Imitation in Large Games

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In games with a large number of players where players may have overlapping objectives, the analysis of stable outcomes typically depends on player types. A special case is when a large part of the player population consists of imitation types: that of players who imitate choice of other (optimizing) types. Game theorists typically study the evolution of such games in dynamical systems with imitation rules. In the setting of games of infinite duration on finite graphs with preference orderings on outcomes for player types, we explore the possibility of imitation as a viable strategy. In our setup, the optimising players play bounded memory strategies and the imitators play according to specifications given by automata. We present algorithmic results on the eventual survival of types.

1 Summary

Imitation is an important heuristic studied by game theorists in the analysis of large games, in both extensive form games with considerable structure, and repeated normal form games with large number of players. One reason for this is that notions of rationality underlying solution concepts are justified by players' assumptions about how other players play, iteratively. In such situations, players' knowledge of the types of other players alters game dynamics. Skilled players can then be imitated by less skilled ones, and the former can then strategize about how the latter might play. In games with a large number of players, both strategies and outcomes are studied using distributions of player types.

The dynamics of imitation, and strategizing of optimizers in the presence of imitators can give rise to interesting consequences. For instance, in the game of chess, if the player playing white somehow knows that her opponent will copy her move for move then the following simple sequence of moves allows her to checkmate her opponent [1]:

`1.e3 e6 2.Qf3 Qf6 3.Qg3 Qg6 4.Nf3 Nf6 5.Kd1 Kd8 6.Be2 Be7 7.Re1 Re8 8.Nc3 Nc6 9.Nb5 Nb4 10.Qxc7#`

On the other hand, we can have the scenario where every player is imitating someone or the other and the equilibrium attained maybe highly inefficient. This is usually referred to as 'herd behaviour' and has been studied for instance in [3].

In an ideal world, where players have unbounded resources and computational ability, each of them can compute their optimal strategies and play accordingly and thus we can predict optimal play. But in reality, this is seldom the case. Players are limited in their resources, in computational ability and their knowledge of the game. Hence, in large games it is not possible for such players to compute their optimal strategies beforehand by considering all possible scenarios that may arise during play. Rather, they observe the outcome of the game and then strategise dynamically. In such a setting again, imitation types make sense.

A resource bounded player may attach some cost to strategy selection. For such a player, imitating another player who has been doing extensive research and computation may well be worthwhile, even if

[1] This is called 'monkey-chess' in chess parlance.

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her own outcomes are less than optimal. What is lost in sub-optimal outcomes may be gained in avoiding expensive strategisation.

Thus, in a large population of players, where resources and computational abilities are asymmetrically distributed, it is natural to consider a population where the players are predominantly of two kinds: optimisers and imitators.[2] Asymmetry in resources and abilities can then lead to different types of imitation and thus ensure that we do not end up with 'herd behaviour' of the kind referred to above. Mutual reasoning and strategising process between optimizers and imitators leads to interesting questions for game dynamics in these contexts.

Imitation is typically modelled in the dynamical systems framework in game theory. Schlag ([12]) studies a model of repeated games where a player in every round samples one other player according to some sampling procedure and then either imitates this player or sticks to her own move. He shows that the strategy where a player imitates the sampled player with a probability that is proportional to the difference in their payoffs, is the one that attains the maximum average payoff in the model. He also gives a simple counterexample to show that the naïve strategy of 'imitate if better' may not always be improving. Banerjee ([3]) studies a sequential decision model where each decision maker may look at the decisions made by the previous decision makers and imitate them. He shows that the decision rules that are chosen by optimising individuals are characterised by herd behaviour, i.e., people do what others are doing rather than using their own information. He also shows that such an equilibrium is inefficient. Levine and Pesendorfer ([7]) study a model where existing strategies are more likely to be imitated than new strategies are to be introduced.

The common framework in all of the above studies is repeated non-zero-sum normal form games where the questions asked of the model are somewhat different from standard ones on equilibria. Since all players are not optimizers, we do not speak of equilibrium profiles as such but optimal strategies for optimizers and possibly suboptimal outcomes for imitators. In the case of imitators, since they keep switching (imitate i for 2 moves, j for 3 moves, then again i for 1 move, etc.) studies consider **stability** of imitation patterns, what types of imitation survive eventually, since these would in turn determine play by optimizers and thus stable subgames, thus determining stable outcomes. Note that, as in the example of chess above, imitation and hence the study of system dynamics of this kind, makes equal sense in large turn based extensive form games among resource bounded players as well.

For finitely presented infinite games the stability questions above can be easily posed and answered in automata theoretic ways, since typically bounded memory strategies suffice for optimal solutions, and stable imitation patterns can be analysed algorithmically. Indeed, this also provides a natural model for resource bounded players as finite state automata.

With this motivation, we consider games of unbounded duration on finite graphs among players with overlapping objectives where the population is divided into players who optimise and others who imitate. Unbounded play is natural in the study of imitation as a heuristic, since 'losses' incurred per move may be amortised away and need not affect eventual outcomes very much. Imitator types specify how and who to imitate and are given using finite state transducers. Since plays eventually settle down to connected components, players' preferences are given using orderings on Muller sets [11]. In this work, we study turn-based games so as to use the set of techniques already available for the analysis of such games.

In this setting we address the following questions and present algorithmic results:

- If the optimisers and the imitators play according to certain specifications, is a global outcome eventually attained?

[2] There would also be a third kind of players, randomisers, who play any random strategy, but we do not consider such players in this exposition.

- What sort of imitative behaviour (subtypes) eventually survive in the game?
- How worse-off are the imitators from an equilibrium outcome?

Infinite two-player turn-based games on finite graphs have been extensively studied in the literature. A seminal result by Büchi and Landweber [4] showed that for Muller objectives, winning strategies exist in bounded memory strategies and can be effectively synthesised. Martin [8] showed that such games with Borel winning conditions are sure-determined (one of the players always has a winning strategy from every vertex). Zielonka [13] gave an insightful analysis of Muller games and provided an elegant algorithm to compute bounded memory winning strategies.

For concurrent-move games, sure determinacy does not hold, and the optimal value determinacy (the values of both the players at every vertex sum to 1) for concurrent-move games with Borel objectives was proved in [9]. Concurrent games with qualitative reachability and more general parity objectives have been studied in [2, 1]. Such games have also been extended to the multiplayer setting where the objectives of the players are allowed to overlap. [5, 6] show that when the objectives are win-lose Borel, subgame perfect equilibria exist. [11] show that bounded memory equilibrim tuples exist in turn based games even when the objectives are not win-lose but every player has preferences over the various Muller sets.

2 Games, Strategies and Objectives

The model of games we present is the standard model of turn based games of unbounded duration on finite graphs. For any positive integer n, let $[n] = \{1,\ldots,n\}$.

Definition 1 *Let $n \in \mathbb{N}$, $n > 1$. An n-player game arena is a directed graph $\mathcal{G} = (V_1,\ldots V_n, A, E)$, where V_i are finite sets of game positions with $V_i \cap V_j = \emptyset$ for $i \neq j$, $V = \bigcup_{i\in[n]} V_i$, A is a finite set of moves, and $E \subseteq (V \times A \times V)$ is the move relation that satisfies the following conditions:*

1. *For every $v, v_1, v_2 \in V$ and $a, b \in A$, if $(v,a,v_1) \in E$ and $(v,b,v_2) \in E$ then $a \neq b$.*
2. *For every $v \in V$, there exists $a \in A$ and $v' \in V$ such that $(v,a,v') \in E$.*

When an initial position $v_0 \in V$ is specified, we call $(\mathcal{G}, v_0)$ an initialised arena or just an arena.

In this model, we assume for convenience that the moves of all players are the same. When $v \in V_i$, we say that player i owns the vertex v. A game arena is thus a finite graph with nodes labelled by players and edges labelled by moves such that no two edges out of a vertex share a common label and there are no dead ends. For a vertex $v \in V$, let vE denote its set of neighbours: $vE = \{v' | (v,a,v') \in E$ for some $a \in A\}$. For $v \in V$ and $a \in A$, let $v[a] = \{v' | (v,a,v') \in E\}$; $v[a]$ is either empty or the singleton $\{v'\}$. In the latter case, we say a is enabled at v and write $v[a] = v'$. For $u \in A^*$, we can similarly speak of u being enabled at v and define $v[u]$ so that when $v[u] = \{v'\}$, there is a path in the graph from v to v' such that u is the sequence of move labels of edges along that path. Given $v \in V$ and $u \in A^*$, if any u-labelled path exists in the graph, it is unique. On the other hand, given any sequence of vertices that correspond to a path in the graph, there may be more than one sequence of moves that label that path.

A play in $(\mathcal{G}, v_0)$ is an infinite path $v_0 \xrightarrow{a_1} \ldots$, such that $v_i \xrightarrow{a_{i+1}} v_{i+1}$ for $i \in \mathbb{N}$. We often speak of $a_0 a_1 \ldots \in A^\omega$ as the play to denote this path. The game starts by placing a token at $v_0 \in V_i$. Player i chooses an action $a \in A$ enabled at v_0 and the token moves along the edge labelled a to a neighbouring vertex $v_1 \in V_j$. Player j chooses an action $a' \in A$ enabled at v_1, the token moves along the edge labelled a' to a neighbouring vertex and so on. Note that since there are no dead ends, any player whose turn it is to move has some available move.

Given a path $\rho = v \xrightarrow{a_0} v_1 \dots \xrightarrow{a_k} v_k$, we call $a_\ell, (1 \leq \ell \leq k)$ the last i-move in ρ, if $v_{\ell-1} \in V_i$ and for all $\ell' : \ell < \ell' \leq k, v_{\ell'} \notin V_i$.

2.1 Objectives

The game arena describes only legal plays, and the game itself is defined by specifying outcomes and players' preferences on outcomes. Since each play results in an outcome for each player, players' preferences are on plays. This can be specified finitely, as every infinite play on a finite graph settles down to a strongly connected component.

For a play $u \in A^\omega$ let $\inf(u)$ be the set of vertices that appear infinitely often in the play given by u. With each player i, we associate a total pre-order $\preceq_i \subseteq (2^V \times 2^V)$. This induces a total preorder on plays as follows: $u \preceq_i u'$ iff $\inf(u) \preceq_i \inf(u')$.

Thus an n-player game is given by a tuple $(\mathscr{G}, v_0, \preceq_1, \dots, \preceq_n)$, consisting of an n-player game arena and players' preferences.

2.2 Strategies

Players strategise to achieve desired outcomes. Formally, a strategy σ_i for player i is a partial function

$$\sigma_i : VA^* \rightharpoonup A$$

where $\sigma_i(vu)$ is defined if $v[u]$ is defined and $v[u] \in V_i$, and if $(v[u])[\sigma_i(vu)]$ is defined.

A strategy σ_i of player i is said to be bounded memory if there exists a finite state transducer FST $\mathscr{A}_\sigma = (M, \delta, g, m_0)$ where M (the 'memory' of the strategy) is a finite set of states, $m_0 \in M$ is the initial state of the memory, $\delta : A \times M \to M$ is the 'memory update' function, and $g : V_i \times M \to A$ is the 'move' function such that for all $v \in V_i$ and $m \in M$, $g(v,m)$ is enabled at v and the following condition holds: given $v \in V_i$, when $u = a_1 \dots a_k \in A^*$ is a partial play from v, $\sigma_i(vu)$ is defined, $\sigma_i(vu) = g(v[u], m_k)$, where m_k is determined by: $m_{i+1} = \delta(a_{i+1}, m_i)$ for $0 \leq i < k$.

A strategy is said to be memoryless or positional if M is a singleton. That is, the moves depend only on the current position.

Definition 2 *Given a strategy profile $\bar{\sigma} = (\sigma_1, \dots, \sigma_n)$ for n players let $\rho_{\bar{\sigma}}$ denote the unique play in $(\mathscr{G}, v_0)$ conforming to $\bar{\sigma}$. A profile $\bar{\sigma}$ is called a Nash equilibrium in $(\mathscr{G}, v_0, \prec_1, \dots, \prec_n)$ if for every player i and for every other strategy σ_i' of player i,* $\inf(\rho_{(\bar{\sigma}_{-i}, \sigma_i')}) \preceq_i \inf(\rho_{\bar{\sigma}})$.

3 Specification of Strategies

We now describe how the strategies of the imitator and optimiser types are specified.

3.1 Imitator Types

An imitator type is again specified by a finite state transducer which advises the imitator whom to imitate when using memory states for switching between imitating one player or another. When deciding not to imitate any other player, we assume that the type advises what to play using a memoryless strategy.

An imitator type τ_j for player j is a tuple $(M, \pi, \mu, \delta, m_0)$ where M is the finite set denoting the memory of the strategy, $m_0 \in M$ is the initial memory, $\delta : A \times M \to M$ is the memory update function,

$\pi : V \to A$ is a positional strategy such that for any $v \in V$, $\pi(v)$ is enabled at v, and $\mu : M \to [n]$ is the imitation map.

Given τ_j as above, define a strategy σ_j for player j as follows. Let $v \in V$ and $u = a_1 \dots a_k \in A^*$ is a partial play from v such that $v[u]$ is defined and $v[u] \in V_j$. Let $m_{i+1} = \delta(a_{i+1}, m_i)$ for $0 \le i < k$. Then $\sigma_j(vu) = a_\ell$, if a_ℓ is the last $\mu(m_k)$ move in the given play and a_ℓ is enabled at vu, and $\sigma_j(vu) = \pi(v[u])$, otherwise.

Note that the type specification only specifies whom to imitate, and how it decides whom to imitate but is silent on the rationale for imitating a player or switching from imitating x to imitating y. In general an imitator would have a set of observables, and based on observations of game states made during course of play, would decide on whom to imitate when. Thus imitator specifications could be given by a past-time formula in a simple propositional modal logic. With any such formula we can associate an imitation type transducer as defined above, so we do not pursue that approach here. See, for instance, [10] for more along that direction.

The following are some examples of imitating strategies that can be expressed using such automata:

1. Imitate player 1 for 3 moves and then keep imitating player 4 forever.
2. Imitate player 2 till she receives the highest payoff. Otherwise switch to imitating player 3.
3. Nondeterministically imitate player 4 or 5 forever.

For convenience of the subsequent technical analysis, we assume that an imitator type $\tau = (M, \pi, \mu, \delta, m_0)$ is presented as a finite state transducer $\mathscr{R}_\tau = (M', \delta', g', m_I)$ where

- $M' = V \times M \times A^{[n]}$.
- $\delta' : A \times M' \to M'$ such that $\delta'(a, \langle v, m, (a_1, \dots, a_n)\rangle) = \langle v', m', (a_1, \dots, a_{i-1}, a, a_{i+1}, \dots, a_n)\rangle$ such that $v \xrightarrow{a} v'$, $\delta(a, m) = m'$ and $v \in V_i$.
- $g' : V \times M' \to A$ such that $g'(v, \langle v, m, (a_1, \dots, a_n)\rangle) = a_i$ iff $\mu(m) = i$ and a_i is enabled at v. Otherwise $g'(v, \langle v, m, (a_1, \dots, a_n)\rangle) = \pi(v)$.
- $m_I = \langle v_0, m_0, (a_1, \dots, a_n)\rangle$ for some $(a_1, \dots, a_n) \in A^{|n|}$.

Figure 1 below depicts an imitator strategy where a player imitates player 1 for two moves and then player 2 for one move and then again player 1 for two moves and so on. She just plays the last move of the player she is currently imitating. Suppose there are a total of p actions, that is, $|A| = p$. She remembers the last move of the player she is imitating in the states m_1 to m_p, and when it is her turn to move, plays the corresponding action.

Given an FST $\mathscr{R}_\tau$ for an imitator type τ, we call a strongly connected component of $\mathscr{R}_\tau$ a subtype of $\mathscr{R}_\tau$. We will often refer to the strategy σ_j induced by the imitator type $\mathscr{R}_\tau$ for player j as $\mathscr{R}_\tau$, when the context is clear.

We define the notion of an imitation equilibrium which is a tuple of strategies for the optimisers such that none of the optimisers can do better by unilaterally deviating from it given that the imitators stick to their specifications.

Definition 3 *In the game $(\mathscr{G}, v_0, \prec_1, \dots, \prec_n)$, given that the imitators $r+1, \dots, n$ play strategies $\tau_{r+1}, \dots, \tau_n$, a profile of strategies $\bar{\sigma} = (\sigma_1, \dots, \sigma_r)$ of the optimisers is called an imitation equilibrium if for every optimiser i and for every other strategy σ'_i of i, $\inf(\rho_{(\bar{\sigma}_{-i}, \sigma'_i)}) \preceq_i \inf(\rho_{\bar{\sigma}})$.*

Remark Note that an imitation equilibrium $\bar{\sigma}$ may be quite different from a Nash equilibrium $\bar{\sigma}'$ of the game $(\mathscr{G}, v_0, \prec_1, \dots, \prec_n)$ when restricted to the first r components. In a Nash equilibrium the imitators

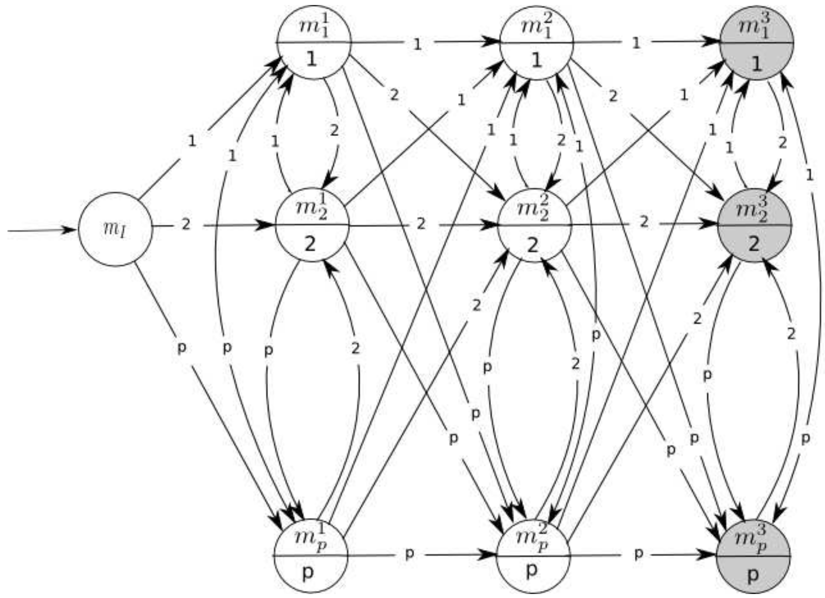

Figure 1: An imitator strategy

are not restricted to play according to the given specifications unlike in an imitation equilibrium. In the latter case, the optimisers, in certain situations, may be able to exploit these restrictions imposed on the imitators (as in the example of 'monkey-chess' discussed in Section 1).

3.2 Optimiser Specifications

One of the motivations for an imitator to imitate an optimiser is the fact that an optimiser plays to get best results. To an imitator, an optimiser appears to have the necessary resources to compute and play the best strategy and hence by imitating such a player she cannot be much worse off. But what kind of strategies do the optimisers play on their part?

In the next section, we show that if the optimisers know the types (the FSTs) of each of the imitators, then it suffices for them to play bounded memory strategies. Of course, this depends on the solution concept: Nash equilibrium is defined for strategy profiles, we need to particularize them for applying only to optimizers.

Thus in the treatment below, we consider only bounded memory strategies for the optimisers.

4 Results

In this section, we first show that it suffices to consider bounded memory strategies for the optimisers. Then we go on to address the questions raised towards the end of Section 1.

First we define a product operation between an arena and a bounded memory strategy.

4.1 Product Operation

Let $(\mathscr{G}, v_0)$ be an arena and σ be a bounded memory strategy given by the FST $\mathscr{A}_\sigma = (M, \delta, g, m_I)$. We define $\mathscr{G} \times \mathscr{A}_\sigma$ to be the graph $(\mathscr{G}', v'_0)$ where $\mathscr{G}' = (V', E')$ such that

- $V' = V \times M$
- $v'_0 = (v_0, m_0)$
-
  - If $g(v,m)$ is defined then $(v,m) \xrightarrow{a} (v',m')$ iff $\delta(a,m) = m'$, $v \xrightarrow{a} v'$ and $g(v,m) = a$.
  - If $g(v,m)$ is not defined then $(v,m) \xrightarrow{a} (v',m')$ iff $\delta(a,m) = m'$ and $v \xrightarrow{a} v'$

Proposition 1 *Let $(\mathscr{G}, v_0)$ be an arena and σ be a bounded memory strategy. Then $\mathscr{G} \times \mathscr{A}_\sigma$ is an arena, that is, there are no dead ends.*

Proof Let $(\mathscr{G}', v'_0) = \mathscr{G} \times \mathscr{A}_\sigma$. $\delta : A \times M \to M$ being a function, $\delta(a,m)$ is defined for every $a \in A$ and $m \in M$. Also by the definition of $\mathscr{G}$, for every vertex $v \in V$ there exists an action $a \in A$ enabled at v and a vertex $v' \in V$ such that $v \xrightarrow{a} v'$. Thus for every vertex $(v,m) \in V'$,

- if $g(v,m)$ is not defined then corresponding to every enabled action $a \in A$ there exists $(v',m') \in V'$ such that $(v,m) \xrightarrow{a} (v',m')$,
- if $g(v,m)$ is defined then by definition the unique action $a = g(v,m)$ is enabled at v. Hence, there exists $(v',m') \in V'$ such that $(v,m) \xrightarrow{a} (v',m')$.

□

Thus taking the product of the arena with a bounded memory strategy σ_i of player i does the following. For a vertex $v \in V_i$, it retains only the outgoing edge that is labelled with the action specified by the corresponding memory state of σ_i. For all other vertices $v \notin V_i$, it retains all the outgoing edges.

Proposition 2 *Let $(\mathscr{G}, v_0)$ be an arena and $\sigma_1, \dots, \sigma_n$ be bounded memory strategies. Then $\mathscr{G} \times \mathscr{A}_{\sigma_1} \times \dots \times \mathscr{A}_{\sigma_n}$ is an arena, that is, there are no dead ends.*

4.2 Equilibrium

Of the n players let the first r be optimisers and the rest $n-r$ be imitators. Let $\tau_{r+1}, \dots, \tau_n$ be the types of the imitators $r+1, \dots, n$. We transform the game $(\mathscr{G}, v_0, \prec_1, \dots, \prec_n)$ with n players to a game $(\mathscr{G}', v'_0, \prec'_1, \dots, \prec'_{r+1})$ with $r+1$ players in the following steps:

1. Construct the graph $(\mathscr{G}', v'_0) = ((V', E'), v'_0)$ as $\mathscr{G}' = \mathscr{G} \times \mathscr{R}_{\tau_{r+1}} \times \dots \times \mathscr{R}_{\tau_n}$.
2. Let $V' = V'_1 \cup \dots \cup V'_r \cup V'_{r+1}$ such that for $i : 1 \leq i \leq r$, $(v, m_1, \dots, m_n) \in V'_i$ iff $v \in V_i$. And $(v, m_1, \dots, m_n) \in V'_{r+1}$ iff $v \in V_{r+1} \cup \dots \cup V_n$. Let there be $r+1$ players such that the vertex set V_i belongs to player i. Thus we introduce a dummy player, the $r+1$th player, who owns all the vertices $(v, m_1, \dots, m_n) \in V'$ such that v was originally an imitator vertex in V. By construction, we know that every vertex $(v, m_1, \dots, m_n) \in V'_{r+1}$ has an unique outgoing edge $(v, m_1, \dots, m_n) \xrightarrow{a} (v', m'_1, \dots, m'_n)$. Thus the dummy player $r+1$ has no choice but to play this edge always. He has a unique strategy in the arena $\mathscr{G}'$: at every vertex of V'_{r+1}, play the unique outgoing edge.
3. Lift the preference orders of the players 1 to r to subsets of V' as follows. A subset W of V' corresponds to the Muller set $F(W) = \{v \mid (v, m_{r+1}, \dots, m_n) \in W\}$ of $\mathscr{G}$. For every player $i : 1 \leq i \leq r$, for $W, W' \subseteq V'$, $W \preceq'_i W'$ if and only if $F(W) \preceq_i F(W')$.

   Since the player $r+1$ has a unique strategy and plays it always, his preference ordering doesn't matter in the game. However, for consistency, we assign the preference of an arbitrary imitator (say imitator n) in the game $(\mathscr{G}, v_0, \prec_1, \dots, \prec_n)$ to the $r+1$th player in the game $(\mathscr{G}', v'_0, \prec'_1, \dots, \prec'_{r+1})$. That is, for $W, W' \subseteq V'$, $W \preceq'_{r+1} W'$ if and only if $F(W) \preceq_n F(W')$.

The game $(\mathscr{G}', v'_0, \prec'_1, \dots, \prec'_{r+1})$ is a turn based game with $r+1$ players (the optimisers and the dummy) such that each player i has a preference ordering $\preceq'_i$ over the Muller sets of V'. Such a game was called a generalised Muller game in [11].

Let L be the set

$$L = \{l \in (V' \cup \{\sharp\})^{|V'|+1} \mid |l|_\sharp = 1 \wedge \forall v \in V' \ (|l|_v = 1)\}$$

where $|l|_v$ denotes the number of occurences of v in l. We have

Theorem 1 ([11]) *The game $(\mathscr{G}', v'_0, \prec'_1, \dots, \prec'_{r+1})$ has a Nash equilibrium in bounded memory strategies, the memory being L.*

Now let $\bar{\sigma}' = (\sigma'_1, \dots, \sigma'_r, \sigma'_{r+1})$ be a Nash equilibrium tuple for $r+1$ players in the game $(\mathscr{G}', v'_0, \prec'_1, \dots, \prec'_{r+1})$. We now construct a bounded memory imitation equilibrium tuple $\bar{\sigma}$ for the r optimisers in the game $(\mathscr{G}, v_0, \prec_1, \dots, \prec_n)$.

For the optimiser $i : 1 \le i \le r$, let $\sigma'_i = (L, \delta', g', l'_I)$. Define $\sigma_i = (M, \delta, g, l_I)$ to a bounded memory strategy in the game $(\mathscr{G}, v_0, \prec_1, \dots, \prec_n)$ as

- $M = M_{r+1} \times \dots \times M_n \times L$ where $M_i,\ r+1 \le i \le n$ is the memory of strategy τ_i of imitator i.
- $\delta : A \times M \to M$ such that $\delta(a, \langle m_{r+1}, \dots, m_n, l\rangle) = \langle m'_{r+1}, \dots m'_n, \delta'(a,l)\rangle$ where $m'_i = \delta_i(a, m_i)$, $r+1 \le i \le n$ such that δ_i is the memory update of strategy τ_i.
- $g : V \times M \to A$ such that $g(v, \langle m_{r+1}, \dots, m_n, l\rangle) = g'(\langle v, m_{r+1}, \dots, m_n\rangle, l)$.
- $l_I = \langle m_I^{r+1}, \dots m_I^n, l'_I\rangle$ where $m_I^i,\ r+1 \le i \le n$ is the initial memory of strategy τ_i.

We then have:

Theorem 2 *$\bar{\sigma} = (\sigma_1, \dots, \sigma_r)$ is an imitation equilibrium in $(\mathscr{G}, v_0, \prec_1, \dots, \prec_n)$.*

Proof Suppose not and suppose player i has an incentive to deviate to a strategy μ in $(\mathscr{G}, v_0, \prec_1, \dots, \prec_n)$. Let $u \in A^\omega$ be the unique play consistent with the tuple $\bar{\sigma}$ where the imitators stick to their strategy tuple $(\tau_{r+1}, \dots, \tau_n)$. Let $u' \in A^\omega$ be the unique play consistent with the tuple $(\bar{\sigma}_{-i}, \mu)$ (that is when player i has deviated to the strategy μ) where again the imitators stick to their strategy tuple $(\tau_{r+1}, \dots, \tau_n)$. Let l be the first index such that $u(l) \neq u'(l)$. Then, $v_0[u_{l-1}] \in V_i$, (where u_{l-1} is the length $l-1$ prefix of u). That is, the vertex $v_0[u_{l-1}]$ belongs to optimiser i since everyone else sticks to her strategy.

Now consider what happens in the game $(\mathscr{G}', v'_0, \prec'_{r+1}, \dots, \prec'_n)$ when all the optimisers except i play the strategies $\sigma'_1, \dots, \sigma'_{i-1}, \dots, \sigma'_{i+1}, \dots, \sigma'_r$ and the imitators stick to their strategy tuple $(\tau_{r+1}, \dots, \tau_n)$. If the optimiser i mimicks strategy μ for $l-1$ moves in the game then the play is exactly u_{l-1} and reaches a vertex $(v, m_{r+1}, \dots, m_n) \in V'_i$ where $v = v_0[u_{l-1}]$. By construction of the product, all the actions enabled at v in the arena $\mathscr{G}$ are also enabled in the arena $\mathscr{G}'$. Hence the optimiser i can play $u(l)$. By similar arguments, optimiser i can mimick the strategy μ in the arena $\mathscr{G}'$ forever.

Thus by mimicking τ in the game $(\mathscr{G}', v'_0, \prec'_{r+1}, \dots, \prec'_n)$, the optimiser i can force a more preferable Muller set. But this contradicts the fact that $\bar{\sigma}'$ is an equilibrium tuple in the game $(\mathscr{G}', v'_0, \prec'_{r+1}, \dots, \prec'_n)$. $\square$

4.3 Stability

Finally, we adress the questions asked in Section 1. Given a game $(\mathscr{G}, v_0, \prec_1, \dots, \prec_n)$ with optimisers and imitators where the optimisers play bounded memory strategies and the imitators play imitative strategies specified by k finite state transducers we wish to find out:

- If a certain stongly connected component W of $\mathscr{G}$ is where the play eventually settles down to.
- What subtypes eventually survive.
- How worse-off is imitator i from an equilibrium outcome.

We have the following theorem:

Theorem 3 *Let $(\mathscr{G}, v_0, \prec_1, \dots, \prec_n)$ be a game with n players where the first r are optimisers playing bounded memory strategies $\sigma_1, \dots, \sigma_r$ and the rest $n-r$ are imitators playing imitative strategies $\tau_{r+1}, \dots, \tau_n$ where every such strategy is among k different types. Let W be a strongly connected component of $\mathscr{G}$. The following questions are decidable:*

(i) Does the game eventually settle down to W?

(ii) What subtypes of the k types eventually survive?

(iii) How worse-off is imitator i from an equilibrium outcome?

Proof Construct the arena $(\mathscr{G}', v'_0) = \mathscr{G} \times \mathscr{A}_{\sigma_1} \times \dots \times \mathscr{A}_{\sigma_r} \times \mathscr{R}_{\tau_{r+1}} \times \dots \times \mathscr{R}_{\tau_n}$.

(i) For the strongly connected component S in $(\mathscr{G}', v'_0)$ that is reachable from v'_0, let S be subgraph induced by the set $\{v \mid (v, m_1, \dots, m_n) \in S'\}$. Collapse the vertices of S that have the same name and call the resulting graph S''. Check if S'' is the same as W and output YES if so.

(ii) For the strongly connected component S in $(\mathscr{G}', v'_0)$ that is reachable from v'_0 do the following:
- For $i: r+1 \leq i \leq n$ take the restriction of S to the ith component for every $(v, m_1, \dots, m_n) \in S$. Let S_i denote this restriction.
- Collapse vertices with the same name in S_i. Let S'_i be this new graph.
- Check if S'_i is a subtype of σ_i. If so output S'_i.

(iii) Compute a Nash equilibrium $\bar{\mu}$ of the game $(\mathscr{G}, v_0, \prec_1, \dots, \prec_n)$ using the procedure described in [11]. Let S' be the reachable strongly connected component of the arena $(\mathscr{G}', v'_0)$. Restrict S' to the first component and call it S. Let $F = occ(S)$. Compare F with $\inf(\rho_{\bar{\mu}})$ according to the preference ordering $\preceq_i$ of imitator i.

□

4.4 An Example

Let us look at an example illustrating the concepts of the previous section. Consider 3 firms A, B and C. Each firm has a choice of producing 2 products, product a or product b repeatedly, i.e., potentially infinitely often. In every batch each of them can decide to produce either of the products.

Now firm A is a large firm with all the technical knowhow and infrastructure and it can change between its choice of production in consecutive batches without much increase in cost. On the other hand, the firms B and C are small. For either of them, if in any successive batch it decides to change from producing a to b or vice-versa, there is a high cost incurred in setting up the necessary infrastructure. Whereas, if it sticks to the product of the previous batch, the infrastructure cost is negligible. Thus in the case where it switches between products in consecutive batches, it is forced to set the price of its product high. This actually favours firm A as it can always set its product at a reasonable price since it is indifferent between producing either of the two products in any batch.

The demand in the market for a and b keeps changing. Firm A being the bigger firm has the resources and knowhow to analyse the market and anticipate the current demand and then produce a or b

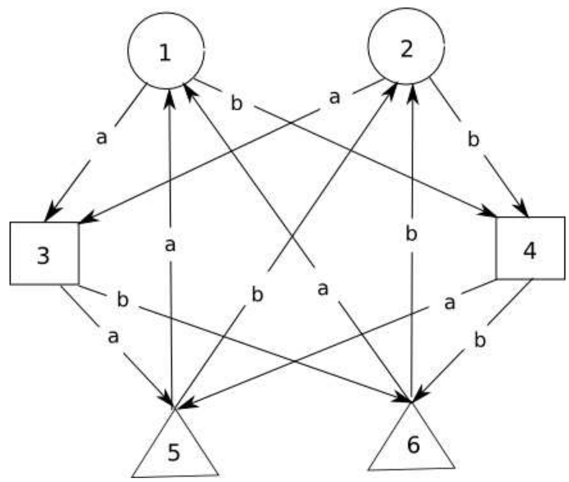

Figure 2: The arena $\mathscr{G}$

accordingly. Also assume that firm A is the first to put its product out in the market. Thus it is tempting for firms B and C to imitate A. But in doing so they run the risk of setting the prices of their products too high and incurring a loss.

We model this situation in the form of the arena $\mathscr{G}$ shown in Figure 2 where the nodes of firm A, B and C are denoted as $\bigcirc$, $\square$ and $\triangle$ respectively. The preferences of each of the firms for the relevant connected components when the market demand is low are given as:

$$\{1,2,3,4,5,6\} >_A X, \text{ for } X \subsetneq \{1,2,3,4,5,6\}$$

$$\{1,3,5\} >_B \{1,4,5\} >_B \{1,3,5,4\} >_B \{2,3,6,4\} >_B Y, \text{ for any other } Y \subsetneq \{1,2,3,4,5,6\}$$

$$\{1,3,5\} >_C \{1,4,5\} >_C \{2,3,6,4\} >_C \{1,3,5,4\} >_C Z, \text{ for any other } Z \subsetneq \{1,2,3,4,5,6\}$$

Thus firm A prefers the larger set $\{1,2,3,4,5\}$ to the smaller ones while B and C prefer the smaller sets. But when the market demand is high their preferences are given as:

$$\{1,2,3,4,5,6\} >_i X, \text{ for } X \subsetneq \{1,2,3,4,5,6\} \text{ and } i \in \{A,B,C\}$$

That is, all of them prefer the larger set.

Now if A produces a and b in alternate batches and B and C imitate A, then we end up in the component $\{1,2,3,4,5,6\}$ which is profitable for A but less so for B and C when the market demand is not so high. But when the demand is high, the component $\{1,2,3,4,5,6\}$ is quite profitable even for B and C and thus in this case, imitation is a viable strategy for them.

5 Discussion

The model that we have presented here is far from definitive, but we see these results as early reports in a larger programme of studying games with player types. The model requires modification and refinement in many directions, being addressed in related on-going work. In games with large number of players, outcomes are typically associated not with player profiles but with distribution of types in the population. Imitation crucially affects such dynamics. Our model can be easily modified to incorporate distributions but the analysis is considerably more complicated. Further, it is natural to consider this model in the context of repeated normal form games, but in such contexts almost-sure winning randomized strategies are more natural. A more critical notion required is that of type based reduction of games, so that analysis of large games can be reduced to that of interaction between player types.

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