On the Existence of Semi-Stable Extensions

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Abstract

In this paper, we describe an open problem in abstract argumentation theory: the precise conditions under which semi-stable extensions exist. Although each finite argumentation framework can be shown to have at least one semi-stable extension, this is no longer the case when infinite argumentation frameworks are considered. This puts semi-stable semantics between stable and preferred semantics. Where stable semantics does not warrant the existence of extensions (even for finite argumentation frameworks) and preferred semantics always warrants the existence of extensions (even for infinite argumentation frameworks), semi-stable semantics warrants the existence of extensions only for finite argumentation frameworks, but not for infinite argumentation frameworks. We illustrate this with a counter-example of the latter. The question is then studied if, even for infinite argumentation frameworks, one can identify specific conditions under which semi-stable extensions do exist.

1 Introduction

Much of the recent work regarding the formal study of argumentation has its origin in Dung's 1995 landmark paper [7]. In this work, the focus is on the mathematical properties of one aspect of argumentation, namely the attack relation between arguments. Dung's analysis of the attack relation uses sets as a central tool. He proposed four kinds of extensions of an argumentation framework: stable, preferred, grounded and complete extensions. Verheij continued the analysis using labellings [13]. He defined labelling analogues of stable and preferred extensions, and added two new kinds of extensions, arising naturally in the setting of labellings: stage extensions and admissible stage extensions. Instead of maximizing the set of arguments, the set of labeled arguments was maximized. In a sense, this meant that the set of arguments taken into account was maximized (whether attacked or not), instead of just the set of unattacked arguments. Verheij continued the labelling analysis of argumentation [16], but in a more expressive setting, namely one in which both support and attack can be analyzed. Recently, Caminada has resumed the analysis of argumentation frameworks in terms of labellings [1, 5]. In Caminada's work, Verheij's admissible stage extensions [13] occur by the elegant name of semi-stable extensions [2]. Although the work of Caminada has been done independent of that of Verheij, both discovered essentially the same concept in their respective formalizations of abstract argumentation semantics. In the current paper, we will use the term *semi-stable extension* (or *semi-stable labelling*) instead of Verheij's original term *admissible stage extension*.

Semi-stable semantics can be located between stable semantics and preferred semantics, in the sense that every stable extension (labelling) is also a semi-stable extension (labelling), and that every semi-stable extension (labelling) is also a preferred extension (labelling) [13, 2]. Moreover, if an argumentation framework has at least one stable extension (labelling) then all of its semi-stable extensions (labellings) are also stable extensions (labellings) [13, 2].

Over the recent years, research on semi-stable semantics has produced complexity analysises [9] as well as an algorithm that computes all semi-stable extensions (labellings) given an argumentation framework [3, 4].[1] In the current paper, we discuss a property that so far has not received any attention: the existence (and possible non-existence) of semi-stable extensions (labellings) of a given argumentation framework. Although semi-stable extensions (labellings) do exist for every finite argumentation framework, we will see that they do not always exist for every infinite argumentation framework.

[1] Slightly modified versions of the algorithm can also be used to computer all preferred or stable extensions (labellings) of a given argumentation framework[3, 4].

The current paper is structured as follows. First, in Section 2 we provide some formal preliminaries on argumentation semantics, and in particular on semi-stable semantics. Then, in Section 3 we will provide an example of an (infinite) argumentation framework without any semi-stable extension (labelling) state some conditions under which semi-stable extensions do exist, even for infinite argumentation frameworks. We will round off in Section 4 with a brief discussion and a description of an open research issue.

2 Formal Preliminaries

In the current section, we state some basic notions of abstract argumentation theory.

Definition 1. *Let U be the universe of all possible arguments. An* argumentation framework *is a pair (Ar, att) where Ar is a subset of U and $att \subseteq Ar \times Ar$.*

We say that an argument A *attacks* an argument B iff $(A, B) \in att$.

An argumentation framework can be depicted as a directed graph in which the arguments are represented as nodes and the attack relation is represented as arrows. For instance, argumentation framework (Ar, att) where $Ar = \{A, B, C, D, E\}$ and $att = \{(A, B), (B, A), (B, C), (C, D), (D, E), (E, C)\}$ is represented in Figure 1.

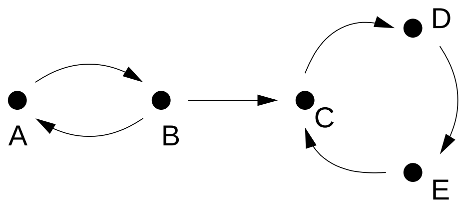

Figure 1: An argumentation framework represented as a directed graph.

The shorthand notation A^+ and A^- stands for, respectively, the set of arguments attacked by argument A and the set of arguments that attack argument A. Likewise, if $\mathcal{A}rgs$ is a set of arguments, then we write $\mathcal{A}rgs^+$ for the set of arguments that are attacked by at least one argument in $\mathcal{A}rgs$, and $\mathcal{A}rgs^-$ for the set of arguments that attack at least one argument in $\mathcal{A}rgs$. In the definition below, $F(\mathcal{A}rgs)$ stands for the set of arguments that are acceptable in the sense of [7].

Definition 2 (defense / conflict-free)**.** *Let (Ar, att) be an argumentation framework, $A \in Ar$ and $\mathcal{A}rgs \subseteq Ar$.*
We define A^+ as $\{B \mid A\ att\ B\}$ and $\mathcal{A}rgs^+$ as $\{B \mid A\ att\ B$ for some $A \in \mathcal{A}rgs\}$.
We define A^- as $\{B \mid B\ att\ A\}$ and $\mathcal{A}rgs^-$ as $\{B \mid B\ att\ A$ for some $A \in \mathcal{A}rgs\}$.
$\mathcal{A}rgs$ is conflict-free *iff $\mathcal{A}rgs \cap \mathcal{A}rgs^+ = \emptyset$.*
$\mathcal{A}rgs$ defends *an argument A iff $A^- \subseteq \mathcal{A}rgs^+$.*
We define the function $F : 2^{Ar} \rightarrow 2^{Ar}$ as
$F(\mathcal{A}rgs) = \{A \mid A$ is defended by $\mathcal{A}rgs\}$.
$\mathcal{A}rgs$ is admissible *iff it is conflict-free and $\mathcal{A}rgs \subseteq F(\mathcal{A}rgs)$.*

When $\mathcal{A}rgs$ is a set of arguments, we refer to $\mathcal{A}rgs \cup \mathcal{A}rgs^+$ as the *range* or $\mathcal{A}rgs$, a term that was first introduced in [13]. Using the concept of admissibility, it then becomes possible to define preferred, stable and semi-stable semantics. The definitions of stable and semi-stable extensions below are not literally the same as in [7] and [2] but can be proved to be equivalent. Our aim is to formulate these notions in such a way to make clear the connection between the extensions-based (Definition 3) and the labelling-based (Definition 5) characterisations of argumentation semantics.

Definition 3 (acceptability semantics)**.** *Let (Ar, att) be an argumentation framework and let $\mathcal{A}rgs \subseteq Ar$ be an admissible set of arguments.*

- *$\mathcal{A}rgs$ is a* preferred *extension iff $\mathcal{A}rgs$ is a maximal (w.r.t. set-inclusion) admissible set.*

- *$\mathcal{A}rgs$ is a* stable *extension iff $\mathcal{A}rgs$ is an admissible set where $\mathcal{A}rgs \cup \mathcal{A}rgs^+ = Ar$.*

- $\mathcal{A}rgs$ *is a* semi-stable *extension iff* $\mathcal{A}rgs$ *is an admissible set where* $\mathcal{A}rgs \cup \mathcal{A}rgs^{+}$ *is maximal (w.r.t. set-inclusion) among all admissible sets.*

As an example, in the argumentation framework of Figure 1 $\{B, D\}$ is a stable (and semi-stable) extension, $\{A\}$ is a preferred extension which is neither stable nor semi-stable, and $\{B\}$ is an admissible set which is not a preferred extension (and also not a stable or semi-stable extension).

The connection between stable, semi-stable and preferred extensions can be stated as follows.

Proposition 1 ([2]). *Let* $AF = (Ar, att)$ *be an argumentation framework.*

1. *Every stable extension of* AF *is also a semi-stable extension of* AF*.*
2. *Every semi-stable extension of* AF *is also a preferred extension of* AF*.*
3. *If* AF *has at least one stable extension, then every semi-stable extension of* AF *is also a stable extension of* AF*.*

The concept of admissibility, as well as that of preferred, stable or semi-stable semantics were originally stated in terms of sets of arguments. It is equally well possible, however, to express these concepts using argument labellings. This approach was pioneered by Pollock [12] has subsequently been applied by Jakobovits and Vermeir [10], Caminada [1, 3], Vreeswijk [18] and Verheij [13, 17]. In the current paper we follow the approach of Caminada [1, 5], where the idea of a labelling is to associate with each argument exactly one label, which can either be `in`, `out` or `undec`. The label `in` indicates that the argument is explicitly accepted, the label `out` indicates that the argument is explicitly rejected, and the label `undec` indicates that the status of the argument is undecided, meaning that one abstains from an explicit judgment whether the argument is `in` or `out`.

Definition 4 ([5]). *Let* (Ar, att) *be an argumentation framework. A* labelling *is a total function* $\mathcal{L} : Ar \longrightarrow \{\mathtt{in}, \mathtt{out}, \mathtt{undec}\}$*. A labelling is called* admissible *iff for every* $A \in Ar$ *it holds that:*

1. *if* A *is labelled* `in` *then all attackers of* A *are labelled* `out`
2. *if* A *is labelled* `out` *then* A *has a attacker that is labelled* `in`*, and*

We write $\mathtt{in}(\mathcal{L})$ for $\{A \mid \mathcal{L}(A) = \mathtt{in}\}$, $\mathtt{out}(\mathcal{L})$ for $\{A \mid \mathcal{L}(A) = \mathtt{out}\}$ and $\mathtt{undec}(\mathcal{L})$ for $\{A \mid \mathcal{L}(A) = \mathtt{undec}\}$. Sometimes, we write a labelling $\mathcal{L}$ as a triple $(\mathcal{A}rgs_1, \mathcal{A}rgs_2, \mathcal{A}rgs_3)$ where $\mathcal{A}rgs_1 = \mathtt{in}(\mathcal{L})$, $\mathcal{A}rgs_2 = \mathtt{out}(\mathcal{L})$ and $\mathcal{A}rgs_3 = \mathtt{undec}(\mathcal{L})$.

Using the concept of an admissible labelling, it becomes possible to define the notions of preferred, stable and semi-stable labellings.[2]

Definition 5. *Let* $\mathcal{L}$ *be an admissible labelling of argumentation framework* $AF = (Ar, att)$*.*

- *We say that* $\mathcal{L}$ *is a* preferred labelling *iff* $\mathtt{in}(\mathcal{L})$ *is maximal (w.r.t. set inclusion) among all admissible labellings.*
- *We say that* $\mathcal{L}$ *is a* stable labelling *iff* $\mathtt{undec}(\mathcal{L}) = \emptyset$*.*
- *We say that* $\mathcal{L}$ *is a* semi-stable labelling *iff* $\mathtt{undec}(\mathcal{L})$ *is minimal (w.r.t. set inclusion) among all admissible labellings.*

The connection between stable, semi-stable and preferred labellings are similar as for the stable, semi-stable and preferred extensions.

Proposition 2. *Let* $AF = (Ar, att)$ *be an argumentation framework.*

1. *Every stable labelling of* AF *is also a semi-stable labelling of* AF*.*
2. *Every semi-stable labelling of* AF *is also a preferred labelling of* AF*.*
3. *If* AF *has at least one stable labelling, then every semi-stable labelling of* AF *is also a stable labelling of* AF*.*

[2] In [1] these were defined using the concept of complete labellings. However, our current formalization based on admissible labellings can be shown to be equivalent.

For preferred, stable and semi-stable semantics, extensions and labellings stand in a one-to-one relation to each other. In essence, in order to convert a labelling to an extension, one simply takes the set of `in`-labelled arguments. Similarly, in order to convert an extension to a labelling, one labels all arguments in the extension `in`, all arguments attacked by the extension `out` and all other arguments `undec`. More details can be found in [5].

3 On the Existence of Semi-Stable Extensions

Although various technical issues regarding semi-stable semantics (like computational complexity [9] and algorithms [3]) have been treated in the literature, there is one particular question that is still to be answered in any reasonable detail: can we guarantee the existence of semi-stable extensions for any argumentation framework? Although we understand that this question might at first appear odd to the reader, it will be explained that answering it is definitely not a trivial task. In fact, the reader might be surprised to learn that for a wide variety of cases, one cannot provide an a priori answer to this question.

To properly understand the nature of the problem, it can be interesting to look at it from the perspective of preferred semantics. Recall that preferred extension can be defined as a maximal admissible sets [7],[3] while semi-stable extension can be defined as admissible sets with a maximal range.[4] When being asked why there always exists a preferred extension, many scholars reply by stating that the empty set is admissible and that one can always keep on adding arguments to it until one has reached a preferred extension. This is stated in remarks like "Every argumentation framework possesses at least one preferred extension (the empty set is always an admissible set)" [6] and "(...) it is always the case that a preferred extension exists since the empty set is always admissible" [8]. While we agree that each argumentation framework has at least a *minimal* admissible set (the empty set), this still does not answer the question of whether each argumentation framework also has a *maximal* admissible set (a preferred extension).

Of course, an easy and straightforward way of ensuring the existence of a preferred extension would be to take into account only argumentation frameworks with a finite set of arguments. If there are only finitely many arguments, then there are also finitely many admissible sets. It then trivially follows that there exists some maximal admissible set. This is for instance the approach taken in [8].

A similar observation holds with respect to the existence of semi-stable extensions, as long as one restricts oneself to finite argumentation frameworks. For finite argumentation frameworks, one can always identify an admissible set $\mathcal{A}rgs$ where $\mathcal{A}rgs \cup \mathcal{A}rgs^+$ is maximal, thus warranting the existence of a semi-stable extension.

When one also allows for argumentation frameworks with an infinite set of arguments, the situation becomes more complex. It should be mentioned that the idea of having an infinite number of arguments is not too far-fetched. When one, for instance, defines an argumentation framework using classical logic (such as [11, 12]) then from the fact that there are infinitely many classical tautologies, it follows that one can construct infinitely many arguments.

Suppose there are infinitely many arguments. Is there then still always a preferred or semi-stable extension? For semi-stable semantics, this question should, unfortunately, be answered negatively. Take the example (taken from [14, 16]) of an argumentation framework where there are infinitely many A-arguments ($A_1, A_2, A_3, \ldots$), infinitely many B-arguments ($B_1, B_2, B_3, \ldots$) and infinitely many C-arguments ($C_1, C_2, C_3, \ldots$). Let each A_i attack itself. Let each B_i attack each A_j with $j \leq i$ as well as each B_k with $k < i$. Furthermore, let each B_i and C_i attack each other. This situation is depicted in Figure 2.

Perhaps the best way of explaining why in this case no semi-stable extension exists is by examining the preferred labellings (recall that every preferred labelling corresponds to a preferred extension). In this case, there exist an infinite sequence of preferred labellings, of which we only provide the first three:

1. each C_i is `in`, each B_i is `out` and each A_i is `undec`
2. C_1 is `out`, all the other C_is are `in`, B_1 is `in`, all the other B_is are `out`, A_1 is `out`, all A_j with $j > 1$ are `undec`.
3. C_2 is `out`, all the other C_is are `in`, B_2 is `in`, all the other B_is are `out`, A_1 and A_2 are `out`, all A_j with $j > 2$ are `undec`.

[3] We write "can be" because it would be equally possible to define a preferred extension as a maximal complete extension [7].

[4] We write "can be" because it would be equally possible to define a semi-stable extension as a complete extension with maximal range [2].

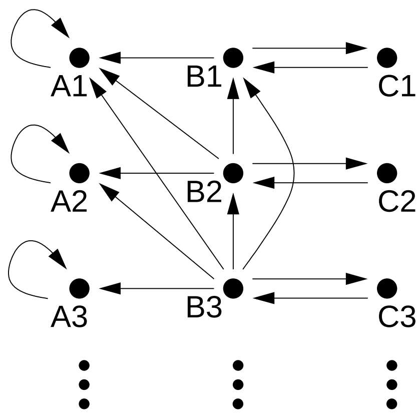

Figure 2: Sometimes, there does not exist a semi-stable extension.

The situation here is as follows. There can be at most one B-argument that is labeled `in` (otherwise one loses conflict-freeness and violates point 1 of Definition 4). Let us assume that B_i is labelled `in` (for some $i \geq 1$). Then, if one wants to minimize `undec`, one should label C_i `out` and all other C_j (with $j \neq i$) `in`. All A_k with $k \leq i$ then become `out` (this is because B_i is `in`) whereas all A_k with $k > i$ remain `undec`.

For instance, if one chooses B_1 to be `in`, then the set of `undec` labelled arguments becomes $\{A_2, A_3, A_4, A_5, \ldots\}$. If one chooses B_2 to be `in` then the set of `undec` labelled arguments becomes $\{A_3, A_4, A_5, \ldots\}$. If one chooses B_3 to be `in` then the set of `undec` labelled arguments becomes $\{A_4, A_5, \ldots\}$, etc. Thus, the larger we choose the i in B_i, the less arguments get labelled `undec`. Nevertheless, we never end up with a *minimal* set of `undec` labelled arguments, since one can always obtain a set that is smaller. There is no admissible set $\mathcal{A}rgs$ where $\mathcal{A}rgs \cup \mathcal{A}rgs^+$ is maximal. Therefore, there exists no semi-stable extension in Figure 2.

As an aside, one may ask the same question regarding preferred semantics. Is it perhaps possible that one can invent an example where the admissible sets keep on increasing, such that there is no admissible set $\mathcal{A}rgs$ where $\mathcal{A}rgs$ is maximal? Suppose there exists an infinite sequence of increasing admissible sets $\mathcal{A}rgs_1, \mathcal{A}rgs_2, \mathcal{A}rgs_3, \ldots$ How can one guarantee the existence of a global maximum?

The first step towards dealing with this is to observe that the union of $\mathcal{A}rgs_1, \mathcal{A}rgs_2, \mathcal{A}rgs_3, \ldots$ where $\mathcal{A}rgs_i$ ($i \geq 1$) keeps getting bigger is again an admissible set. This is not difficult too see, as the union is conflict-free (otherwise at least one $\mathcal{A}rgs_i$ ($i \geq 1$) would not be conflict-free) and defends all its elements (otherwise at least one $\mathcal{A}rgs_i$ ($i \geq 1$) would not defend all its arguments). Nevertheless, this is still not enough to warrant the existence of a global maximum. What if the union (say $\mathcal{A}rgs'$) is in itself again the starting point of an ever increasing sequence of admissible sets?

The key to the existence of preferred extensions is to be found in Zorn's Lemma, which can be stated as follows: "Every non-empty partially ordered set (S) of which every totally ordered subset (T) has an upper bound contains at least one maximal element". Let S be the set of all admissible sets, where the admissible sets are ordered according to the subset relation. As every totally ordered subset T (that is: every sequence of increasing admissible sets) has an upper bound (that is: its union), one can apply Zorn's Lemma and obtain the existence of at least one maximal element (a preferred extension). Although not explicitly mentioned in [7], this is in fact the reason why there always exists a preferred extension.

As for semi-stable semantics, one cannot perform the same trick. The point is that the union of a sequence of admissible sets $\mathcal{A}rgs_1, \mathcal{A}rgs_2, \mathcal{A}rgs_3, \ldots$ where $\mathcal{A}rgs_i \cup \mathcal{A}rgs_i^+$ ($i \geq 1$) keeps getting bigger might not be an admissible set itself. Again, an example can be found in Figure 2, where this union is not conflict-free (since it contains more than one B-argument). Thus, we cannot apply Zorn's Lemma for semi-stable semantics.

To summarize: the existence of extensions is not as straightforward as it may appear at a first sight. For stable semantics, the situation is clear: there may not be stable extensions regardless of whether there are finitely or infinitely many arguments. For preferred semantics, the situation is quite the opposite: there is always at least one preferred extension, again regardless of whether there exists finitely or infinitely many arguments. For semi-stable semantics, however, the situation is somewhere in between: extensions are guaranteed to exist for finite argumentation frameworks, but not for infinite argumentation frameworks. So also here, it can be seen that semi-stable semantics has a position between stable semantics and preferred semantics.

Overall, some properties with respect to the existence of semi-stable extensions can be identified as follows.

1. There exist (infinite) argumentation frameworks without semi-stable extensions (labellings).
2. Every finite argumentation framework has at least one semi-stable extension (labelling).
3. Every argumentation framework with a finite number of preferred extensions (labellings) has at least one semi-stable extension (labelling).
4. Every argumentation framework with at least one stable extension (labelling) has at least one semi-stable extension (labelling).

Point 4 follows from Proposition 1 and Proposition 2. Points 2 and 3 are actually special cases of the following theorem.

Theorem 1. *If an argumentation framework (Ar, att) does not have an infinite sequence of preferred extensions with strictly increasing ranges (or equivalently, does not have an infinite sequence of preferred labellings with strictly decreasing sets of* `undec`*-labelled arguments) then it has at least one semi-stable extension (labelling).*

Proof. We prove this by modus tollens. Suppose (Ar, att) has no semi-stable extension. Now pick an arbitrary preferred extension (say P_1). It is not semi-stable, so there exists an admissible set A_2 with a larger range (that is, the range of A_2 is a proper superset of the range of P_1). Let P_2 be a preferred extension that is a superset of A_2 (from [7] it follows that such a preferred extension always exists). From the fact that P_2 has a larger (or equal) range than A_2, together with the fact that A_2 has a larger range than P_1, it follows that P_2 has a larger range than P_1. P_2 is not semi-stable either, so using the same reasoning there exists a preferred extension P_3 with a larger range. Repeating this process gives (by induction) an infinite sequence of preferred extensions with strictly increasing ranges. □

The validity of point 3 above follows directly from Theorem 1. The validity of point 2 above follows from the fact that it is a special case of point 3 above.

4 Discussion

Although Theorem 1 does provide a guideline regarding the existence of semi-stable extensions (labellings) it does so by examining its preferred extensions (labellings). An interesting question is whether one can also warrant the existence of semi-stable extensions based on the topological properties of the argumentation framework. One possible candidate would be to consider only *finitary* argumentation frameworks, as defined in [7]. Recall that in a finitary argumentation framework each argument has a finite number of atackers. It is not too difficult to see that the argumentation framework of Figure 2 is not finitary. This is because each A_i ($i \geq 1$) has an infinite number of atackers (each B_j with $j \geq i$) and each B_i has an infinite number of atackers (each B_j with $j > i$).

The fact that there exists no semi-stable extension for the argumentation framework of Figure 2 is closely related to the fact that it is not finitary. In fact, we have been unable to construct an example of a finitary argumentation framework that still does not have any semi-stable extensions. Still, this does not mean that there exists an easy and straightforward proof of the existence of semi-stable extensions for finitary argumentation frameworks. It appears that such a proof would have to use Zorn's Lemma, and it is not obvious how such should be done while making use of the specific properties of finitary argumentation frameworks. The following conjecture should therefore be seen as an open research issue in abstract argumentation.

Conjecture 1. *Every finitary argumentation framework has at least one semi-stable extension (labelling).*

In our view, the above conjecture is currently one of the main technical open issues in the theory of abstract argumentation.[5]

[5]We encourage people who are interested to work on this to contact us, in order to prevent double work from being done.

Epilogue: Historic Context and Terminology

The issue of the existence of semi-stable extensions was first examined in [14, 16]). The argumentation framework associated to example 5.8 of [16][p. 338][6] has no semi-stable extension.[7] The result is obtained using the DefLog language, a straightforward generalization of Dung's attack graphs. DefLog[8] is a logical language in which attack is interpreted as a kind of conditional relation. The language adds support, nested conditionals and — what might be called — negation-as-defeat[9] to the expressiveness of Dung's attack graphs. Analogues of Dung's stable and preferred extensions are defined, and shown to be faithful generalizations (in the sense that translating an attack graph into DefLog does not affect its stable and preferred extensions). Next to the semi-stable semantics, Verheij [13, 16] adds a second kind of semantics that is new with respect to Dung's definitions, namely the stage semantics. A stage extension is a conflict-free set of arguments, with maximal range [13].[10] For the sake of completeness of the analysis, Verheij [16] adds maximal conflict-free sets to the comparative analysis (using the term "compatibility class"). Table 1 contains an overview of the different uses of terminology.

Dung [7]	Verheij [13]	Verheij [14, 16]	Caminada [2]	Encompassing proposal
stable extension	complete stage extension	extension, dialectical interpretation	stable extension	stable extension
preferred extension	preferred stage	dialectically preferred stage	preferred extension	preferred extension
grounded extension	-	-	grounded extension	grounded extension
complete extension	-	-	complete extension	complete extension
-	admissible stage extensions	maximal dialectically preferred stages	semi-stable extension	semi-stable extension
-	stage extension	maximal stage	-	stage extension
-	-	compatibility class (in [14]: satisfiability class)	-	conflict-free extension

Table 1: Comparison of terminology.

As an aside, the example of Figure 2 is also a counterexample against the existence of stage extensions for infinite argumentation frameworks. Identifying topological properties that warrant the existence of extensions (labellings) is therefore not only an issue for semi-stable semantics, but for stage semantics as well, and to some extent even for stable semantics.

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[6] Example 7.12 in [14].

[7] Somewhat confusingly, Verheij [16] refers to semi-stable extensions using a different term than the 1996 term "admissible stage extensions". In [16], semi-stable extensions are called "maximal dialectically preferred stages."

[8] The basis of [16] is a technical report [14] containing extensive additional material. The first peer reviewed publication on DefLog is [15].

[9] Called "dialectical negation" in [16].

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