Extracting scenarios from a Bayesian network as explanations for legal evidence

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Abstract. In order to make an informed decision in a criminal trial, conclusions about what may have happened need to be derived from the available evidence. Recently, Bayesian networks have gained popularity as a probabilistic tool for reasoning with evidence. However, in order to make sense of a conclusion drawn from a Bayesian network, a juror needs to understand the context. In this paper, we propose to extract scenarios from a Bayesian network to form the context for the results of computations in that network. We interpret the narrative concepts of scenario schemes, local coherence and global coherence in terms of probabilities. These allow us to present an algorithm that takes the most probable configuration of variables of interest, computed from the Bayesian network, and forms a coherent scenario as a context for these variables. This way, we take advantage of the calculations in a Bayesian network, as well as the global perspective of narratives.

Keywords. Legal reasoning, Bayesian networks, narrative

Introduction

In a criminal trial, a judge or jury ideally forms an idea of what may have happened concerning the supposed crime before making a decision. In any case, a conclusion needs to be drawn about the indictment: did the suspect fire the gun? Was the killing premeditated? Better yet is if the fact-finder forms a scenario, not just about the specific details in the indictment, but about the sequence of events that, as a whole, accounts for the evidence that was found. In this paper we discuss how a combination of Bayesian networks and narrative tools can help to draw a more informed conclusion: whereas a Bayesian network can be used to calculate the posterior probability of the variables in the indictment, scenarios can be formed around these variables to provide a context.

The process of reasoning with evidence to draw conclusions about what happened is typically modeled with one of three approaches: a probabilistic approach, an argumentative approach or a narrative approach [1]. Bayesian approaches have recently become popular as a probabilistic tool for reasoning with evidence [2,3]. For combining multiple pieces of evidence, Bayesian networks are particularly suitable. A Bayesian network consists of a directed graph which models the (in)dependencies between variables and

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probability tables, together representing a joint probability distribution over the variables in the domain. Methods exist for eliciting the large number of probabilities typically required for a Bayesian network [4]. For building the structure, methods specifically for the legal domain have been developed in [5,6]. In our previous work we presented a design method for Bayesian networks based on narrative [7]. In this current paper the focus lies on understanding the results of a Bayesian network rather than the construction. The relevance of understanding the results of Bayesian methods is emphasized by the debate that followed the recent UK Court of Appeal ruling R v T.3 The court decided that Bayes’ theorem should not be used in evaluating evidence, except for DNA and ‘possibly other areas where there is a firm statistical base’. The ruling led to extensive scholarly debate (see, e.g., the 2012 special issue of Law Probability and Risk [8]).

Whereas a Bayesian network is a good tool to find out specifically how probable certain hypotheses of interest are, simply presenting the outcomes of computations does not provide a judge or jury with much insight into the case. As shown by Pennington and Hastie [9], jurors tend to use stories or scenarios to organize the evidence and make sense of a case. A scenario, which is a coherent sequence of states and events [10], allows for a more global perspective on a case and can be used to explain the evidence. We aim to present a judge or jury with a scenario that forms an explanation of the evidence, while also incorporating the outcome of the computations resulting from the Bayesian network. This combines the advantages of the solid mathematical foundation provided by Bayesian networks with the global perspective of scenarios. This approach differs from existing methods for understanding Bayesian networks because of the narrative point of view. To the best of our knowledge, there is no previous research on extracting scenarios from Bayesian networks.

Extracting scenarios from a Bayesian network is not a trivial task. A scenario should be sufficiently detailed such that it can account for the evidence that was observed, but it should not include unrelated details that lead to a loss of coherence. In this paper we use scenario schemes to ensure coherence, as scenario schemes outline the general structure of a scenario. A main contribution of this paper is the interpretation of several concepts from the narrative field, including that of scenario schemes and coherence, in the context of Bayesian networks (Section 2). Based on this, we present an algorithm (Section 3) that extracts scenarios from a Bayesian network by ‘filling’ the structure of a scenario scheme with nodes from the network, resulting in a coherent scenario around the results of computations from a Bayesian network.

1. Prerequisites

1.1. Bayesian networks

A Bayesian network is a representation of a joint probability distribution (JPD). It consists of a directed, acyclic graph, and conditional probability tables for all nodes in the graph. The graph models the (in)dependencies between nodes in the network. When there is no arrow between nodes, the variables are either independent or conditionally indepen-

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2This work is part of the research project Designing and Understanding Forensic Bayesian Networks with Arguments and Scenarios, funded in the NWO Forensic Science program. See www.ai.rug.nl/~verheij/nwofs.

dent. Each node can have several values and a value-assignment \( \{V_1 = v_1, \ldots, V_k = v_k\} \) denotes an assignment of values \( v_1, \ldots, v_n \) to nodes \( V_1, \ldots, V_k \). Each node in the network has a conditional probability table (CPT) which holds the probability for each value to occur, conditioned on the values of direct predecessors. When there are no predecessors, the CPT holds the prior probabilities for that node.

After constructing a Bayesian network, any prior or posterior probability of interest can be computed, including, for example, the probability of a hypothesis \( h \) given the evidence \( e \): \( P(h|e) \). For a set of variables of interest \( \{I_1, \ldots, I_k\} \), a MAP-computation (for Maximum A Posteriori) [11] can find a value-assignment \( \{I_1 = i_1, \ldots, I_k = i_k\} \) that maximizes the probability \( P(I_1 = i_1, \ldots, I_k = i_k|e) \) of the variables of interest given the evidence.

1.2. Explanations

A hypothesis \( h \) forms an explanation for evidence \( e \) if assuming \( h \) increases our degree of belief in evidence \( e \). To measure the quality of an explanation in a probabilistic setting, one can consider the \textit{explanatory power} of a hypothesis for a set of evidence. Several measures of explanatory power in terms of probabilities have been proposed, which all lead to ordinally equivalent results (see [12]). The following definition is based on the measure proposed in [13]:

\textbf{Definition 1.1 (Explanatory power).} The \textit{explanatory power} of a hypothesis \( \{H_1 = h_1, H_2 = h_2, \ldots, H_n = h_n\} \) for evidence \( e \) is

\[
\frac{P(e|H_1 = h_1, \ldots, H_n = h_n)}{P(e)}.
\]

A hypothesis has explanatory power for evidence \( e \) iff the fraction is larger than 1.

As is well-known, explanations with high explanatory power are not necessarily the most probable explanations in the sense of posterior probabilities. Explanatory power connects the prior and posterior probability of a hypothesis via Bayes’ rule: \( P(h|e) = P(h) \frac{P(e|h)}{P(e)} \).

2. Defining scenarios in a Bayesian network

In this section some notions from the narrative field are formalized in the context of a Bayesian network, namely scenario schemes (Section 2.1), global coherence (Section 2.2) and local coherence (Section 2.3). In what follows, a Bayesian network is assumed to be given by a graph \( (V, E) \) with nodes \( V = \{V_1, \ldots, V_n\} \) and edges \( E = \{(V_i, V_j), \ldots\} \).

2.1. Scenarios and scenario schemes

Various definitions of stories (such as those in [14] and [15]) convey related ideas, namely that a story is a coherent sequence of states and events. In a Bayesian network, states and events are denoted by variables assigned to a certain value. When working with Bayesian networks, we define a scenario as a value-assignment to a collection of nodes:

\textbf{Definition 2.1 (Scenario).} A \textit{scenario} \( S(K) \) is a value-assignment \( \{K_1 = k_1, K_2 = k_2, \ldots, K_n = k_n\} \) to a subset of nodes \( K = \{K_1, \ldots, K_n\} \subseteq V \) in the Bayesian network.
Any value-assignment can be a scenario, but the main feature of a good scenario is coherence. In the narrative field, the term coherence is used to express that the components of a story somehow belong together. As a formalization of coherence, several authors have proposed to consider underlying patterns of stories, in the form of story grammars [16], scripts [17] or schemes [9]. Essentially, the idea of such patterns is that a story should always consist of certain components, and a story should ‘have all of its parts’ in such a way that there is some connection between these parts. The underlying patterns can be specified on various levels of detail: Schank and Abelson [17] speak of scripts specifically tailored to a certain situation, such as their famous restaurant script for a story about a restaurant, whereas Pennington and Hastie [9] utilize an ‘intentional action scheme’ that is applicable to stories on various topics. A simplified version of the intentional action scheme is shown in Figure 1.

A scenario scheme organizes how various types of states and events are put together to form a coherent scenario. A scheme consists of slots that can be filled with states or events of a certain type, and connections between these slots. The restaurant script can be filled with events of type ‘payment’ or ‘being seated’, etcetera, whereas the intentional action scheme requires the less specific types ‘initiating state/event’, ‘goal’, ‘action’ and ‘consequence’. The general notion of a scenario scheme can be formalized as follows:

**Definition 2.2 (Scenario scheme).** A scenario scheme $P$ is a pair $(T, C)$ where

- $T = \{t_1, \ldots, t_n\}$ is a set of which the elements express event types, and
- $C = \{(t_i, t_j), \ldots\}$ is a set of ordered pairs representing directed connections between event types.

According to this definition, a scenario scheme is a graph with nodes representing the various types and edges between nodes representing the connections between these states or events. The intentional action scheme may be formalized as $(T, C)$ where

- $T = \{\text{initiating states/events (i), goals (g), actions (a), consequences (c)}\}$
- $C = \{(i, g), (g, a), (a, c)\}$

Other scenario schemes can have a more complex structure (e.g. non-linear), or consist of more specific types (e.g. specific types of actions such as ‘forced entry’ and ‘removing items’ for a burglary scenario). As will become clear from the definitions in the following sections, each slot in the scenario scheme can contain a block of several nodes, which we call a component (a definition of the term component is in Section 2.2). For example, in the intentional action scheme, several actions may together form the action-component.

The nature of connections between components has been subject of debate: some researchers assume that all connections in a scenario are causal [18], while others speak of underlying common sense generalizations [19,10], or sometimes merely temporal con-
nections between states and events [17]. With our current goal of extracting narratives as explanations, we propose to interpret the connections as explanatory: there is a connection from one component to another if the first component explains the second. In the intentional action scheme, this means that the initiating states and events should explain the goals (since they have a connection in the scheme), the goals should explain the actions and the actions should explain the consequences. This explanatory interpretation of connections within a scenario will be discussed in Section 2.3.

For a scenario to ‘match’ a scenario scheme, there are two factors that play a role:

1. each type in the scenario scheme must correspond to some component in the scenario and vice versa; and
2. for each directed connection in the scenario scheme there must be a connection between the corresponding components.

In the following subsections, these two factors will be defined as two separate kinds of coherence: (1) global coherence (a scenario must ‘have all of its parts’, Section 2.2) and (2) local coherence (components of a scenario must have some connection, Section 2.3).

2.2. Global coherence

In order to find a correspondence between components in a scenario and types in a scenario scheme, information is needed about what type each node in a Bayesian network may correspond to. We therefore assume that the graph of the Bayesian network has been labeled with appropriate types, where a labeling is defined as follows:

**Definition 2.3 (Labeling).** Given a set of scenario schemes \( \{(T_1, C_1), ..., (T_n, C_n)\} \), a labeling \( L \) on a graph \( (V, E) \) is a well-defined function \( L : V \rightarrow \bigcup_{i=1}^{n} T_i \) which maps each node in the graph to a type present in (at least one of) the scenario schemes.

With such a labeling, each node in the network is mapped to a type. A scenario has global coherence with respect to a scenario scheme when for each type in the scheme there is some component in the scenario with nodes of that label. This matches the intuition that a scenario should ‘have all of its parts’. In addition, to exclude irrelevant states or events from a scenario, each component of the scenario should be of a type that is in the scenario scheme.

**Definition 2.4 (Global coherence).** A scenario \( S(K) \) has global coherence with respect to labeling \( L \) and scenario scheme \( P = (T, C) \) iff the following two conditions hold:

1. \( \forall t_i \in T \exists \text{ a node } K_i \in K \text{ such that } L(K_i) = t_i \)
2. \( \forall K_i \in K \text{ it holds that } L(K_i) \in T \)

The definition of global coherence only requires at least one node for each type in the scheme. However, as mentioned above, a scenario could include several nodes labeled with a certain type, which together correspond to that type in the scheme. Such a collection of nodes of the same type \( t \) is called the \( t \)-component of a scenario:

**Definition 2.5 (t-component).** Given a scenario \( S(K) \), a labeling \( L \) and type \( t \), the \( t \)-component \( S(K)_t \) of that scenario is the value-assignment to all nodes labeled with \( t \):

\[
S(K)_t = \{ K_j = k_j | K_j \in K \text{ and } L(K_j) = t \}.
\]
2.3. Local coherence

In addition to global coherence, a good scenario must also have local coherence. The local coherence of a scenario depends on the strength of connections between components of the scenario. In Section 2.1 we have proposed to interpret connections in a scenario as explanatory, which allows us to use the measure of explanatory power (defined in Section 1.2) as a measure for local coherence:

**Definition 2.6 (Local coherence between components).** The local coherence between the \( t_i \)-component and the \( t_j \)-component of a scenario \( S(K) \) with respect to scenario scheme \( P = (T, C) \) with \( (t_i, t_j) \in C \), is given by the explanatory power of the \( t_i \)-component \( S(K)_{t_i} \) for the \( t_j \)-component \( S(K)_{t_j} \):

\[
\frac{p(S(K)_{t_j} | S(K)_{t_i})}{p(S(K)_{t_j})}.
\]

A scenario has local coherence when for each connection \((t_i, t_j)\) in the scenario scheme the local coherence \(1\) between the \( t_i \)-component and the \( t_j \)-component is greater than \(1\):

**Definition 2.7 (Local coherence).** A scenario \( S(K) \) has local coherence with respect to labeling \( L \) and scenario scheme \( P = (T, C) \) iff \( \forall (t_i, t_j) \in C \) the local coherence between the \( t_i \)-component \( S(K)_{t_i} \) and the \( t_j \)-component \( S(K)_{t_j} \) is greater than \(1\).

The definition of local coherence only requires some explanatory power between components. When applying these ideas to extract scenarios from a Bayesian network, an algorithm can optimize the resulting scenario by searching for high explanatory power between components, and thus high local coherence.

3. An algorithm for extracting scenarios

With the definitions from Section 2, value-assignments in a Bayesian network can be matched to scenario schemes. Based on this idea, we made an implementation\(^4\) that first calls for a computation of the most probable configuration of the variables of interest, and then extracts a coherent scenario including the found configuration for the variables of interest. Currently, our implementation extracts scenarios based on the intentional action scheme, but the ideas underlying the algorithm should extend to other scenario schemes as well (see also the discussion in Section 4). In this section the algorithm is described and illustrated with an example.

Consider a burglary case with three pieces of evidence: a window is broken, fingermarks matching the prints of person X were found on the windowsill and stolen items were found at the home of person Y. Two alternative scenarios are (where the first scenario fails to explain the items being found at the home of Y): (1) ‘Person X had no money. X decided to steal something. X broke the window and took some items’ and (2) ‘Person X had no money. X was friends with Y, and together they decided to steal something. X broke the window and Y stole the items.’ These two scenarios and their

\(^4\)Our implementation uses the java library from SamIam: reasoning.cs.ucla.edu/samiam
connections to the evidence were modeled in a single Bayesian network, shown in Figure 2, where each node has values true/false (T/F). The graph was labeled as follows for use with the intentional action scheme:

- \( L : \) X had no money, X and Y are friends \( \rightarrow i \) (initiating states/events)
- \( L : \) X wanted to steal, X and Y wanted to steal \( \rightarrow g \) (goals)
- \( L : \) X broke window, X stole items, Y stole items \( \rightarrow a \) (actions)
- \( L : \) Broken window, X’s fingerprints, Items at Y \( \rightarrow c \) (consequences)

Furthermore, variables of interest need to be specified. In the burglary case, these may be (in a trial for suspect X) X broke window and X stole items. Given the evidence, our algorithm calls for a MAP-computation (standard part of the SamIam-software, see also Section 1) on the variables of interest. For the burglary case, the MAP-assignment is \{X broke window = T, X stole items = T\}. Our goal is to form a scenario around this assignment, as a context to explain the evidence.

The main idea of the algorithm is to fill the slots in the scenario scheme with components that lead to the highest local coherence. By making sure that each slot is filled, global coherence is guaranteed simultaneously. For the intentional action scheme, starting with a given consequences-component, the algorithm works its way back through the actions that best explain the consequences (amounting to high local coherence), the goals that best explain the actions and the initiating states/events that best explain the goals. In the burglary case, the available evidence forms the consequences-component: \( S(K)_c = \{\text{Broken window} = T, \text{X’s fingerprints} = T, \text{Items at Y} = T\} \). In general, we assume that the evidence is of the consequences-type, since legal evidence cannot consist of intentions, goals or actions, but only of observations (e.g., a witness testimony or a consequence of an action). Starting with the consequences \( S(K)_c \), the algorithm compiles a list of candidate actions-component because the scenario scheme has a connection \((a, c)\) from actions to consequences. First, the algorithm checks whether there are variables of interest of type \( a \), and what their MAP-assignments are. This leads to a minimal candi-
date component, in this case \( \{ X \text{ broke window} = T, X \text{ stole items} = T \} \). The list of candidates then consists of components extending this minimal component. In the burglary case, this results in the following list of \( a \)-components:

\[
\begin{align*}
\{ X \text{ broke window} = T, X \text{ stole items} = T \} \\
\{ X \text{ broke window} = T, X \text{ stole items} = T, Y \text{ stole items} = T \} \\
\{ X \text{ broke window} = T, X \text{ stole items} = T, Y \text{ stole items} = F \}
\end{align*}
\]

For each of the candidates on the list, the algorithm computes the local coherence between each candidate and the \( c \)-component. After computing the local coherence for each candidate, if there is more than one candidate resulting in local coherence for the \( c \)-component, the candidate with the highest local coherence is selected, namely: \( S(K)_a = \{ X \text{ broke window} = T, X \text{ stole items} = T, Y \text{ stole items} = T \} \).

The algorithm now proceeds to the next connection in the scenario scheme, \((g,a)\). A list of the candidate \( g \)-components is compiled and the candidate with the highest local coherence with respect to \( S(K)_a \) is selected to be the goals-component \( S(K)_g \). Repeating this for the last connection \((i,g)\), the resulting scenario is as follows:

\[
\begin{align*}
S(K)_i &= \{ X \text{ and Y are friends} = T, X \text{ had no money} = T \} \\
S(K)_g &= \{ X \text{ wanted to steal} = T \} \\
S(K)_a &= \{ X \text{ broke window} = T, X \text{ stole items} = T, Y \text{ stole items} = T \} \\
S(K)_c &= \{ X \text{ broke window} = T, X \text{ stole items} = T \}
\end{align*}
\]

This scenario includes the outcome of the MAP-computation about the variables of interest (\( X \text{ broke window} \) and \( X \text{ stole items} \)) and additionally provides a context.

4. Discussion and related work

The goal of this paper was to formalize narrative concepts for the extraction of a scenario from a Bayesian network. By forming a coherent scenario around a MAP-assignment for the variables of interest, a context is provided that can help make sense of the results. The burglary example in Section 3 showed what such an approach can do: while the MAP-assignment only concluded that \( X \text{ broke the window and stole items} \), the extracted scenario tells the story of \( X \) and \( Y \) breaking in together, which explains why the items were found at the house of person \( Y \).

In future research, the algorithm should be tested thoroughly on more complex examples. It will also be worth investigating whether this approach indeed helps to make the results of the Bayesian network more insightful to a juror. Other future research could involve the exploration of other schemes, in particular differently structured schemes (e.g. non-linear) that require the algorithm to go through the connections in a smart order.

Related work on Bayesian networks modeling legal evidence comprises methods for constructing such networks for legal cases, e.g. [5,6]. Our previous work [7] proposed a construction method using narrative as a basis for building Bayesian networks for legal evidence. In this current paper our focus lies on understanding a Bayesian network rather than building one.

An overview of research on explanation methods for understanding Bayesian networks is available in [20]. Much of this research is about explaining why a Bayesian network was modeled in a certain way, or why certain inferences can be made in the net-
work. Our goal was rather to explain (or provide a context for) the evidence. Some work on explanations of evidence in Bayesian networks with scenarios was done by Druzdzel [21], who also considers scenarios to be configurations of nodes in the network. Shen and colleagues [22] worked on a decision-support system for crime investigation with Bayesian networks and scenarios, where scenarios are interpreted as ‘situations’ that can explain the evidence. A crucial difference between these previous works and our current paper is our use of scenario schemes, by which we ensure narrative properties in our scenarios, in particular local and global coherence.

Other related work on understanding Bayesian networks for legal evidence includes work by Keppens [23], who proposed an algorithm for the extraction of argument diagrams from Bayesian networks and work by Timmer [24], who recently developed an algorithm to extract a system of arguments that shows how various arguments can support or attack competing conclusions in a case. Related to our use of narrative concepts is work by Bex [10], who also uses scenario schemes in his hybrid theory of scenarios and arguments for reasoning with legal evidence. Finally, whereas our work and that of Keppens, Timmer and Bex all propose some combination of scenarios and probabilities, probabilities and arguments or arguments and scenarios, Verheij aims for an integrated theory of scenarios, arguments and probabilities in [25].

Conclusion

In this paper we have proposed to aid the decision making of a judge or jury with a combination of probabilistic calculations in a Bayesian network and tools from the narrative field. While a Bayesian network forms a precise tool for computing the most probable configuration of variables of interest, simply reporting the results of such a computation will not provide the judge or jury with any insight into the case. Narrative concepts can be used to provide a context for the results from a Bayesian network.

The main contribution of this paper is the interpretation of a number of key concepts from narrative research in the probabilistic context of Bayesian networks. In particular, definitions of scenario schemes, local and global coherence were provided. Using these definitions, it is now possible to match nodes in a Bayesian network to a scenario scheme to obtain coherent scenarios from a network. An algorithm to extract a coherent scenario from a Bayesian network was presented. The goal of this algorithm is to produce a coherent scenario, using the concepts described above, around the most probable outcome of the variables in the indictment. With such a combination of Bayesian networks and narrative tools, one can take advantage of both the solid mathematical framework of Bayesian networks and the global perspective provided by narratives that helps to make sense of a case.

More research is required to evaluate our algorithm and to find whether this approach indeed provides a judge or jury with more insight. Further development of the algorithm could explore different scenario schemes and we ultimately aim to test the algorithm on a larger case study.

References