Exercises for Computability and Complexity, Spring 2019, Sheet 4 – Solutions

Please return on Tuesday March 5 in class. As usual you are invited but not requested to work in teams of size at most 2.

Exercise 1 Consider the set $\boldsymbol{T}$ of all single-tape TMs with tape alphabet $\Sigma = \{0, 1, \sqcup, \rhd\}$. Design a coding scheme by which every TM M in $\boldsymbol{T}$ becomes coded by a codeword $<M> \in \{0, 1, \#\}^*$. Describe your coding scheme in formal notation and use it to encode the the ultra-simple TM M with tape alphabet $\Sigma = \{0, 1, \sqcup, \rhd\}$ and states $\{s, yes, no\}$ that has the following transition table:

$p \in K$	$\sigma \in \Sigma$	$\delta(q, \sigma)$
s	0	$(yes, 0, -)$
s	1	$(s, 1, \rightarrow)$
s	$\sqcup$	$(no, \sqcup, -)$
s	$\rhd$	$(s, \rhd, \rightarrow)$

Solution. Here is one of a zillion of possibilities: Let $M \in \boldsymbol{T}$ have l states $s_1, s_2, ..., s_l$ (including the h, yes, no states). Encode a state s_i by $<s_i> := \#\text{bin}(i)$, where bin(i) is the binary representation of the number i. Encode the three cursor move directions by $<\rightarrow> = \#01$, $<\leftarrow> = \#10$, $<-> = \#00$, and tape symbols by $<0> = \#0$, $<1> = \#1$, $<\sqcup> = \#00$, $<\rhd> = \#11$. Let $R = s_i\ \sigma\ (s_j, \sigma, d)$ be a row in a transition table Tab(M) of M, where d is one of $\rightarrow$, $\leftarrow$, $-$. Code R by $<R> = <s_i><\sigma><s_j><\sigma><d>$. Let $R_1, ..., R_m$ be the rows of Tab(M). Code the transition table by $<\text{Tab}(M)> = <R_1>...<R_m>$ and we are done, because the TM is uniquely specified by this table. For the example, put $s_1 = s$, $s_2 = h$, s_3 = "yes", s_4 = "no", leading to $<s> = \#1$, $<h> = \#2$, etc. Then the four rows given in the table in Exercise 1 translate to

$<M>$ = #1#0#11#0#00#1#1#1#1#01#1#00#100#00#00#1#11#1#11#01