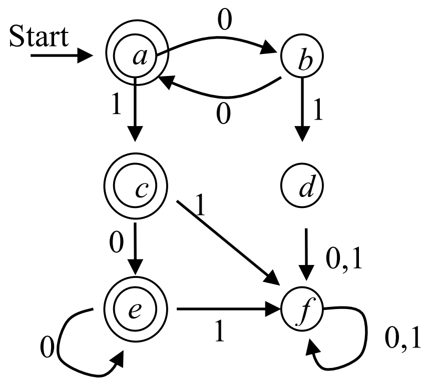
Exercises for FLL, Fall 2018, sheet 4 – Solutions

Return Wed Oct 11, in class. As always you may work in teams of two if you wish – submitting one solution per team with both names on it.

Exercise 1. Minimize the DFA shown in the figure by using the table filling method. Deliverables: the filling table, the set of states of the minimal DFA, and a graph representation of the minimal DFA.

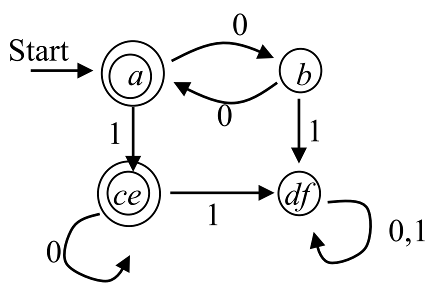


Solution. Manual labour by accurate following of the recipe...

Table:

b	x_1				
c	x_2	x_1			
d	x_1	x_2	x_1		
e	x_2	x_1		x_1	
f	x_1	x_2	x_1		x_1
	a	*b*	*c*	*d*	*e*

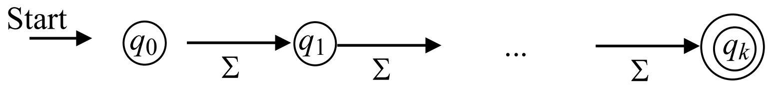
Minimal DFA:



New states: $\{a\}$, $\{b\}$, $\{c,d\}$, $\{e,f\}$

Exercise 2. Let L be a regular language specified by a DFA, NFA, ε-NFA, or regexp. Show that it is decidable whether $L = \Sigma^k$ for some $k > 0$.

Solution. There are many ways of how this can be decided. One elegant way is to first construct the minimal DFA A for L. Then obviously $L = \Sigma^k$ for some k iff A has the form



Note. The use of the word "obviously" in mathematical proofs is a delicate affair. One never knows what the reader is ready to accept as obvious. Here I think we have a borderline case, and one might feel the need to prove that if $L = \Sigma^k$ then the minimal automaton actually has the given form (it is really obviously obvious that this kind of DFA accepts $L = \Sigma^k$, so the only possibly questionable claim is its minimality). Minimality of DFAs of the shown form could be proven by going through the table-filling algorithm and showing that all the shown states are distinguishable. This would be a case for extra grading points.