

A quantitative approach to proportionality

Giovanni Sartor¹

¹Affiliation not available

April 2, 2018

Abstract

This chapter is meant to address the extent to which value-based reasoning –as involved in balancing and proportionality— may include quantitative reasoning, according to arithmetic constraints. Relying on some work on cognitive and evolutionary psychology it is argued that processing non-symbolic approximate magnitudes is a fundamental cognitive capacity, which is deployed also when we are reasoning with goals and values. A model is developed for determining the impact of alternative choice on multiple values, assessing the utilities so produced and merging these utilities into an overall evaluation, which may be used in comparisons. The model is applied to assessing legislative choice on rights and social values, according to proportionality. Finally, it is shown how proportionality assessments may be constrained by the requirement of consistency with precedents.

1 Introduction

The present contribution discusses the extent to which value-based reasoning, as deployed in proportionality arguments, may include quantitative reasoning and constraints.¹ It will therefore address from a different angle the proportionality debate, which has been examined from a legal and philosophical perspectives in Chapter 5 of part III, by Giorgio Bongiovanni and Chiara Valentini, and link proportionality to teleological reasoning, as addressed in Chapter 5 of part II by Lewis Kornhauser. Assessing the merit of alternative choices relatively to values involves considering the magnitudes that quantify the impacts of such choices on the realisation of these values, and the weights of such values. Even though this reasoning does not use numerical symbols, it still deals with quantities (it subtracts, multiplies divides, etc., such quantities), and is therefore subject to the basic laws of arithmetic. In fact, according to research on cognitive and evolutionary psychology, processing non-symbolic approximate magnitudes is a fundamental cognitive capacity, which seems to be deployed also when we are reasoning with values. This capacity needs to be integrated with logic and argumentation to provide a comprehensive account of value-based reasoning.

On the basis of this assumption, a conceptual framework is developed for reasoning with values. Ways to determine the impacts of a choice on single values are presented, and ways to determine the associated utilities and merge these utilities into a single measure of the merit of that choice are introduced. Some issues pertaining to the comparison of alternative choices are addressed.

It is shown how the standards of proportionality assessments —suitability, necessity, and proportionality in a strict sense— fit in the proposed framework.

Finally, it is considered how value-based assessments may be constrained by the requirement of consistency with precedent assessments of the same kind.

This framework is applied to examples of legal reasoning in judicial decision-making.

¹On the approach presented in this chapter see [Sartor \(2010, 2013\)](#).

2 Quantitative reasoning without (symbolically expressed) numbers

When we are to assess whether a decision α duly realises some values, we need to compare the extent to which these values are implemented by α and the extent they would be implemented if a different choice β were made instead of α (where β may consist in not interfering with the status quo, or in changing it in a different way). On this basis, as we shall see, we determine the differential merit of making choice α .

This raises the issue of how we are going to determine the impact of a choice on all values at stake and aggregate such impacts into a determination of the overall benefit or loss that is provided by that choice, as compared with different possible choices. If we could obtain appropriate numbers,² it seems that some mathematics should provide the answer on the merit of our choices. For this purpose, we should need numbers expressing the different impacts of our choices (in all possible scenarios) on the implementation of the values at stake and functions determining, for each such impact, the corresponding gain or loss in the overall benefit being delivered. However, in most legal cases (at least when constitutional adjudication is at issue), we do not have sensible ways for assigning numbers and constructing the corresponding functions. Nor have we an exhaustive set of preferences between all possible combinations of the different impacts on values, which may be represented as a utility function, in accordance with the so-called representation theorems used in economics.³ This makes quantitative methods used in decision theory and cost-benefit analysis not directly applicable to many legal contexts, and in particular, to constitutional decisions involving impacts on different values.⁴

One explanation of our ability to assess the impacts of our choices on the relevant values—though we cannot sensibly not express these impacts through numbers—is that people possess some, inborn or acquired, capacity to reason with non-numerical quantities.⁵ In fact, experiments have shown that we can make computations with quantities without associating numerical symbols to such quantities (see [Gallistel and Gelman, 2005](#)). This capacity is not limited to ordinal comparisons, namely, to assessing whether a certain object is more or less than another (with regard to dimensions such as length, volume, weight, speed, etc.). It also covers cardinal measures: even without numbers we are able to assess, though in a very approximate way, the size (the cardinal measure) of an object or the extent of its difference from another. To express such non-numerical cardinal evaluations, we often refine our ordinal assessment with adverbs. For instance, we may say that this object is a little, fairly, a lot larger, or smaller, or quicker, than that object. We can sometimes map such approximate cardinal evaluations into quantitative proportions and relations without referring to a general unit of measure and without engaging in explicit numerical computations. For instance, we may say that an entity is about a half, two times, three times larger, or smaller, or quicker than another. Thus, not only can we compare two lines and establish which one is longer, but we can say that one line is twice longer than the other, or that a line is the sum of the two lines of different sizes, without making numerical calculations.

Apparently, this kind of mathematical competence is quite widespread in the animal kingdom. Animals not only are able to order objects according to their size, but they can also perform tasks that involve processing magnitudes: they compute distances by summing up the extent of successive displacements, they make visits

²I use the term “number” to refer only to the cases where a quantity is expressed with the symbols (the numerals) or a particular number system. When a quantity is represented (e.g. graphically, or mentally) without the use of such symbols, I use the term “magnitude”.

³According to the so-called Morgenstern-Von Neumann representation theorem, if we have a set of preferences among alternatives, and these preferences are complete, transitive, independent and continuous, then we can build a utility function assigning a numerical utility to each alternative, in such a way that any alternative strictly preferred to another would have a higher utility than the latter.

⁴This does not exclude that the methods of decision theory and cost-benefit analysis can be usefully deployed in many cases; for a technical account of multi-criteria decision-making, see [Keeney and Raiffa \(1993\)](#).

⁵This assumption is not meant to exclude that other ways of reasoning may also be significant for this purpose, such as the capacity of making analogies out of cases, or of building arguments. We rather integrate these different skills in complex value assessment.

to caches according to the differences between the time when the food was stored and its expected rotting time, they remain in different locations according to ratios between time spent and rewards obtained, etc.

Research with vertebrates, some of which have not shared a common ancestor with man since before the rise of the dinosaurs, implies that they represent both countable and uncountable quantity by means of mental magnitudes [...] The system of arithmetical reasoning with these mental magnitudes is closed under the basic operations of arithmetic, that is, mental magnitudes may be mentally added, subtracted, multiplied, and divided without restriction. ((Gallistel et al., 2006, 259)Gallistel)

Thus, it seems that there exists an inborn ability to represent and mathematically process mental magnitudes, which is deployed without translating these magnitudes into the linguistic symbols (the numerals) of a number system. Contrary to a famous statement by the mathematician Leopold Kronecker (“God made the integers; all else is the work of man”) it seems nature has endowed us, and other animals, with the primitive ability to store and process continuous (though approximated, or noisy) mental magnitudes,⁶ quantities that are only mappable into real numbers (since they include also negative magnitudes, fractions, and even irrational magnitudes, such square roots). On the top of this ability, humans have the additional possibility of using symbols for expressing such quantities and making them more precise. Our mind, however, continues to map numerical values into analogical magnitudes, as we do when making quick, unreflected, judgments. We may possibly say, using the terminology of Kahneman (2011), that reasoning with analogical magnitudes pertains to our fast (parallel, intuitive and apparently effortless) thinking, while the corresponding numerical processes pertain to our slow (sequential, reflective and demanding) thinking.

According to John Pollock (2006, Ch. 3) this capacity for intuitive cardinal assessment of quantities, which he calls “analogical quantitative cognition”, applies not only to lengths, weights or volumes, but also to our likes and dislikes, and to the realisation of our values.

I shall accept the assumption that humans have a basic (and largely inborn, though improvable by training and experience) intuitive capacity for non-symbolic quantitative reasoning, a capacity that includes not only assessing and comparing magnitudes, but also performing on such magnitudes approximate mathematical operations: sums, subtractions, proportions, multiplications and divisions (and even approximate differentiation and integration). I shall argue that exactly this capacity is involved in assessing impacts on values according to proportionality. We can deploy it in choices concerning our private life (choosing a car or a computer by balancing design, performance, and cost; choosing a restaurant by considering quality of food, service and price, choosing a course of studies balancing interest and work-opportunities, etc.) but also when public choices have to be taken or assessed. For engaging in this kind of intuitive, or “analogical” quantitative reasoning, we do not need to translate quantities into numbers through measurement (which is an ability that only humans possess, and in many domains only after adequate schooling): we just rely on our intuitive appreciation of the quantities involved and of their relations. When more precision is needed, and numerical quantification makes sense, we may move to symbolically expressed numbers, to test and refine our intuitions.

The nature of this mathematical capacity entails that mathematical relationships do not hold only among symbolically expressed numbers: they also constrain the process of our intuitive-analogical quantitative reasoning. Thus, such relationships can be used as a standard of rationality for that reasoning, and for facilitating the transition to numerical quantification, when possible and convenient. Finally, note that the assumption that we can reason with approximate quantities does not entail that we can precisely assess such quantities, nor that we can always determine with certainty whether one object’s magnitude is bigger than

⁶Thus, apparently, these findings of contemporary cognitive science seem to validate Leibniz’s principle of continuity (often expressed by the saying *natura non facit saltus*), at least with regard to the mental processing of quantitative information: “I also take it for granted that every created being is subject to change [...] and even that this change is continuous in each.” (Rescher, 1991, Section 10). There are various methods for dealing with approximate quantities, but here I cannot even attempt at discussing them, and moreover the general account here provided is meant to be neutral in this regard as much as possible. For a review of methods for reasoning for uncertainty, see for instance, Parsons (2001).

another's. For instance, we may sometimes (though not in most cases) remain uncertain when comparing the lengths of two twisted lines, or the volumes of two solid objects. Similarly, we may sometimes (though not in most cases) remain in doubt concerning the impacts of our choices on our values, and the comparative merits of such choices.

In the following section, I shall examine teleological reasoning in law as an instance of non-numerical quantitative reasoning and I shall derive some implications of this idea.

3 Basic concepts

I shall specify certain notions that are needed in order to proceed in the analysis. First of all, I assume that we can quantify the level of the realisation of a value in a particular situation (where a situation is an actual or possible set of circumstances, including social and institutional arrangements).

Definition 1 (Realisation-quantity of a value) The realisation-quantity of a value v in a particular situation is the extent up to which v is realised in case that situation obtains. Let us write $\text{Real}_v(s)$ to denote the realisation-quantity of value v in situation s . We correspondingly denote as $\text{Real}_v(s_c)$, or simply Real_v , the level of realisation of v in the current situation s_c (the present state of affairs). Thus $\text{Real}_v = q$ means that in the current situation the value v is realised in quantity q .

We can express our assessment of the realisation-quantity of a value in non-numerical term (e.g., we may say that privacy is protected to a sufficient extent in Country x , while its protection is low in Country y , that a large freedom of speech is enjoyed by the citizen of Country w , etc.) or in numerical term, when appropriate numerical indicators are available (as for GDP per head, employment rate, etc.). For some values (transparency, democracy, economic freedom, equality, non-discrimination, etc.) proxies are available according to various measurements, such as those that are used for ranking countries according to their levels of welfare or protection of human rights. However, even when no such proxies are available we still engage in quantitative assessments, while being aware that such assessments are inevitable noisy, approximate and revisable.

Such assessments may be different according to different conceptions of the values at issue and individual attitudes and experience, but different people would show usually some consistency in making them. For instance, I think that very few people would disagree that a 50% increase in the revenue per head (with the same distribution) would provide a higher welfare, or that that storing personal data for a longer time would involve an additional limitation to privacy, or that extending the time for detention without judicial authorisation would additionally restrain individual liberty.

We may wonder, however, if it really possible to compare situations where values are realised in different ways. Assume for instance that we have to compare a situation where privacy is well protected against governmental interference but much less protected against commercial interference, and a situation where privacy is well-protected against commercial interference, and much less protected against governmental interference. those cases in which there is really a competition between two different aspects of value (e.g., given a fixed amount of resources available for the welfare of dependant people, we can increase the welfare of old people only by decreasing welfare for children), we could see the two aspects as distinct values, to be comparatively assessed, as it were a conflict between different values .

When a value is realised up to a certain extent in a certain situation, a certain amount of benefit or utility is delivered.

Definition 2 (Utility-quantity concerning a value) The utility-quantity concerning a value v is the amount of utility provided by the realisation of v . We write $\text{Ut}_v s$ to denote the utility which is obtained with regard to value v in situation s , where v is realised up to the extent $\text{Real}_v s$. Thus, $\text{Ut}_v s = q$ means that situation s , consequently to the realisation of v , utility q is obtained.

Note that here I use *utility* as a “neutral” term denoting the amount of goodness (or badness, when the utility is negative) that is provided by a choice, without making any assumption on the nature or distribution of such goodness. Thus the “utility” of a choice includes the assessment all of its aspects and consequences that affect the relevant values, increasing or decreasing their realisation.⁷

In the following I shall consider how to move from the realisation $\text{Real}_v s$ of a value v to the utility $\text{Ut}_v s$ that is provided by that realisation. Obviously, people’s assessment of the utility of the realisation of a value may be quite variable, and in particular, more variable than their assessment of the realisation-quantity of a value. However, some relations between such assessments may be considered to be invariant.

First of all, since values are by definition good things, we can assume that the utility provided by the realisation of a value, increases as the realisation of that value increases. Thus, we assume that relation between the realisation of a value and the corresponding utility is a monotonic function, and indeed a strictly increasing one. We take this as a defeasible assumption, which expresses what is usually the case, and does not exclude that in certain cases over-realisation of a value can be counterproductive.

Assumption 1 (Increasing utility from values) A higher realisation of a value provides a higher utility. In other words, when the realisation-quantity of value v increases, also its utility-quantity increases: if the realisation-quantity of v in a situation s_2 is higher than the realisation-quantity of v in s_1 , then also the utility-quantity of v in situation s_2 is higher than its utility-quantity in s_1 . In other words, if $\text{Real}_v(s_1) < \text{Real}_v(s_2)$ then $\text{Ut}_v(s_1) < \text{Ut}_v(s_2)$.

Thus, the utility resulting from the realisation of a value v will increase progressively, when v ’s realisation-quantity increases. For instance, a higher level of a value such as health, or environmental quality, or privacy, or freedom of speech, etc., gives more utility than a lower level of the same value, all the rest (the realisation-quantity of the other values) remaining equal. Moreover, as we shall see in the following the extent of this increase progressively diminishes as the realisation-quantity of the value gets higher (there is a diminishing marginal utility), but the relationship above still holds.

On the basis of the notions introduced, we can address impacts of actions (choices) on the realisation of values. We use Greek letters α, β, \dots as variables ranging over actions. We assume that actions have outcomes, namely, they make a change in the status quo, with one exception: the null action Nil consists in letting things as they are (letting the status quo), or better, letting things evolve without our intervention.

To simplify things, let us assume a deterministic framework, where each action has only one outcome (otherwise we have to expand the current framework with probabilities): the unique outcome of an action α is the situation

$\text{Out}(\alpha)$ that would result from performing α , in the current situation. We are now able to specify the impact of an action α to a value v , namely, the change the action α can make to the realisation of v . This is the difference between the extent up to which v would be realised by α , and the extent up to which it would be realised by not doing anything, i.e., by the null action Nil.

Definition 3 (Realisation Impact) The realisation impact of an action α on a value v , denoted as $\Delta\text{Real}_v(\alpha)$, is the difference between the realisation-quantities of v resulting from α and from Nil. Let us denote the outcome of action α , namely, the situation resulting from its performance, in the current situation, as

$\text{Out}(\alpha)$ and the realisation impact (the differential realisation) of an action α on a value v , as $\Delta\text{Real}_v(\alpha)$.

⁷In particular, I do not assume a utilitarian approach, according to which utility is to be viewed as happiness or preference satisfaction. On the contrary, here “utility” refers to the sum of all impacts on all (legally relevant) communal and individual values, since I assume that such impacts are independent. In principle, such utility might also be specified in such a way that the distribution of individual opportunities is subject to some fairness requirements. Then, such opportunities, as they follow from the realisation of the relevant values, according to their importance, would need to be allocated according to a scheme that is fair enough, in the sense that it balances the requirement of distributive fairness against the importance of increasing the total realisation of the concerned values. Here however, I shall not examine whether including fairness requirement in the model here proposed would require a relaxation of some of its assumption, such as the assumption of the independence of the utilities obtained by realising different values.

Then $\Delta\text{Real}_v(\alpha) = \text{Real}_v$
 $\text{Out}(\alpha) - \text{Real}_v$
 $\text{Out}(\text{Nil})$.

For instance, if α is a law prohibiting the use of a polluting substance which is currently in use in industrial processes, the realisation impact of α on health is the increased level of health that results from not having any longer the pollution caused by that substance, while α 's realisation impact on productivity, is the decreased level of productivity which results from not using the substance in production processes.

The notion of realisation impact, allows us to define what it means to promote or demote a value: promoting means increasing (having a positive impact on) the value's level of realisation and demoting means decreasing (having a negative impact on) it, as compared to Nil.

Definition 4 (Promotion and demotion of a value) An action α promotes a value v if its realisation impact on v is positive ($\Delta\text{Real}_v(\alpha) > 0$); it demotes v if its realisation impact on v is negative ($\Delta\text{Real}_v(\alpha) < 0$).

Thus, a legislative choice which prohibits the use of a polluting substance may promote health and demote productivity; a legislative measure that makes internet provider liability for violations of data-protection by their subscribers, may promote data protection and demote freedom of speech, etc. We can also characterise the utility of an action with regard to a value, as the differential utility-impact provided by that action with regard to that value: this is a measure of the difference in utility provided by the fact that the value is realised to a higher or lower extent.

Definition 5 (Utility-impact of an action on a value) The utility-impact of an action α on a value v , is the difference between the utility-quantity by v resulting from α and from Nil: $\Delta\text{Ut}_v(\alpha) = \text{Ut}_v$
 $\text{Out}(\alpha) - \text{Ut}_v$
 $\text{Out}(\text{Nil})$

Thus, in the above case of the prohibition α of the use of a polluting substance, we can say that since α , as compared with the status quo, promotes the value of health, while demoting the value of productivity, it increases the health-related utility concerning, and decreases the productivity-related utility: $\Delta\text{Ut}_v(\text{Nil}) = \text{Ut}_v$
 $\text{Out}(\text{Nil}) - \text{Ut}_v$
 $\text{Out}(\text{Nil}) = 0$

Corollary 1 (Realisation- and utility-impact of Nil) The above definitions entail that the realisation impact of Nil on any value is 0, and so is Nil's utility impact.

4 Impacts on single values

The notions we have described enable us to compare the impact of different choices on different values. First of all, we need to introduce a way to express that a choice α is superior to β with regard to its aggregate impact on a set of values.

Definition 6 (Superiority with regard to a set of values) We say that choice α is superior to choice β with regard to a set of values $\{v_1, \dots, v_n\}$ and write $\alpha \succ_{\{v_1, \dots, v_n\}} \beta$, if α 's utility-impact on this set is higher than β 's utility-impact on the same set. In other words, $\alpha \succ_{\{v_1, \dots, v_n\}} \beta$ if and only if $\Delta\text{Ut}_{\{v_1, \dots, v_n\}}\alpha > \Delta\text{Ut}_{\{v_1, \dots, v_n\}}\beta$.

Note that since the utility-impact of Nil is null (0), then a choice α is superior to Nil with regard to a set of value, wherever the choice has a positive utility-impact on that set.

Consider, for instance, an environmental-protection measure α that prohibits the use of polluting substance, and in this way promotes health and demotes productivity. Measure α is superior to Nil, in case its utility-

impact with regard to the combination of health and productivity is positive. In such a case we would write: $\alpha \succ_{\{health, productivity\}} Nil$.

We will come back later on how to establish superior utility with regard to a set of values. Let us first address impacts on a single value. When we are considering just one value, we can say that whenever the realisation-impact on that value is positive, then the utility-impact on it is positive, given Assumption 1 (higher realisation of a value provides a higher utility by that value). But a higher utility by a value entails superiority with regard to that value. In other words, since (a) $\Delta Real_v \alpha > \Delta Real_v \beta$ entails $\Delta Ut_v \alpha > \Delta Ut_v \beta$ and (b) the latter entails $\alpha \succ_v \beta$, we can conclude (c) that $\Delta Real_v \alpha > \Delta Real_v \beta$ entails $\alpha \succ_v \beta$. This leads us to the following corollary.

Corollary 2 (Superiority (with regard to a value), according to contribution) Whenever α 's realisation-impact on value v is higher than β 's, then α is superior to β with regard to v ($\alpha \succ_v \beta$). In other words, $\Delta Real_v \alpha > \Delta Real_v \beta$ entails $\alpha \succ_v \beta$.

Note that this corollary also applies to the comparison of a choice α with Nil. Since Nil provides 0 differential contribution to the realisation of any value, any choice giving a positive marginal contribution would be better than Nil, and any choice giving a negative marginal contribution would be worse than it.

Consider, for instance, the enactment α of a law allowing wiretapping only on the basis of a judicial warrant, when the current regulation (the status quo, i.e., Nil), allows police authorities to wiretap any communication in their criminal investigations. Under such circumstances, enactment α has a positive impact on privacy and a negative impact on crime prevention: $\alpha \succ_{privacy} Nil$, while $Nil \succ_{crime_prevention} \alpha$.

The utility-impact of a choice α on a set of values is just the sum of the utility-impacts that α delivers by affecting of each of these values.

Corollary 3 (Utilities from different values) Given a choice α having an impact on values v_1, \dots, v_n , the utility-impact of α with regard to the set of those values is the sum $i_1 + \dots + i_n$ of the utility-impacts i_1, \dots, i_n of α with regard to each of such values. In other words, $\Delta Ut_{\{v_1, \dots, v_n\}} \alpha = \Delta Ut_{v_1} \alpha + \dots + \Delta Ut_{v_n} \alpha$.

For instance, consider a law exempting host providers from liability for the privacy violations committed by their users, as compared to a situation where providers are considered to be liable for such a violation. The total utility provided by such a law results from the sum of the utility-impacts it provides on the different values involved, its positive utility-impact on freedom of expression, freedom of information and economic efficiency, and its negative utility-impact on privacy.

5 Pareto superiority

Let us now extend our analysis to choices having an impact on multiple values. The easy case is when α , as compared to β , provides a higher realisation of some values, and does not provide a lower realisation of any other value. In this case we say that α is Pareto-superior to β .

Definition 7 (Pareto superiority) We say that choice α is Pareto superior to β if there exists a value v_1 such that the utility impact of α on v_1 is higher than β 's and for no value v_2 , the utility impact of β on v_2 is higher than α 's. In other words, α is Pareto-superior to β with regard to $\{v_1, \dots, v_n\}$ if (a) there exists a $v_i \in \{v_1, \dots, v_n\}$ such that $\Delta Ut_{v_i} \alpha > \Delta Ut_{v_i} \beta$ and (b) there exists no $v_j \in \{v_1, \dots, v_n\}$ such that $\Delta Ut_{v_j} \beta > \Delta Ut_{v_j} \alpha$. In this case we also say that β is Pareto-inferior to α .

Given that a sum $x_1 + \dots + x_n$ is bigger than a sum $y_1 + \dots + y_n$, whenever some x_i is bigger than y_i and no x_j is bigger than y_j , and that a higher realisation of a value is assumed to provide a higher utility, we get the following corollary.

Corollary 4 (Pareto superiority entails overall superiority.) If α is Pareto superior to β with regard to a set of values then α is superior tout court to β with regard to the same set. In other words, if α is

Pareto-superior to β with regard to $\{v_1, \dots, v_n\}$ then $\alpha \succ_{\{v_1, \dots, v_n\}} \beta$.

Establishing Pareto-superiority involves comparing the utilities resulting from the impacts of the alternative choices on each value at stake. However, this assessment can be simplified according to Assumption 1 above, namely, the idea that the increased realisation of a value always involves, also across different choices, an increased utility resulting from the realisation of that value. Then to determine Pareto superiority, it is sufficient to examine the extent to the same values are realised through the alternative choices.

Consider for instance that a legislator is discussing whether to raise the length of copyright from the status 70 years (the status quo) to 90 years after the death of the author and assume that the two lengths are equivalent with regard to the incentive to produce new works, but the shorter term contributes more to the value of knowledge. In such a case, we can say that the shorter term is Pareto superior, and thus superior tout court to the longer one. A legislator's choice which, like this one, is Pareto inferior to Nil (to the status quo) is particularly condemnable: it makes things worse in some regards, while providing no advantage in any other regards. Such choices may however take place, as a consequence of mistakes in appreciating the social impacts of a new regulation, or as a consequence of the fact that the legislators are being pushed by private interests representing no public value.

6 Comparative evaluations without Pareto superiority

In many cases however, legislative choices are not Pareto inferior to the status quo: they promote some value and demote some other values. For instance, a regulation increasing privacy protection may likely decrease freedom of speech, or a regulation increasing environmental protection may decrease productivity and economic freedom.

To evaluate choices having such impacts, we need to find a way of adding up gains and losses, providing a single outcome, on the basis of which to evaluate each choice as a whole. This means that the utilities provided by impacts on distinct values must somehow and subject to elementary arithmetical operations (sum, subtraction, comparison).

Let us assume, as above, that we have an approximate way for assessing the current realisation-quantity of a value v (such as privacy, freedom of speech, welfare, environmental quality, transparency, political freedom, etc.), which may or not be expressed in numerical way, and a way of assessing the impact of a particular action α on the realisation of v . Given this information, we want to establish the utility-impact of α on v , namely, we want to assess the utility-impact of the fact that v 's realisation has been increased or decreased by α . And we want to express this utility-assessment in an absolute cardinal quantity, namely, a quantity that is homogenous to the quantities through which we express the utility-impacts of this choice on other values at stake, so that these quantities can be added to make up the overall assessment of the utility generated or destroyed by this choice. We want to find a way of accomplishing this task that not only makes sense in principle, but is also psychologically plausible, as a way to perform intuitive quantitative reasoning.

Let us distinguish two steps in the determination of the utility-impact (gain or loss) of a choice on a value. First, we assess the impact of that choice on the realisation of the value, in a way that is independent of any particular unit of measure. Second, we determine the change in the utility that corresponds to a change in the realisation of the value. We intuitively express an assessment of the extent to which a value is affected by a choice, when we say that the choice would provide a (very) big or a (very) small gain or loss concerning the value.

Two different frames of reference seem to be usable for such a judgment. On the one hand, we could quantify increases or decreases as proportions of what the full realisation of that value would be. On the other hand, we could quantify the same increases or decreases as proportions of the current realisation level of that value. We use both frames when using numbers, but also when deploying analogical magnitudes. Thus, we may say that the GDP per head in a poor county has increased a little in absolute terms (viewing the increase

as a fraction of the GDP per head in the richest countries), but that it has increased a lot relatively its previous level. Similarly, we may say that a liberalisation measure in an authoritarian regime provides a little increase in freedom of the press in absolute terms, but a huge increase relatively to the previous level.

I would argue that in practical situation an intermediate position can be taken. We assess the level of realisation of a value as a proportion of what might seem the maximum realisation that is concretely available under the existing conditions, within the constraints that we see as unsurpassable (the maximum realisation resulting from actions we view as practicable). As a common-sense example, consider a person who is considering what career to undertake, and is considering what kind of revenue and work satisfaction he or she may obtain from different professions. The range of revenue-quantities and satisfaction-quantities the person is considering would probably end at the top of the levels of revenue and satisfaction that person considers to be reasonably achievable.

The same takes place also with regard to public choices, whose impact on the relevant values are to be considered within this feasibility horizon: changes in the GDP of a country would be assessed with reference to the maximal achievable GDP for that country, and similarly changes in privacy or freedom of speech.

Thus, an action α 's proportional impact on the realisation level of value v could be defined as the proportion between the increase or decrease in the realisation of v brought about by α 's and the maximum amount of such realisation that is viewed as realisable by the agent.

Definition 8 (Proportional impact on the realisation of a value) The proportional impact of an action α on the realisation of a value v is the proportion between α 's realisation impact on v and the reasonably achievable maximum level of v , denoted as $\text{MaxReal}_v\alpha$. In other words, $\Delta\text{PropReal}_v\alpha = \frac{\Delta\text{Real}_v\alpha}{\text{MaxReal}_v\alpha}$.

Similarly, we need to define the proportional contribution an action to the utility deriving from the realisation of a value, as a proportion of the utility that can be obtained by the maximal feasible realisation of that value.

Definition 9 (Proportional impact on the utility by a value) The proportional impact of an action α on the utility provided by the realisation of value v is the proportion between α 's utility-impact on v and the utility provided by the maximal, reasonably achievable, realisation of v , denoted as $\text{MaxUt}_v\alpha$. In other words, $\Delta\text{PropUt}_v\alpha = \frac{\Delta\text{Ut}_v\alpha}{\text{MaxUt}_v\alpha}$.

The second step consists in moving from a change in the proportional realisation of a value determines to the corresponding change in proportional utility. The relation between the two changes is not constant, since the realisation of a value has decreasing marginal utility: this means that the same change in the realisation of a value will provide less (more) utility the higher (the lower) the position of the realisation interval at issue.

Assumption 2 (Decreasing marginal utility of the realisation of a value) A change in the realisation-quantity of value v , from quantity q_i to quantity q_j (the difference between q_i and q_j being constant) provides a smaller utility-difference the higher is the position of interval $[q_i, q_j]$.⁸

Thus, for instance, a proportional loss in the realisation of revenue (or of privacy) of 1/10 determines a higher loss if it is the passage from 5/10 to 4/10 than if it is the passage from 9/10 to 8/10.

Corollary 6 (From decreasing marginal utility) The hypothesis of the decreasing marginal utility has the following implications:

- The utility loss resulting from a diminution in the realisation of a value is higher than the utility gain which is provided by an equal increase in the realisation of the same value.
- A greater decrease in the realisation of a value causes a proportionally greater decrease in the utility generated by the value; a greater increase in the realisation of a value causes a proportionally smaller

⁸In mathematical terms, we would say that the function connecting a value to its utility, is such that its second derivative is negative. This too, however, has to be taken as what happens in most of the cases, namely, as a defeasible assumption.

increase in the utility by that value

After establishing the proportional contribution of a choice to the utility provided by a value (note that the approximate magnitudes would be located in a range from 0 to 1, being proportions of the maximum achievable utility) we need to find a way of having homogeneous quantities for the utilities provided by the realisation of different values. For this purpose, we need to assign weights to values.

Definition 10 (Weight of a value) The weight of value v , denoted as w_v , is a quantity expressing the importance of value v relatively to the other values.

Obviously, more important values, such as personal freedom, or freedom of speech will have a higher weight, while less important values, such as privacy or transparency will have a lower weight. The idea of assigning weights to values may introduce arbitrariness in balancing, due to the difficulty of comparing different values. However, when we engage in such comparisons we often come to determinations (approximate quantities) that are sufficient to support our choices, and even to sharing them. As Sen (2009, 297) observes, the “reasonable variations (or inescapable ambiguities) in the choice of relative weights” does not exclude that a shared assessment, with a sufficient precision, can be made under many circumstances.

We are now in a condition to provide a quantitative characterisation of the absolute utility of an action with regard to a value.

Definition 11 (Absolute utility-impact on a value) The absolute utility-impact of action α on value v , is the proportional impact of α on the utility concerning v , multiplied by the weight of v . In other words, $\Delta Ut_v \alpha = \Delta Prop Ut_v \alpha * w_v$.

This allows us to give content to the idea that the utility of a choice is the sum of its impacts on all relevant values at stake. The elements to be summed up consist in the absolute utility-impacts concerning each value, which are obtained by multiplying the proportional utility-impact on that value, for the weight of the value.

Definition 12 (Utility of an action) The utility of action α with regard to a set of values $\{v_1, \dots, v_n\}$, is the sum $i_1 + \dots + i_n$ of the absolute utility-impacts of α on each of such values. In this sum each element i_j is the differential utility of α 's impact on value v_j multiplied by the weight of v_j . In other words, $\Delta Ut_{\{v_1 \dots v_n\}} \alpha = \Delta Ut_{v_1} \alpha + \dots + \Delta Ut_{v_n} \alpha$

By separating positive and negative elements, in the set of the utility-impacts of α we get the notion of outweighing: the positive impacts of α outweigh its negative impacts, if their sum is higher than the sum of the negative elements.

Definition 13 (Positive impact, negative impact and outweighing,) The positive impact of action α on value-set $\{v_1, \dots, v_n\}$, is the sum of its impacts on the values whose realisation it increases; α 's negative impact on $\{v_1, \dots, v_n\}$, is the sum of its impacts on the values whose realisation it decreases. In other words, the positive impact can be expressed as: $\Delta Pos Ut_{\{v_1 \dots v_n\}} \alpha = \sum_{1 \leq i \leq n | Ut_{v_i} > 0} \Delta Ut_{v_i}$. The negative impact is correspondingly: $\Delta Neg Ut_{\{v_1 \dots v_n\}} \alpha = \sum_{1 \leq i \leq n | Ut_{v_i} < 0} |\Delta Ut_{v_i}|$.

We use positive quantities for negative impacts (given that the absolute value $|-x|$ of a negative number $-x$ is the positive number x), since we want to express the negative impact through a positive quantity, which can be compared with the quantity of the positive impact.

Corollary 6 (From the notion of outweighing) The following statements are equivalent:

- α 's utility is larger than 0, i.e., $\Delta Ut_{\{v_1, \dots, v_n\}} \alpha \geq 0$;
- α 's positive utility-impact on values in $\{v_1, \dots, v_n\}$ is larger than α 's negative utility-impact on values in $\{v_1, \dots, v_n\}$, i.e., $\Delta Pos Ut_{\{v_1, \dots, v_n\}} \alpha > \Delta Neg Ut_{\{v_1, \dots, v_n\}} \alpha$;
- α 's positive utility-impact on values in $\{v_1, \dots, v_n\}$ outweighs α 's negative utility-impact on values in $\{v_1, \dots, v_n\}$;

- the proportion between α 's positive utility-impact on values in $\{v_1, \dots, v_n\}$ and α 's negative utility-impact on values in $\{v_1, \dots, v_n\}$ is bigger than 1, i.e., $\frac{\Delta \text{PosUt}_{\{v_1, \dots, v_n\}} \alpha}{\Delta \text{NegUt}_{\{v_1, \dots, v_n\}} \alpha} > 1$

The last item of Corollary 6, in its negative form provides a generalisation of the so-called weight formula proposed by Robert Alexy (2003b, 43). In fact, Alexy's formula, which provides the proportion between negative and positive impacts, has the form: $W_{[v_i, v_j]} \alpha = \frac{I_{v_i, \alpha} * W_{v_i}}{I_{v_j, \alpha} * W_{v_j}}$. In our terms $W_{[v_i, v_j]} \alpha$, which Alexy calls the concrete weight of the (demoted) value v_i as opposed to the (promoted) value v_j , in case α , corresponds to the proportion between the negative impact of α on v_i and its positive impact on v_j (which are obtained by multiplying the importance of the impact on the value for its weight), namely, to $\frac{|\Delta \text{Ut}_{v_i} \alpha|}{\Delta \text{Ut}_{v_j} \alpha}$, which amounts to $\frac{|\Delta \text{PropUt}_{v_i} \alpha| * w_{v_i}}{\Delta \text{PropUt}_{v_j} \alpha * w_{v_j}}$. According to Alexy a choice is wrong when the proportion between its negative impacts and its positive impacts is higher than 1, i.e., when $\frac{\Delta \text{NegUt}_{\{v_1 \dots v_n\}} \alpha}{\Delta \text{PosUt}_{\{v_1 \dots v_n\}} \alpha} > 1$.

Finally, we can define the utility of an action α relatively to an alternative action β .

Definition 14 (Utility of an action relatively to another action) The utility of action α relatively to action β , with regard to a set of values $\{v_1, \dots, v_n\}$ is the difference between the absolute utility of α and β with regard to those values. In other words, $\Delta \text{Ut}_{\{v_1, \dots, v_n\}} (\alpha, \beta) = \Delta \text{Ut}_{\{v_1, \dots, v_n\}} \alpha - \Delta \text{Ut}_{\{v_1, \dots, v_n\}} \beta$

This entails that superiority can also be specified on the basis of relative utility.

Corollary 7. Action α is superior to action β when the utility of α relatively to β is positive. In other words, $\alpha \succ_{\{v_1, \dots, v_n\}} \beta$ if and only if $\Delta \text{Ut}_{\{v_1, \dots, v_n\}} (\alpha, \beta) > 0$.

Another interesting corollary is that it may happen that given a set of actions, the action that is superior to all actions in the set is not superior to any of them with regard to any single value.

Corollary 8: Superiority does not necessarily require maximality with regard to a single value, when at least three choices are compared with regard to at least two values. More precisely, given an option set $\{o_1, o_2, \dots, o_m\}$ and a value set $\{v_1, \dots, v_n\}$ it is possible that there is an option $o^* \in \{o_1, o_2, \dots, o_m\}$ such that $o^* \succ_{\{v_1, \dots, v_n\}} o_i$ for every $o_i \neq o^*$ but there is no $v_j \in \{v_1, \dots, v_n\}$ such that for every o_i , $o^* \succ_{\{v_j\}} o_i$.

For instance, given three possible choices α, β, γ , it may be the case that γ is superior to both α and β with regard to value set $\{v_1, v_2\}$ while being inferior to α with regard to v_1 and to β with regard to v_2 . In this case γ represents an adequate compromise between the values that are best promoted α and β : (a) γ outweighs α relatively to v_2 more than α outweighs γ relatively v_1 and (b) γ outweighs β relatively to v_1 more than β outweighs γ relatively to v_2 .⁹ For instance, with regard to the conflict between privacy and security, the best choice may be one that does not maximise neither of the two values, providing a compromise between them. For instance, the intermediate choice of keeping DNA data from suspects only for a short time, with appropriate warranties, may be preferable, all things considered, to both the most privacy favourable option (not storing the data at all) and the most security favourable option (keeping the data indefinitely).

7 Assessing Compliance with Value Norms

We can now deploy the concepts just defined in order to assess compliance with norms dealing with values, or principles, in the terminology of Alexy (2002), e.g., the norms that establish constitutional rights and collective goals in today's constitutions (see Stone Sweet and Mathews, 2008). We shall focus on norms requiring the respect of a value, though the analysis can easily be extended to norms requiring the promotion or the irrelevance of a value.

A norm requiring the respect of a value is satisfied if the agent never chooses a course of action that sacrifices the value, unless the sacrifice is needed for obtaining a more significant increase in the satisfaction of other

⁹ $\Delta \text{Ut}_{\{v_1\}} (\alpha, \gamma) < \Delta \text{Ut}_{\{v_2\}} (\gamma, \alpha)$ and $\Delta \text{Ut}_{\{v_2\}} (\beta, \gamma) < \Delta \text{Ut}_{\{v_1\}} (\gamma, \beta)$

values. This means that such a norm is violated if the agent makes a choice that demotes the value, and the overall utility sum—considering all impacts in all relevant values, included the negative impact on the value at issue—is negative. In this sum, we have to include all values whose consideration is prescribed by the legal system, plus those values that have been chosen by the decision maker, to the exclusion of the values whose consideration is prohibited. The weight to be attributed to such values is the weight that is prescribed by the legal system, and for the permissible values chosen by the decision maker, the importance that is given to them by the decision maker, within the boundaries established by the legal system.

This idea can be expressed by the following two conditions. A value norm prescribing the respect of value v is violated by legislative measure α in case that:

- measure α demotes value v , and
- the total utility-impact of α , with regard to all relevant values is negative, relatively to the null action Nil, or in some cases with regard to an alternative measure β .

Let us see how this idea can be matched with the traditional proportionality texts. According to the reconstruction proposed by Alexy (2003a, 135), a legislative norm interfering with rights protected through a constitutional value-norm—a principle, in Alexy’s terminology—is only legitimate when it meets the following tests:

- Suitability, which excludes “the adoption of means obstructing the realisation of at least one principle without promoting any principle or goal for which they were adopted”;
- Necessity, which requires, with regard to principles P_1 and P_2 , “that of two means promoting P_1 that are, broadly speaking, equally suitable, the one that interferes less intensively in P_2 ought to be chosen”;
- Balancing in strict sense, which requires that “the greater the degree of non-satisfaction of, or detriment to, one principle, the greater the importance of satisfying the other.”

The three tests provide independently necessary and jointly sufficient conditions for teleological correctness. For instance, as Alexy observes, a legislative norm requiring tobacco producers to place health warnings in their products passed the proportionality test, since the German Constitutional Court considered that (1) this norm served a suitable end, i.e., health, (2) there were no alternative measures achieving that end that would be less interfering upon the economic freedom of tobacco producers; (3) the advantage this measure provided with regard to health outweighed the minor interference it caused on economic freedom.

Let us specify the three tests using the concept introduced above, starting with choices affecting only two values: the goal-value v_g pursued by the agent and the prescribed-value v_p to be respected according to a value-norm.

- Suitable choice. A choice α is suitable if it has a positive realisation-impact on a permissible goal-value v_g . In other words, α is suitable iff $\alpha \succ_{v_g} \text{Nil}$.
- Necessary choice. A choice α having a negative impact on a prescribed value v_p is necessary if it has a positive impact on a permissible goal-value v_g and there exists no alternative choice β , having a non inferior impact of on the goal-value v_g and a better impact on the prescribed goal v_p . In other words, choice α is necessary iff $\alpha \succ_{v_g} \text{Nil}$ and there exist no β such that $\beta \succ_{v_g} \alpha$ and $\beta \succ_{v_p} \alpha$.
- Balanced choice (first definition). A choice α having a negative impact on a prescribed value v_p and a positive impact on a goal value v_g is balanced in a strict sense if the positive utility-impact on the goal-value v_g is not outweighed the negative utility-impact on the prescribed value v_p . In other words, choice α is balanced in a strict sense iff implementing it is better than not doing anything: $\alpha \succ_{\{v_g, v_p\}} \text{Nil}$.
- Balanced choice (second definition). A choice α having negative impact on a prescribed value v_p and a positive impact on the goal value v_g is balanced if there exists no alternative β such that β would

have a smaller negative impact on the prescribed goal v_p , and would at the same time provide an higher overall utility—the overall utility being the difference between the utility of the impact on v_g and the disutility provided by the negative impact on the v_p . In other words, choice α is balanced iff there exists no β such that $\beta \succ_{v_p} \alpha$ and $\beta \succ_{v_g} v_p, v_g \alpha$.

I have distinguished two notions of a balanced choice, because the second one provides a much stricter standard than the first: It requires that the decision having a negative impact on a prescribed value should provide a superior overall utility to any possible decision interfering with the prescribed value to a lesser extent. If brought to the extreme, it would almost completely undercut the possibility for a decision-maker to adopt a decision that has a negative impact on a prescribed value and escape censorship. The reviewer would be free to imagine possible alternative decisions which the decision maker did not consider, speculate on their possible effects and merits, and to condemn the decision-maker as soon as the latter's decision could be shown to be suboptimal. Thus, I believe, this kind of review needs to be strongly constrained, for instance by requiring that the adoption of the chosen alternative α appears to have been an unreasonable mistake, given the evidence available when α was adopted.

By denying the conditions above, we get three conditions under which a choice infringes a value-norm.

- **Unsuitable choice.** A choice α having a negative impact on a prescribed value is unsuitable if it has no a positive impact on a permissible goal-value. Thus the unsuitable choice is Pareto-inferior to the status quo, the null action Nil. This may depend on the fact that the chosen action is incapable of reaching that goal—its adoption is based on mistaken factual assumptions—or on the fact that the pursued goal is impermissible, and thus irrelevant according to a value-norm.
- **Unnecessary choice.** A choice α having a negative impact on a prescribed value is unnecessary if there exists an alternative choice β , which is better than α with regard to the prescribed value and a non-inferior with regard to the goal value. Thus the unnecessary α is Pareto-inferior to the alternative β .
- **Unbalanced choice (first definition).** A choice α having a negative impact on a prescribed value is unbalanced if the positive utility of its impact on the goal-value is outweighed by the disutility of its impact on the prescribed value
- **Unbalanced choice-(second definition).** A choice α having a negative impact on a prescribed value and a positive impact on the goal value is unbalanced if there exists an alternative β such that β has a smaller negative impact on the prescribed value, and provides an higher overall utility than α .

We can generalize these notions to the case of decisions affecting more than two values and introduce further refinements and specifications of the notions just introduced. For instance, a different notion of necessity is needed for covering cases where a choice is qualified as “necessary” even though it has a small negative impact on the goal-value, while having a much more significant negative impact on the value to be respected. This will be left to further research.

As these definitions should have been made clear, what is at issue in a proportionality assessment concerning a decision α affecting values v_1 and v_2 is not a comparison of the weights of v_1 and v_2 , but rather a comparison of α 's impacts on such values. Consequently, the fact that value v_1 is more important, i.e., has a higher weight than value v_2 , does not necessarily entail that α 's utility-impact on v_1 is larger than its utility-impact on v_2 : The utility-impact on a value depends on both (1) the proportional utility-impact on that value—the extent to which the benefit deriving from the realisation of the value is increased or decreased—and (2) the weight the value.¹⁰ This is affirmed with particular clarity by the Israeli judge Aharon Barak:

¹⁰More exactly, in the terminology use used before, the absolute utility-impact of a decision α on two values is the sum of its utility impacts on each of them, where each utility-impact is the result of the proportional impact on a value for the weight of that value: $\Delta Ut_{\{v_1, v_2\}} \alpha = \Delta PropUt_{v_1} \alpha * w_{v_1} + \Delta PropUt_{v_2} \alpha * w_{v_2}$. Thus, the condition for the first term (in hypothesis, the negative impact) to be higher than the second is that $|\Delta PropUt_{v_1} \alpha * w_{v_1}| > \Delta PropUt_{v_2} \alpha * w_{v_2}$ which is equivalent to $\frac{w_{v_1}}{w_{v_2}} > \frac{\Delta PropUt_{v_2} \alpha}{|\Delta PropUt_{v_1} \alpha|}$. This inequality can be falsified even when the weight w_{v_1} of value v_1 is much larger than the weight w_{v_2}

[T]he comparison is not between the advantages gained by realizing the goal in contrast to the effect brought by limiting the right. Nor is it between security and liberty. The comparison is between the marginal benefit to security and the marginal harm to the right caused by the restricting law and as such, the comparison is concerned with the marginal and the incremental (Barak, 2010, 8).

Thus, it may happen that in a certain case the impact of a measure α on value v_1 outweighs α 's impact on v_2 , while in another case the impact of a different measure β on v_2 outweighs β 's impact on v_1 . To explain this, it is not necessary to assume that the weights of v_1 and v_2 have changed, being "context dependent". A more plausible explanation may be provided by the fact that the proportional impacts on v_1 and on v_2 were different in the two cases, the weights remaining the same, namely, that in the first case v_1 was affected by α more than it was affected by β in the second case, or that v_2 was affected by β in the second case more than it was affected by α in the first case.

8 Teleological Reasoning and the Choice of Rules

The evaluation of value-impacts pertains not only to the adoption of individual decisions but also to the adoption of general rules.

A value-based choice of rules takes place in the teleological interpretation of legislative texts. Teleology requires choosing the interpretation that most realizes the legislator's goals and the legal values at stake. Within constitutional review, a similar reasoning pattern is used in the so-called "definitional balancing", i.e., when a court does not only affirm that a certain law is disproportionate, but explains this statement considering that any law having a certain kind of content would be disproportionate and would therefore violate the constitution (Aleinikoff, 1987, (Alexy, 2002, 80ff)). It seems that two teleological arguments are involved in this reasoning. First, the teleological assessment of a specific legislative choice, according to its impacts on the values at issue in the individual case. Second, the teleological choice of a rule that generalizes the outcome of the case.

Let me give you an example to explain how a court may engage in definitional balancing, or on the contrary refrain from it. The European Court of Human Rights has recently addressed a case concerning a man and a woman who were both unaffected carriers of mucoviscidosis, a very serious genetic disease, and thus had a high risk (1/4) of having children affected by this illness (Costa and Pavan v. Italy, application no. 54270/10). The claimants, who already had generated an affected child, attacked an Italian law which prohibited pre-implantation tests (Law 40/2007). They argued that this law impeded them from avoiding the risk of having an affected child, through a medical procedure involving the in-vitro production of embryos and the implantation of a non-affected one. The Court affirmed that this law violates art. 8 of the Charter of Human Rights by disproportionately affecting the right to private life of the claimants, in comparison to its alleged benefits to other interests at stake, such as protecting the life of embryos and preventing eugenic practices.

In this case., the judges did not state any general rule to explain or justify their decision, even though they had many possible rules available to them. Such possible rules include the following ones: (1) Any prohibition of pre-implantation testing for mucoviscidosis is disproportionate, with regard to couples who have already generated an affected child, (2) any prohibition of pre-implantation testing for mucoviscidosis is disproportionate for carriers of the disease, including those not having already generated an affected child, (3) any prohibition of pre-implantation testing for any genetic disease is disproportionate, even when concerning genetic problems different from mucoviscidosis, (4) any prohibition of pre-implantation testing is disproportionate, even when the test serves non-therapeutic purposes, such as sex selection, (5) any

of v_2 . This happens when the proportion between $\Delta\text{PropUt}_{v_2}\alpha$, and $\Delta\text{PropUt}_{v_1}\alpha$ is larger than the proportion between w_{v_1} and w_{v_2} .

prohibition of pre-implantation interventions is disproportionate, even when the intervention goes beyond mere testing, as for cloning or genetic engineering.

The adoption of any one of these “definitional” rules by the competent court would have enabled subsequent judges to decide similar cases through rule-based reasoning, rather than through balancing. The subsequent judge, if a definitions rule had been established, could then simply check whether a prohibition of genetic standing has the properties which would make it disproportionate according to the rule and decide the case accordingly.

Since disproportionateness of a legislative measure entails that it should not be taken, such definitional rules could be re-expressed as prohibitions of adopting laws having the indicated content. For instance, assume that the judges in this case, rather than being silent, had ruled any prohibition of pre-implantation testing for genetic diseases is disproportionate. This ruling, in combination with the fact that legislator should not adopt any disproportionate law, entails the following rule: “The legislator should not adopt any law which prohibits pre-implantation testing for genetic diseases.”¹¹

It seems that the judges—having established that a law produces a disproportionate outcome in a particular case—face a choice concerning how to frame their opinion. Their options include choosing one of many available rule-based explanations or choosing not to provide any such explanations. On what grounds should they make such a choice? The answer, I think, requires another appeal to teleological reasoning. They should adopt the generalization that could best realize the legal values at issue, through the subsequent application of the generalization by judges, legislators, and citizens, in the given institutional framework—as characterized by applicable norms, judicial powers, legislative competences, interpretive practices, existing precedents, and social norms. However, on the basis of the same teleological reasoning judges could also conclude that perhaps stating no rule is the best solution, for instance, given the high uncertainty of the matter at stake, which prevents them from stating with sufficient confidence even a highly defeasible general rule.

Thus, the proportionality review of a legislative measure may involve two teleological assessments.

The first assessment concerns establishing that the legislative decision α in the current case has a negative overall impact on the values at stake and is therefore disproportionate. In our example, this is the assessment that the prohibition of pre-implantation testing in the case of a man and a woman both carrier of mucoviscidosis, who already had an affected child, has a disproportionate negative impact on their private life.

The second assessment involves two steps. The first step consists in developing, through abductive reasoning, a set of possible explanations as to why α ’s utility-impact is negative in that case, each explanation appealing to a (different) ruling stating that legislative actions having certain features are disproportionate. The second step consists in selecting, for the rule-based justification of the decision, the rule whose future adoption by courts, legislators, and citizens, in the given institutional and socio-economic context, is likely to provide the highest utility impact, or choosing to provide no rule-based justification in case no advantageous rule can be found.

Thus, this second teleological assessment takes place at a meta-level, since it concerns the choice between alternative patterns for decision in future cases. It compares the utility impacts provided by the future application of different rules, and also the utility-impact of not having any such rule (thus entrusting future decisions to a case-by-case assessments).

The formulation of rules based on proportionality assessments is also at the basis of the attempts to specify the essential content, or the core, of a right. The essential core of a right is indeed identified by indefeasible prohibitions and obligations of performing certain actions, for the sake of the right at issue. Whether a certain

¹¹This is an instance of logical inference according to which premises $A \rightarrow B$ and $B \rightarrow C$ entail $A \rightarrow C$ (instantiating A with “the law prohibits pre-implantation tests for genetic diseases”, B with “the law is disproportionate”, and C with “the law is forbidden”). This inference may be assumed to hold, though only defeasibly, also for defeasible conditionals.

rule protecting a right could be viewed as defeasible or rather infeasible can also be established through teleological reasoning. It should be preferable that the rule is viewed as infeasible—i.e., as concerning the essential core of the right at issue—when the following conditions hold: (1) The action it prohibits, e.g., torture, has a negative impact on a value, this impact being so significant that it is very unlikely that it will be compensated by positive impacts on other values, and (2) the costs of mistaken exceptions to the rule, i.e., of failing to apply it when its application would provide a higher benefit—are presumably higher than the costs of its overreaching applications, i.e., of applying it when its non-application would be more beneficial. Under such conditions we should indeed accept the view that the rule should be treated as being infeasible and we should be ready to pay the costs of the rare disutility of its counterproductive application in extreme cases for the benefit of preventing the disutility of its counterproductive disapplication in a larger or more important set of cases. In other terms, proportionality analysis would not be used at the stage of the application of the rule, but rather at the stage of the justification of its infeasibility.

9 Consistency in balancing

The idea that quantitative reasoning with non-numerical magnitudes has a valuable function in the application of the law can be challenged by pointing to the arbitrariness of the inputs of such a reasoning: even though balancing is constrained by arithmetic, it operates on magnitudes that are idiosyncratic contents of the minds of individual decision-makers (or reviewers of their decisions). How can there be convergence in the outcomes of such reasoning, and how can any social control over such outcomes be effective if any outcome would be possible by changing subjective input quantities?

A certain degree of convergence may be explained by our natural inclinations, social environment, and cognitive capacity. We learn the magnitudes that are associated to our values (the proportional utilities of their realisation and their weights) by processing the inputs we get from our inborn attitudes, education, and personal experience, possibly through inductive/abductive patterns of reasoning which deliver both adjustments and explanations of our intuitive assessments. Moreover, we may consider what reasons support or attack the quantitative assessments we are inclined to make, and thus subject them to critical review, though monological or dialogical argumentation. When we are assumed to adopt a single and shared point of view (the legal point of view), our assessments are additionally constrained by the need that they fit with the past expressions—value norms contained in constitutions and legislative documents, other explicit statements on the absolute and comparative importance of impacts and values, decisions of individual cases involving impacts on such values, legislative rules addressing value conflicts—of the point of view we are adopting.

I cannot here provide an analysis of how we can determine the measure of fit of a new assessment with a certain past history of teleological reasoning, which may involve also incompatible decisions, giving conflicting clues.¹²

Let us first mention two basic cases, where reasoning *a fortiori* on the basis of previous assessments may give clear indications. Assume that α , involving a demotion of value v_d and the promotion of value v_p , was assessed as being proportionate (v_d and v_p , being the only values at stake). Now consider a new decision β involving a smaller (in absolute number) demotion of v_d and an equal or greater promotion of v_p : clearly, β must be considered proportionate as well.

Assume on the contrary that choice α was assessed as non-proportionate. Consider a decision β involving a greater demotion of v_d and a smaller or equal promotion of v_p : clearly, β must be considered disproportionate as well.

These ideas can be further generalised, as this simple example will show. Assume that I have to assess the proportionality of the choice β to store DNA samples of all citizens for 20 years, which demotes their

¹²On the idea of fit, see [Dworkin \(1986\)](#). On the connection between value-based reasoning and the interpretation of rules or the determination of their priorities see in particular [Bench-Capon and Sartor \(2003\)](#) and [\(Prakken, 2000\)](#).

privacy (v_1) and promotes their security (v_2). Assume that in the past a choice α which involved storing DNA samples of all citizens accused of a crime for 10 years was considered to be unacceptable, since its negative impact on privacy outweighed the gain in security. Assume also that it is agreed that by increasing the conservation time (by adopting β rather than α), the damage to privacy is proportionally increased to a larger extent than the gain in security: $\frac{\Delta\text{PropUt}_{v_1}\beta}{\Delta\text{PropUt}_{v_1}\alpha} > \frac{\Delta\text{PropUt}_{v_2}\beta}{\Delta\text{PropUt}_{v_2}\alpha}$. Given such premises, any assessment according to which the new law would provide a positive balance (by subtracting losses and adding gains) would be inconsistent with the previous decision. In fact, in our example, any assignments of weights to v_1 and v_2 that would satisfactorily explain the disproportionality of the 10 years term, would also determine the disproportionality a longer term.

10 Conclusions

I have argued that teleological reasoning includes the assessment of impacts upon relevant values, which may be viewed as a kind of approximate quantitative reasoning, even when we are unable to assign symbolic numerals to the concerned magnitudes. We engage in this reasoning both when making common-sense private choices, and when participating in public decision-making. Non-numerical quantitative reasoning involves certain rationality conditions, and first of all it should normally respect the usual arithmetical relationships, which indicate general constraints for processing of quantitative information. Thus, arithmetical relationships can also be viewed as default standards of rationality to be applied by legal reasoners (legislators, interpreters and judges) when engaging in proportionality assessments.

These quantitative assessments, express intuitive appreciations of the importance of positive or negative impacts on the values at stake, but can be supported or attacked through arguments. These arguments may address all legally relevant aspects at stake (the identification of values at stake, the determination of impacts on their realisation, the assessment of their weights, etc.) and they can appeal to consistency with precedents. Thus, intuitive quantitative assessments are subject to some degree of discursive control.

References

- Aleinikoff, T. (1987). Constitutional law in the age of balancing. *Yale Law Journal* 96, 943–1005.
- Alexy, R. (2002). *A Theory of Constitutional Rights*. Oxford University Press.
- Alexy, R. (2003a). Constitutional rights, balancing, and rationality. *Ratio Juris* 16, 131–40.
- Alexy, R. (2003b). On balancing and subsumption: A structural comparison. *Ratio Juris* 16, 33–49.
- Barak, A. (2010). Proportionality and principled balancing. *Law and Ethics of Human Rights* 4, 2–18.
- Bench-Capon, T. J. M. and G. Sartor (2003). A model of legal reasoning with cases incorporating theories and values. *Artificial Intelligence* 150, 97–142.
- Dworkin, R. M. (1986). *Law's Empire*. Kermode.
- Gallistel, C. R. and R. Gelman (2005). Mathematical cognition. In K. J. Holyoak and R. G. Morrison (Eds.), *The Cambridge Handbook of Thinking and Reasoning*, pp. 559–88. Cambridge University Press.
- Gallistel, C. R., R. Gelman, and S. Cordes (2006). The cultural and evolutionary history of the real numbers. In S. C. Levinson and P. Jaisson (Eds.), *Evolution and culture*, pp. 247–74. MIT.
- Kahneman, D. (2011). *Thinking: fast and slow*. Allen Lane.
- Keeney, R. and H. Raiffa (1993). *Decisions with Multiple Objectives: Preferences and Trade Offs*. Cambridge University Press.

- Parsons, S. (2001). *Qualitative Methods for Reasoning Under Uncertainty*. MIT.
- Pollock, J. L. (2006). *Thinking about Acting: Logical Foundations for Rational Decision Making*. Oxford University Press.
- Prakken, H. (2000). An exercise in formalising teleological case-based reasoning. In H. Prakken and R. Winkels (Eds.), *Proceedings of the Thirteenth Annual Conference on Legal Knowledge and Information Systems (JURIX)*, pp. 49–57. IOS.
- Rescher, N. (1991). *G. W. Leibniz's Monadology: An Edition for Students*. University of Pittsburgh Press.
- Sartor, G. (2010). Doing justice to rights and values: Teleological reasoning and proportionality. *Artificial Intelligence and Law* 18, 175–215.
- Sartor, G. (2013). The logic of proportionality: Reasoning with non-numerical magnitudes. *German Law Journal* 14, 1419–57.
- Sen, A. (2009). *The Idea of Justice*. Belknap.
- Stone Sweet, A. and J. Mathews (2008). Proportionality balancing and global constitutionalism. *Columbia Journal of Transnational Law* 47, 72–164.