Exercises for FLL, Fall 2018, sheet 1 – Solutions

Return Thursday Sep 13, in class

Note: you may work in teams of 2 if you wish. If you do, hand in a single solution sheet for both of you.

Exercise 1 (a) How many words exist over the alphabet Σ = {1}? and over the alphabet Σ = {a, b}? **(b)** How many words of length n exist over an alphabet of size k? **(c)** How many languages exist over the alphabets from (a) and (b)? **(d)** How many languages of words of length n exist over an alphabet of size k? **(e, a bit more difficult, optional)** Show that there are countably infinite many *finite* languages over Σ = {a, b}. *Hint: show two things. First, that there are **at least** as many finite languages as there are natural numbers – show this by giving an injective map from* ℕ *to the set of finite languages. Second, show that there are **at most** as many finite languages as there are natural numbers – show this by giving an injective map from the set of finite languages to* ℕ. *The second part is more difficult than the first.*

Solution: (a) The words over S = {1} are ε, 1, 11, 111, ... – that is, as many as there are integers, that is, countably infinite many. The words over S = {a, b} can be listed in a sequence, shortest first, sorted alphebetically for same size: ε, a, b, aa, ab, ba, bb, aaa, aab, ... – that is, again countably many. **(b)** k^n many. **(c)** In both cases, |Σ*| = | ℕ |, that is, there are are | $2^{\mathbb{N}}$ | = | ℝ | many languages over these alphabets – indeed, over any finite alphabet there are | ℝ | many languages. **(d)** Since there are k^n many words of length n over a symbol set of size k, there are $2^{(k^n)}$ many such languages. **(e)** Let F be the set of finite languages over {a, b}. (I) "***at least** as many finite languages as there are natural numbers":* there are many ways to define an injection β: ℕ → F. For instance, $\beta(n) := \{a, a^2, ..., a^n\}$ does it. (II) "***at most** as many finite languages as there are natural numbers":* It is clear that a finite language over over Σ = {a, b} can be written as a word over the over Σ' = {a, b, _}. Example: L = {a, aa, ab, bbba} can be written as a_aa_ab_bbba. We know that there are | ℕ | many words over Σ' = {a, b, _}, and each finite language over Σ corresponds to one of these words, so there are no more finite languages over Σ than words over Σ', that is no more than | ℕ | many.