning, Spring 2019: Exercise Sheet 5 – with solutions

*This problem sheet is a refresher for basic probability concepts. You can easily find solutions for these basic problems on the web, even on Wikipedia, – it's of course a much more profound learning experience when you work out the derivations yourself.*

**Problem 1** Give a derivation for the formula  $\text{Cov}(X, Y) = E[XY] - E[X] E[Y]$ .

**Problem 2.** Prove that the mean minimizes the quadratic loss, that is, for a random variable  $X$  with values in  $\mathbb{R}$ ,

$$E[X] = \operatorname{argmin}_{x \in \mathbb{R}} E[(x - X)^2]$$

(this is another good reason for why the quadratic loss is so popular!)

**Problem 3.** Show that for two RVs  $X, Y$  with values in  $\mathbb{R}$ , it holds that  $-1 \leq \text{Corr}(X, Y) \leq 1$ . (Assuming that both RVs don't have zero standard deviation, and that their joint distribution is characterized by a pdf  $f(x, y)$ ). You may use the following fact (a special case of the so-called *Cauchy-Schwarz* inequality):

$$\left( \int_{\mathbb{R}^2} xy f(x, y) d(x, y) \right)^2 \leq \int_{\mathbb{R}^2} x^2 f(x, y) d(x, y) \cdot \int_{\mathbb{R}^2} y^2 f(x, y) d(x, y)$$

where  $f$  is a pdf on  $\mathbb{R}^2$ .