Logic, Context and Valid Inference
Or: Can there be a Logic of Law?

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Abstract

The question is addressed whether it makes sense to speak of a logic of law. It is shown that what counts as valid inference depends to a large extent on context-dependent choices. This suggests that our question has a simple answer, namely that a logic of law can exist. After noticing that one logic can serve as the background of another, it is explicated that a more subtle answer can be given. On the one hand a logic of law can exist, and on the other hand it can be possible to reduce such a logic to a set of legal premises in a more abstract logic. It is posited how a 'contextual logic' approach and an 'abstract logic' approach can lead to different priorities in the formalization of legal reasoning.

1 Introduction

Recently, a lot of research has been done on the formalization of legal reasoning (see e.g. the work of Hage (1996) and Prakken & Sartor, (1996)). Among the topics addressed are exceptions to rules, rule applicability, inconsistency handling, reasoning with priorities, the weighing of reasons, the role of rules and principles, argument attack and defeat, role-dependency (such as burden of proof) and the dynamics of reasoning.

In this research, many formal patterns of legal reasoning have been explicated, which has resulted in a deeper understanding of these reasoning patterns. Nevertheless – and notwithstanding the offhand claims of some of the researchers involved – no formalism has gained the status of the canonical logic of law. In this paper, I try to clarify what could be meant by a logic of law, and whether it is to be expected that a canonical theory will arise.

Some of my experiences during several years of research on formalizing legal reasoning have led me to write the present paper. First in my collaboration with Jaap Hage I have seen many versions of Reason-Based Logic[1]. Would it have been the case that the version history of Reason-Based Logic simply consisted of the gradual addition of new features, there would not be reason to ponder on its development. However, the version history of Reason-Based Logic is *not* that simple. The signs of changes of opinion, shifts of

1 Reason-Based Logic was initiated by Hage and elaborated in cooperation with me. See, e.g., Hage (1996, 1997) and Verheij (1996a).

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focus and personal biases are abundant.[2] As a result, a recurring theme in my discussions with Jaap Hage has been the question whether in the end Reason-Based Logic would converge to a stable theory of legal reasoning. Question: Can a canonical logic of law be expected?

Second, I have always felt a stimulating tension between my research work in a legal faculty and my mathematically-oriented background. On the one hand there was the requirement of practical or at least theoretical relevance for the field of law. On the other hand there was the urge to look for the simple, abstract core underlying the confusing richness of legal reasoning. The tension is discernible in the structure of my dissertation (Verheij, 1996a). It is divided into two parts, the first on Reason-Based Logic, a rich theory of rules and reasons, developed with an eye on the law, the second on CumulA, an abstract theory of argumentation with arguments and counterarguments, of which the legal relevance was less clear. Question: Can an abstract theory, such as CumulA, be of any relevance for the study of legal valid inference?

Third, not long ago I have written a review of the revised edition of Prakken's dissertation (Prakken, 1997). Prakken distinguished 'justifying force as a matter of form' and 'justifying force as a matter of content' in an attempt to distinguish deductive reasoning from reasoning by analogy. Elsewhere (Verheij, 1998a, 1998b) I argued that Prakken's argument did not lead to a successful distinction between the two. My main difficulty was that I consider 'formally' valid inference to be determined to a large extent by context-dependent choices of logic. Question: In what way is 'formally' valid inference determined by the logical context?

Fourth, recently I visited a symposium on legal argumentation, where Prof. A. Soeteman who is known for his work on the relation of logic and law (see, e.g., Soeteman, 1991), gave a lecture on the meaning of principles for legal argumentation. In his interesting talk, he made some thought-provoking remarks. One of his claims was that as a topic there is no such thing as a logic of law. According to Soeteman, the 'lawness' of valid legal reasoning is entirely in the legal premises. Question: Can there be a logic of law, or is it just a matter of legal premises?

In the present paper, I attempt to find answers to the questions raised above by studying the relation between logic, context and valid inference. Some preliminary remarks on these two notions are in place.

By a logic, I mean here a formal theory of valid inference.[3] In a logic, valid inference is often dually explicated in terms of formal operations, such as proof construction using rules of inference, and of the interpretations of sentences (or of sets of sentences), such as logical models. Ideally, the former syntactic and the latter semantic characterization of valid inference are equivalent (cf. e.g. Haack, 1978). For instance, in first-order predicate logic, a proof theory (e.g., in terms of rules of inference) is shown to be sound and complete with respect to model-theoretic interpretations of the logical language.

2 E.g., Hage (1997) argues that rules and principles are to be formally distinguished, while Verheij and Hage (1994) presented an integrated view on rules and principles (see also Verheij *et al.*, 1998).

3 I use the notion of logic in a rather broad sense. It is certainly not my intention to imply that this interpretation of logic is the right one or the only one.

The use of the term 'context-dependent valid inference' implies that I take it for granted that valid inference is context-dependent. Note that I consider the *validity* of reasoning to be possibly context-dependent, and not just reasoning itself; actual reasoning clearly depends on the actually obtaining facts and held beliefs, e.g., expressed in logic by a set of premises.

Two obvious observations lead me to take the existence of context-dependent valid inference for granted. First there is the common observation that reasoning by analogy can be forbidden in the context of criminal law (on the basis of the principle of *nulla poena*), while it is allowed in the context of civil law, as is indeed the case in the Dutch legal system.[4] In other words, whether reasoning by analogy is allowed as a means to reach a legal decision depends on the legal domain of the case at hand.[5]

The second observation is that even in what might be the least context-dependent domain of reasoning, viz. mathematics, there are different contexts of valid inference. This is shown by the differences between mathematical results that are based on classical reasoning and those based on intuitionistic reasoning. For example, the well-known intermediate-value theorem[6] obtains in classical real analysis, but has a counterexample in intuitionistic real analysis (see Troelstra and Van Dalen, 1988, p. 292).

In the next section, I more extensively discuss the idea of logic as formalized context-dependent valid inference. In section 3, it is noted that one logic can serve as the background of another logic. The findings are used to answer the question whether a logic of law can exist (section 4).

2 Logic as formalized context-dependent valid inference

In this section, I consider logic as a formal theory of context-dependent valid inference.[7] I claim that what is regarded as valid inference depends to a large extent on contingent choices and is therefore context-dependent.

To some this claim may seem strange if for instance inferring P from $P \wedge Q$ or $P \vee Q$ from P is regarded to be valid because of the 'logical form' of the sentences involved. It is however important to note that the idea of 'logical form' only makes sense *given the choices* made in the characterization of standard valid inference involving the classical connectives $\wedge$ and $\vee$. To those who do not know that '$\wedge$' is standardly used to express conjunction and '$\vee$' to express disjunction, this context-dependency is obvious. You have to be *told* what valid inference involving $\wedge$ and $\vee$ looks like, for instance in terms of a logical semantics or in terms of rules of inference, in order to see the 'formality' of inferring P from $P \wedge Q$.

Therefore one should clearly keep in mind that what counts as 'logical form' is determined by the actual logic considered. *Each logic can be regarded as a context determining formally valid inference in the context of that*

4 Van Bemmelen and Van Veen (1995), p. 31.

5 One might be of the opinion that this observation is beside the point if one does not consider reasoning by analogy as a *valid* form of reasoning, not even in the context of civil law. I argue elsewhere (Verheij, 1998a, 1998b) that reasoning by analogy can be considered as a valid form of reasoning, depending on the context. The main point is in section 2.

6 The theorem reads as follows. If f is a continuous real function on a closed interval [a, b] and d is any number between f(a) and f(b), then there is a number c in the open interval (a, b) such that f(c) = d.

7 In this section, some of the material of Verheij (1998a, 1998b) is used.

logic. In the rest of the present section, this point is further explained by means of some example logics.
As a first example of context-dependent valid inference I discuss valid inference in the contexts of strict rules and of rules with exceptions. The starting point is a logical language in which three non-elementary types of sentences can be distinguished:

1. As a rule, if *P*, then *Q*
2. There is an exception to the rule that if *P*, then *Q*
3. It is not the case that *P*

All three are sentence schemes. Sentences are obtained by instantiating the sentence variables *P* and *Q*. Three examples of sentences are the following:

As a rule, if Peter has violated a property right, then Peter has committed a tort
There is an exception to the rule that if Peter has violated a property right, then Peter has committed a tort
It is not the case that there is an exception to the rule that if Peter has violated a property right, then Peter has committed a tort

Note that the sentences above are used as *logical* expressions (which explains the unnatural repetition of 'Peter'), and not as expressions in natural language.

As yet, the three sentence types have been left *uninterpreted*, even though the notation suggests an intended meaning. Indeed, the first type of sentence is meant to express that there is a conditional relation between two facts, and the second that there is an exception to such a conditional relation. The third is meant to express negation. It should be noted however that by choosing the logical language there is not yet a commitment to any interpretation. For instance, rules can still be interpreted as strict rules or as rules with exceptions. Whether the rule sentences are actually interpreted as strict rules or as rules with exceptions depends on a *choice*, that can be made explicit by semantic constraints, e.g., in terms of truth values.

For the negation sentences, the classical semantic constraint for negation is used:

I *Negation*
'It is not the case that *P*' is true if and only if '*P*' is false.

The semantic constraint should be read as a constraint on an interpretation of sentences in the language. If one prefers, the constraint can be read as the constraint that 'It is not the case that *P*' is true in an interpretation M if and only if '*P*' is false in M. Here a possible interpretation is an assignment of truth values true or false to the sentences of the logical language.[8]

Which semantic constraints do we want to obtain if rules are interpreted as *strict* rules? If we take strict rules as rules that cannot have exceptions,

8 Interpretations are here not assignments of truth values to only the sentential constants of the logical language since the truth value of sentences of our language is not a function of the truth-values of its subsentences. In other words, our logic is not truth-functional.

the semantic constraints (in terms of truth values) would include the following:

II *Strict rules are followed*
If 'As a rule, if *P*, then *Q*' and '*P*' are both true, then '*Q*' is true.

III *Strict rules have no exceptions*
'There is an exception to the rule that if *P*, then *Q*' is false.

Let's consider the following two sentences.

a. Peter has violated a property right
b. As a rule, if Peter has violated a property right, then Peter has committed a tort

If these sentences are both interpreted as true sentences, it follows, using the semantic constraint II, that the sentence 'Peter has committed a tort' must also be true. Note also that by the constraints I and III any sentence of the form 'It is not the case that there is an exception to the rule that if *P*, then *Q*' is true in any interpretation.

Next we can look for semantic constraints on the *same* types of sentences, i.e., for the same logical language, but this time interpreted for rules with exceptions. Negation sentences are still interpreted subject to the constraint I. Obviously, the two constraints II and III for strict rules should not hold for rules with exceptions. The semantic constraint III does not have a counterpart for rules with exceptions, but the counterpart of constraint II could be as follows:

IV *Rules are followed if there is no exception*
If 'As a rule, if *P*, then *Q*' and '*P*' are both true, and 'There is an exception to the rule that if *P*, then *Q*' is false, then '*Q*' is true.

If we now again look at the two sentences a. and b. above, and interpret them as true sentences, but use the semantic constraint for rules with exceptions, it does no longer follow that the sentence 'Peter has committed a tort' is true. Using the semantic constraint IV requires the falsity of the sentence 'There is an exception to the rule that if Peter has violated a property right, then Peter has committed a tort'. So from 'Peter has violated a property right', 'As a rule, if Peter has violated a property right, then Peter has committed a tort' and 'It is not the case that there is an exception to the rule that if Peter has violated a property right' it follows (using I and IV) that 'Peter has committed a tort'.[9]

Summarizing, we now have two semantics for the logical language with the three sentence types 1, 2 and 3. In the first, 'strict' semantics, the rule sentences are interpreted as strict rules, and the semantic constraints I, II and III must obtain. In the second, 'exception' semantics, the rule sentences are interpreted as rules with exceptions, and the semantic constraints I and IV must obtain.

9 Note that the notion of valid inference on the basis of rules with exceptions as modeled here is *monotonic*. See for instance Verheij (1996a, p. 92 ff) for ways to deal with nonmonotonic inference with rules with exceptions.

The semantic characterization to valid inference has its dual in the proof-theoretic characterization, e.g., in terms of rules of inference. Let's consider two rules of inference.

Modus ponens
Premises:
As a rule, if *P*, then *Q*
P
Conclusion:
Q

Modus non excipiens[10]
Premises:
As a rule, if *P*, then *Q*
P
It is not the case that there is an exception to the rule that if *P*, then *Q*
Conclusion:
Q

The second rule of inference *Modus non excipiens* differs only in its third premise 'It is not the case that there is an exception to the rule that if *P*, then *Q*' from the well-known *Modus ponens* (that here may look a bit odd due to our unusual logical language). It says that the consequent of a rule follows from its antecedent, provided that there is no exception to the rule.

It is now important to see that the sentences in the rules of inference *Modus ponens* and *Modus non excipiens* can again be interpreted both for *strict* rules and for rules *with exceptions*. Let's investigate whether these two rules of inference are truth-preserving (sound) with respect to the strict and the exception semantics. Which rules of inference are truth-preserving is, *just as what we are used to in classical logic*, determined by the obtaining semantic constraints.

If the rule sentences are interpreted as expressing *strict* rules, the constraint that strict rules are followed determines that the rules of inference *Modus ponens* and *Modus non excipiens* are both truth-preserving. However, for strict rules, *Modus non excipiens* does not allow consequences that cannot already be derived using *Modus ponens* alone, since the additional premise of *Modus non excipiens* ('It is not the case that there is an exception to the rule that if *P*, then *Q*') is – as was already noted – always fulfilled in the strict semantics. As a result, Occam's razor suggests that, for strict rules, *Modus ponens* suffices as a rule of inference.

Similarly, if the rules are interpreted as rules *with exceptions*, the constraint that rules are followed if there is no exception, determines that the rule of inference *Modus non excipiens* is truth-preserving. *Modus ponens* interpreted for rules with exceptions is not truth-preserving since its conclusion does not obtain in all interpretations in which its premises obtain: in a case that the rule's antecedent obtains, while there is an exception, the consequent does not always obtain.[11]

10 I thank Bram Roth for suggesting the name of this rule of inference.

11 Though *Modus ponens* is not truth-preserving with respect to the exception semantics, it still seems to play a central role in *nonmonotonic* inference with rules →

Just as in classical logic, the truth-preserving rules of inference and the semantic constraints seem to be six of one and half a dozen of the other.

To make this point as clear as possible, I give an example concerning the relation of love and hate, a topic not very fashionable in logic. In a semantics of love and hate[12], the following semantic constraint could make love and hate mutually excluding:

Hate and love are mutually excluding
'P hates Q' and 'P loves Q' are not both true.[13]

(This is not to imply that the constraint holds in our world.) With respect to this semantic constraint (plus a classical interpretation of negation sentences 'It is not the case that P' as above), the following rule of inference is truth-preserving:

Modus odii
Premises:
P hates Q.
Conclusion:
It is not the case that P loves Q.

The example concerning love and hate attempts to show that unusual semantic constraints can make unusual rules of inference truth-preserving. Similarly, the unusual constraints on rules with exceptions make the unusual *Modus non excipiens* truth-preserving.

Let's now return to the remarks on logical form at the beginning of this section. It can be concluded that *in the logic of love and hate*, inferring 'It is not the case that P loves Q' from 'P hates Q' is determined by the logical form of the sentences involved, just as *in classical logic* inferring P from $P \wedge Q$ is.

Summarizing, we have seen three contexts with different notions of valid inference, viz. the contexts of strict rules, of rules with exceptions and of love and hate. It was shown how the semantic constraints determine which rules of inference are truth-preserving. Especially the latter example on love and hate may give the reader the uneasy feeling that there is no 'logicality' left if one considers valid inference in the rather non-logical context of love and hate. It should be borne in mind however that what kinds of valid inference deserve the name of *logical* valid inference, is not the concern of this paper.[14] The next section contains other examples of context-dependent valid inference, viz. classical and intuitionistic valid inference, the logicality of which is hardly subject to discussion.

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with exceptions. In my opinion, this role is as yet not fully satisfactorily clarified. See Prakken (1997) and my comments (Verheij, 1998b).

12 I already hinted at a semantics of love and hate in my dissertation (Verheij, 1996a, p. 22). The present exposition is taken from Verheij (1998a, 1998b).

13 In this case, P and Q do not express propositions, but refer to persons. It does not follow from the example on love and hate that I consider the relation of love and hate to be a matter of logic proper.

14 The reader is referred to the work of e.g. Gabbay (1994) and Sher (1991). Gabbay has developed the notion of a labelled deductive system in his search for the essence of logical systems. Sher proposes invariance under isomorphism as a key to logicality.

3 One logic as the background of another

In the previous section, we saw that it is possible to specify context-dependent valid inference in terms of a dedicated logic. Particular choices of semantic constraints and rules of inference resulted in different specifications of valid inference.
On the one hand this suggests that logics for context-dependent valid inference exist. However, a slight worry remains, since somehow it seems that there are contextual premises that 'underlie' a particular choice of context-dependent logic. For instance, the semantic constraint that in the logic of love and hate '*P* hates *Q*' and '*P* loves *Q*' are not both true in an interpretation, seems to be based on the context-specific premise that no one both hates and loves someone else[15]. In this section, this intuition is made precise.

An important initial step towards more precision is to note that the premise that no one both hates and loves someone else will again have to be interpreted. This requires again a logic! And indeed, it can make sense to consider one logic against the background of another logic. We recall the semantic constraint that hate and love are mutually excluding:

'*P* hates *Q*' and '*P* loves *Q*' are not both true.

This semantic constraint can be read as a sentence in a logic characterizing negation ('not') and conjunction ('both'). Using standard notation for negation and conjunction, the semantic constraint above can be translated to the following:

$\neg(P$ hates $Q \wedge P$ loves $Q)$ is true.

The sentence $\neg(P$ hates $Q \wedge P$ loves $Q)$ can now be read as a sentence of any logic with negation and conjunction. If such a logic is taken as the background, '*P* hates *Q*' and '*P* loves *Q*' are regarded as logical constants. As a result, in the background logic, the 'logical form' of the semantic constraint that hate and love are mutually excluding is reduced to $\neg(\varphi \wedge \psi)$. Within the background logic, valid inference is governed only by the semantic con straints and rules of inference of the background logic.

It should be noted that the translation of the semantic constraint that hate and love are mutually excluding, to a background logic with negation and conjunction is not a *reduction* of the logic of love and hate to the background logic. Instead language elements of the metalevel of the logic of love and hate, viz. 'not' and 'both', have been translated to the elements of the object level language of a background logic, viz. $\neg$ and $\wedge$. Instead of to a reduction, the translation leads to a further *explication* of the logic of love and hate. *Different* logics of negation and conjunction lead to *different* explications of the logic of love and hate. E.g., interpreting negation classically or intuitionistically leads to different relations between love and hate.[16]

15 Or oneself. Note that in an interpretation of '*P* hates *Q*' *P* and *Q* might refer to the same person.

16 I became aware of the possibility of different readings of a semantics because of different background logics by Troelstra and Van Dalen (1988, p. 78, 79), who speak of the classical and intuitionistic reading of an intuitionistic Kripke semantics. See also note 17.

It also occurs that one logic *can* be reduced to an axiomatic set of premises against the background of another logic. An example is the reduction of classical valid inference to a set of premises against the background of intuitionistic logic. The following holds, denoting classical inference as $\vDash_{cl}$ and intuitionistic inference as $\vDash_{int}$:

(*) S $\vDash_{cl}$ φ if and only if S, T $\vDash_{int}$ φ, where S is a set of sentences, φ is a sentence and T denotes the set of sentences of the form $\neg\neg P \rightarrow P$.

In other words, classical valid inference coincides with intuitionistic valid inference from an extended set of premises. The added set of sentences T, expressing the law of double negation that does not hold in intuitionistic valid inference, can be regarded as an axiomatic specification of classical valid inference against the background of intuitionistic valid inference.[17]

Let's now consider the question *whether there is a logic of classical valid inference*. Answering this question helps us in finding an answer to the question whether there is a logic of law, that is discussed in the next section. Most of us would probably answer the question concerning the existence of a logic of classical valid inference, with a loud and clear yes. Given the standard text-book logics, this is a sensible answer. Another sensible answer can however be given. An archetypal follower of Brouwer, believing that only intuitionistic inference is 'genuinely' valid, could deny the existence of a logic of classical valid inference, and state *that it is a matter of classical premises*, viz. those of the theory T above, against the background of intuitionistic valid inference. Since he could point to the property (*) above, his answer would also make sense. Paraphrasing Soeteman (see the introduction), it could be claimed that as a topic classical logic does not exist, and that its 'classicality' is merely in the classical premises that express the law of double negation.

Summarizing, if a logic L_1 can be reduced against the background of another logic L_2 in analogy with (*) above, it will depend on one's taste or purpose whether L_1 is conceived as an 'independent' logic, or whether L_1's notion of valid inference is regarded as being determined by a set of premises against the background of L_2's notion of inference. As the example of classical and intuitionistic logic shows, both points of view can make sense.

4 Can there be a logic of law?

Let me recapitulate the main points of the previous sections. First it has been attempted to show that valid inference is context-dependent. It was e.g. noted that the same logical language (considered as an uninterpreted set of sentences) can be the vehicle of different notions of valid inference. The logical language of section 2 with negation, rule and exception sentences was shown to allow notions of valid inference based on the interpretation of rules as strict rules and of rules with exceptions. In section 3, clas-

17 I do not know whether the topic of logics against background logics has been thoroughly investigated. The general situation is as follows. Assume two notions of valid inference $\vDash$ and $\vDash^*$ in the languages L and L*, respectively, and a translation of sentences of L into L*, mapping a sentence φ in L to φ^* in L*. Then $\vDash$ is reduced to an axiomatic set of premises T against the background $\vDash^*$ if it holds that S $\vDash$ φ if and only if S*, T* $\vDash^*$ φ^* (where S* is the image of S under the translation).

sical and intuitionistic valid inference (using the same standard logical language) were discussed.

Second it was argued that a notion of valid inference can be reducible to another in the sense that a more concrete notion of valid inference can be specified against the background of a more abstract notion in terms of a context-dependent axiomatic theory. As an example, it was noted that classical valid inference is reducible to intuitionistic valid inference, in terms of a theory that expresses the law of double negation.

The above can now be used to answer the question whether there can be a logic of law. The question can be answered positively and negatively, depending on one's point of view, just as the answer to the question concerning the existence of a logic of classical valid inference at the end of section 3.

The point of view in which the question is answered positively, i.e., *there can be a logic of law*, corresponds to the account of logics of context-dependent valid inference as it was exposited in section 2. I call this the *contextual logic* point of view. In this point of view, a logic of law can exist in the sense that it is possible to look for semantic constraints and/or rules of inference that are specific for the legal context.

In section 2, the example of a logic for rules with exceptions was discussed. Given the abundance in the law of rules that have exceptions, such a logic can already be regarded as a simple logic of law, in the sense that it is a specification of one typical aspect of legal reasoning. The most characteristic example of a logic of law in the sense discussed here is Reason-Based Logic. As is not surprising given the richness of legal reasoning, the logical language of such a logic that is explicitly dedicated to *legal* reasoning, is rich, which can lead to the use of a baroque set of logical constants.[18] In the example of Reason-Based Logic, a series of dedicated terms and predicates is available, expressing for instance rule validity, rule exclusion, rule applicability and application, and the weighing of reasons. A typical semantic constraint in Reason-Based Logic (taken from Verheij, 1996a, p. 36) is the following:

EXCLUSION
For all sentences *Fact* and *State_of_affairs*,
If *Fact* and Valid(rule(*condition*, *conclusion*)) are true, then either
Excluded(rule(*condition*, *conclusion*), *fact*, *state_of_affairs*)
or
Applicable(rule(*condition*, *conclusion*), *fact*, *state_of_affairs*)
is true.

Informally, the semantic constraint EXCLUSION states that a rule is either excluded or applicable if its condition is satisfied. Here *Fact* stands for the fact that satisfies the condition (expressed by the term *condition*) of the rule. For details, the reader is referred to extensive accounts of Reason-Based Logic, as given by Hage (1996, 1997) or Verheij (1996a). An exposition of Reason-Based Logic in terms of context-dependent rules of inference is for instance given by Verheij and Hage (1994).

18 Here I paraphrase Trevor Bench-Capon, who once spoke of Reason-Based Logic as a 'baroque' system of logic.

The point of view in which the question is answered negatively, i.e., *there cannot be a logic of law*, can be explained using the account of the reduction of one logic to another, as it was given in section 3. I call this the *abstract logic* point of view. Since legal valid inference is bound to be reducible to a more abstract notion of valid inference in terms of a set of legal premises, in the abstract logic point of view it can be argued that there is no logic of law.

I know of one presentation of Reason-Based Logic, that chooses this approach, viz. Verheij (1996b). There Reason-Based Logic is characterized in terms of an axiomatic set of premises against the background of standard first-order predicate logic. A typical axiom is the following:

$$\begin{aligned} &\mathrm{Valid}(\mathrm{rule}(\mathit{condition}, \mathit{conclusion})) \\ &\qquad \wedge\ \mathit{Condition} \\ &\qquad \wedge\ \neg\mathrm{Exception}(\mathrm{rule}(\mathit{condition}, \mathit{conclusion})) \\ &\rightarrow \mathrm{Reason}(\mathit{condition}, \mathit{conclusion}) \end{aligned}$$

The connectives $\wedge$, $\neg$ and $\rightarrow$ are interpreted in the background logic, viz. first-order predicate logic. The axiom expresses that, if the condition of a rule is satisfied and there is no exception to the rule, then the (state of affairs expressed by the) condition of the rule is a reason for the (state of affairs expressed by the) conclusion.

Also the work of Prakken and Sartor (1996) seems to take this second approach. The elements of their work can be roughly divided into two classes. The first class consists of the more abstract elements of, as they call it, a dialectical model of assessing conflicting arguments. The abstract notions of argument defeat and priority handling belong to this class. The second class consists of the more typically legally-oriented part. Applications of the abstract model to conflict handling using legal principles, such as Lex superior, and to applicability statements can be regarded as belonging to the second class.

Note that the contextual logic point of view and the abstract logic point of view, though each leads to a different answer to the question whether a logic of law can exist, are themselves *not* incompatible. An analogous point was made at the end of section 3 with respect to classical and intuitionistic inference.

I think that the different points of view do lead to a different practice of formalization. I distinguish two tasks that have to be undertaken if one wants to formalize legal valid inference. On the one hand there is the *empirical task*, in which a search for notions of inference that correspond to actual legal reasoning, is central. Topics such as reasoning with rules and principles, rule applicability, and the purpose of rules, are part of the empirical task. On the other hand there is the *technical task*, in which the solution of technical difficulties as they are encountered during the formalization of legal reasoning, is central. Topics such as the defeasibility and the dynamics of reasoning, and the dialogical characterization of valid inference, belong to the technical task.

In my opinion, the two tasks are given different priority dependent of one's point of view. In the *contextual logic* point of view, the empirical task will be given highest priority. Technical solutions are in the first place used as tools for the study of actual legal reasoning. Each resulting logic is regarded as a 'special-purpose logic', consisting of a set of formal patterns of reasoning that are valid in a legally relevant context. Making such formal patterns explicit is seen as the first-and-foremost task in the research on the logic of law. A logic is considered as a theory to be tested.

In the *abstract logic* point of view, the technical task will be given highest priority. The technical inadequacies of standard logics for the modeling of legal reasoning receive most attention. The search is for solutions of technical difficulties, while formalizing typically legal aspects of reasoning, such as those based on rule applicability and the purpose of rules, is considered as a separate task of knowledge representation, against the background of an abstract notion of valid inference.

Hage is probably the most typical representative of the contextual logic approach to the logic of law, and Prakken of the abstract logic approach.

5 Conclusions

The above investigation of the idea of a logic of law, in the sense of a formalization of legal valid inference, leads to the following conclusions.

It was shown that the idea of a context-dependent logic, such as a logic of law, makes sense. As examples, simple logics of strict rules and of rules with exceptions were discussed. It turned out that the contingent, context-dependent choices of semantic constraints and corresponding rules of inference, determine what counts as valid inference.

It was noted that a logic can serve as the background of another logic. The example of classical and intuitionistic inference showed that it can even be the case that one notion of valid inference is reduced to an axiomatic set of premises against the background of another inference notion. It was shown that as a result of such a reduction it becomes a matter of taste or purpose whether a particular notion of valid inference counts as a 'genuine' logic in itself or is regarded as a context-dependent set of axioms against the background of a more abstract inference notion.

The subtle conclusion is that on the one hand a logic of law can exist, and on the other hand it can be possible to reduce such a logic to a set of legal premises in a more abstract logic.

In an attempt to distinguish two styles of research in the logic of law, two points of view with regards to the formalization of legal valid inference were distinguished, viz. the *contextual logic* point of view and the *abstract logic* point of view. I suggested that from the contextual logic point of view the empirical task of formalizing legal reasoning is given first priority, while in the abstract logic point of view the technical task is first attended to.

Once the technical difficulties with regards to the formalization of legal reasoning have been solved satisfactorily, the two styles will merge. Only after that it will become apparent whether a canonical logic of law is achievable.

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