Exercises for Computability and Complexity, Spring 2019, Sheet 5

Please return on Tuesday, March 12, in class. As usual you are invited but not requested to work in teams of size at most 2.

Exercise 1 (rather easy) Prove that $H_2 = \{<M>;x \mid Code(<M>)$ and $Standard(x)$ and there exists some y with $Standard(y)$ such that $M(x) = y\}$ from Proposition 6.3 is undecidable.

Exercise 2 (medium difficult) Show that the language

$$L = \{<M> \in \{0, 1, \#\}^* \mid M \text{ halts on no input}\}$$

is not recursively enumerable. *Hint: in addition to a reduction argument, you might wish to also work in Proposition 3.1 from the lecture notes.*

Challenge problem (optional, not easy) Prove the following claim: If L is recursively enumerable but not recursive, then there exists another language L' which is likewise r.e. but not recursive, such that $L \cup L'$ is recursive.