Exercises for FLL, Fall 2018, sheet 9 – Solutions

Return Thursday Nov 15 in class

Exercise 1. Here is a famous photo[1]:

(a) Imagine you would have to describe this picture to a blind friend in 5 sentences. Write these 5 sentences down in plain English and in a FOL rendering. Specify the type and arity of your symbols; you may use parenthesis-saving conventions.
(b) This photo invites aesthetic and philosophical thinking (take a look at the website it was taken from!). Think of one such "deep" thought and argue why it can't be formalized in FOL.
(Alternatively, argue that all aesthetic and philosophical thoughts can be expressed in FOL). You think this is a strange exercise? Well, it has been tried to formalize legal reasoning in FOL, for juridical expert systems... and legal reasoning is "deep".

Solution. Here are some ad hoc descriptive FOL sentences to give the flavour, in ordinary English and some less or more detailed FOL renderings.

"The ground is covered with water":

is_covered_with_water ground   [is_covered_with_water: unary predicate, ground: constant]
is_covered_with ground water   [is_covered_with: binary relation; ground, water: constants]
$\forall x$ (ground-location $x \rightarrow \exists y$ (substance-of y = water $\wedge$ 3-dim-shape-of y = horizontal_sheet $\wedge$ on-top y x))
[ground-location: unary predicate; substance-of, 3-dim-shape-of: unary functions; water, horizontal_sheet: constants; on-top: binary relation]

"A ladder is lying on the ground":

[1] http://www.dienes-and-dienes.com/Cartier-Bresson.html

∃ x (is_visible x ∧ physical-object x ∧ ladder x ∧ orientation-of x = horizontal ∧ on-top x ground)
[is_visible, physical-object, ladder: unary predicates; orientation-of: unary function; horizontal, ground = constant; on-top: binary relation]

Exercise 2. List all subformulas and terms that occur in

$$\exists x ((Qxy \lor \forall y (\neg Pffa \to Qaa)) \lor Ryxa),$$

where Q is a binary predicate symbol, P is a unary predicate symbol, f is a unary function symbol, a is a constant symbol and R is a ternary predicate symbol. Determine for each occurrence of a variable whether it is free or bound.

Solution. The terms that occur are a, x, y, fa, and ffa. The subexpressions are Qxy, $Pffa$, $\neg Pffa$, Qaa, $(\neg Pffa \to Qaa)$, $\forall y (\neg Pffa \to Qaa)$, $\forall y (\neg Pffa \to Qaa)$, $(Qxy \lor \forall y (\neg Pffa \to Qaa))$, $Ryxa$, $((Qxy \lor \forall y (\neg Pffa \to Qaa)) \lor Ryxa)$. Free/bound occurrences: marked by indices here: $\exists x ((Qx_{bound}y_{free} \lor \forall y (\neg Pffa \to Qaa)) \lor Ry_{bound}x_{bound}a)$.