The Sheaf-Theoretic Structure Of Non-Locality and Contextuality

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Locality and non-contextuality are intuitively appealing features of classical physics, which are contradicted by quantum mechanics. The goal of the classic no-go theorems by Bell, Kochen-Specker, et al. is to show that non-locality and contextuality are necessary features of *any* theory whose predictions agree with those of quantum mechanics. We use the mathematics of sheaf theory to analyze the structure of non-locality and contextuality in a very general setting. Starting from a simple experimental scenario, and the kind of probabilistic models familiar from discussions of Bell's theorem, we show that there is a very direct, compelling formalization of these notions in sheaf-theoretic terms. Moreover, on the basis of this formulation, we show that the phenomena of non-locality and contextuality can be characterized precisely in terms of obstructions to the existence of global sections. We give linear algebraic methods for computing these obstructions, and use these methods to obtain a number of new insights into non-locality and contextuality. For example, we distinguish a proper hierarchy of strengths of no-go theorems, and show that three leading examples — due to Bell, Hardy, and Greenberger, Horne and Zeilinger, respectively — occupy successively higher levels of this hierarchy. We apply our linear algebraic methods for constructing global sections to the issue of giving local hidden-variable realizations using negative probabilities. We show that, for all multipartite devices with two dichotomic measurements per site, there is an equivalence between the existence of such realizations, and the no-signalling property.

The structure uncovered by this analysis is very general. The same methods can be applied in a range of other situations of an apparently very different nature, including social choice theory and relational databases.

References

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