Exercises for FLL, Fall 2018, sheet 6 – Solutions

Return Thu Oct 30, in class.

Exercise 3. Design a PDA that accepts $L = \{0^n1^m2^k \mid n, m, k \geq 1 \text{ and } n \neq m\}$ by accepting state. Describe the idea behind your PDA in words and specify its transition function. Be kind to the TAs: your plain English description of the working principles of your PDA should be clear and complete.

Solution.
Idea: The PDA for L always ends dead when the input is not of the form $0^n1^m2^k$, where $n, m, k \geq 1$. This can be achieved by ensuring that the set S_0 of states that are entered after reading 0 are disjoint from the set S_1 of states that are entered after reading 1, and again both sets are disjoint from the 2-reachable states S_2. Furthermore, states from S_1 can only be reached from states of S_0 or S_1, and states from S_2 only from S_1 or S_2 states. Accepting states are all in S_2. To check the conditions $n \neq m$, the PDA first memorizes the number of read 0's by copying them on the stack. When it starts reading 1's, it cancels 0's from the stack until one of the following occurs:

(i) It reads a 1 but the stack has no more 0's. Then $n \neq m$. The PDA enters a mode where it only checks whether the rest of the word is of form $1^{m'}2^k$, where $m' \geq 0$, $k \geq 1$.

(ii) It reads the first 2 but the stack still has 0's. Then again $n \neq m$. The PDA enters a mode where it only checks whether the rest of the word is of form $2^{k'}$, where $k' \geq 0$.

(iii) It reads the first 2 exactly after it has deleted the last 0 from the stack. Then $n = m$ and the PDA is halted in a dead end.

Here is a list of the required transitions. The stack top symbol is Z_1, and the start state is q_1. The single accepting state is q_{accept}.

1. $\delta(q_1, 0, Z_1) = \{(q_1, 0Z_1)\}$
2. $\delta(q_1, 0, 0) = \{(q_1, 00)\}$
3. $\delta(q_1, 1, 0) = \{(q_2, \varepsilon)\}$; entering the downcounting of 0's
4. $\delta(q_2, 1, 0) = \{(q_2, \varepsilon)\}$
5. $\delta(q_2, 1, Z_1) = \{(q_3, Z_1)\}$; entering mode (i)
6. $\delta(q_3, 1, Z_1) = \{(q_3, Z_1)\}$; continue reading 1's
7. $\delta(q_3, 2, Z_1) = \{(q_{\text{accept}}, Z_1)\}$
8. $\delta(q_{\text{accept}}, 2, \#) = \{(q_{\text{accept}}, \#)\}$; # is any stack symbol
9. $\delta(q_2, 2, 0) = \{(q_{\text{accept}}, \varepsilon)\}$; entering mode (ii)
10. $\delta(q_2, 2, Z_1) = \{\}$; case (iii)