

## PSM SPRING 2019, HOMEWORK 4

1. Prove the immediate consequences listed in the LN after Definition 7.1.1:

$$\begin{aligned}P(\emptyset) &= 0 \\P(A^c) &= 1 - P(A) \\A \subseteq A' &\Rightarrow P(A) \leq P(A')\end{aligned}$$

2. Consider the uniform distribution on the unit interval  $S = [0, 1]$ . Since this is a part of the real line, this sample space is equipped with the Borel  $\sigma$ -field  $\mathfrak{B}([0, 1]) = \sigma(\{(a, b) \mid 0 \leq a \leq b \leq 1\})$ . For each interval  $(a, b]$  in this generator of  $\mathfrak{B}([0, 1])$ , we have  $P(X \in (a, b)) = b - a$ . Use this to show that  $P(X = a) = 0$  (consider only the case  $0 < a < 1$ ).
3. Show that

$$P(X \in A, Y \in B, Z \in C) = P(X \in A) P(Y \in B \mid X \in A) P(Z \in C \mid X \in A, Y \in B).$$