Appendix A. Signal-to-Noise Ratio

The modulation of a feature across different strokes of a handwriting pattern can be considered as a signal. This signal can be estimated by stroke-wise averaging of the stroke features across a series of replications of a pattern. The patterns may need to be rescaled or normalized. The difference of each replication with respect to this average pattern can be considered as additive motor noise. A series of replications of a handwriting pattern allows the estimation of the signal-to-noise ratio (SNR), defined here as the ratio of the standard deviation of the modulation of the signal and the standard deviation of the noisy variations per stroke feature:

$$SNR = (Var(Signal) \ / \ Var(Noise))^{1/2} \qquad (1)$$

The SNR can be estimated as follows. Assume X_{ij} describes the data set of features for all strokes i (I strokes) and replications j (J replications). Averages across replications j are denoted as X_{i*}, etc. The analysis of variance schema for the main effect of strokes (I levels) and J replications yields:

$$Var(Noise) = [Sum\ j=1,J]\ [Sum\ i=1,I]\ \ (X_{ij} - X_{i*} - X_{*j} + X_{**})^2 \ / \ (I-1)(J-1) \qquad (2)$$

and

$$Var(Signal) = [Sum\ i=1,I]\ \ (X_{i*} - X_{**})^2 / (I-1) - \ Var(Noise) / J \qquad (3)$$

Var(Signal) is negative in rare cases, and should be set zero then. This dimensionless measure is useful when comparing invariances of patterns of different features.

References

Benecke, R., Rothwell, J.C., Day, B.L., Dick, J.P.R., & Marsden, C.D. (1986). Motor strategies involved in the performance of sequential movements. Experimental Brain Research, 63, 585-595.

Bernstein, N.A. (1967). The coordination and regulation of movements. London: Pergamon Press.

Cutting, J.E., & Kozlowski, L.T. (1977). Recognizing friends by their walk: Gait perception without familiarity cues. Bulletin of the Psychonomic Society, 9, 353-356.

Denier van der Gon, J.J., & Thuring, J.Ph. (1965). The guiding of human handwriting movements. Biological Cybernetics, 2, 145-148.

Edelman, S., Flash, T., & Ullman, S. (1990). Reading cursive handwriting by alignment of letter prototypes. International Journal of Computer Vision, 5, 303-331.

Gentner, D.R. (1987). Timing of skilled motor performance: Tests of the proportional duration model. Psychological Review, 94, 255-276.

Guberman, Sh.A., & Rozentsveig, V.V. (1976). Algorithm for the recognition of handwritten text. Avtomatika i Telemekhanika, 5, 122-129.

Heuer, H., (1984). On the re-scalability of force and time in aiming movements. Psychological Research, 466, 73-86.

Heuer, H., (1988a). Testing the invariance of relative timing: Comment on Gentner (1987). Psychological Review, 95, 552-557.

Heuer, H., (1988b). Adjustment and readjustment of the relative timing of a motor pattern. Psychological Research, 50, 83-93.

Heuer, H., & Schmidt, R.A. (1988). Transfer of learning among motor patterns with different relative timing. Journal of Experimental PSychology: Human Perception and Performance, 14, 241-252.

Ivry, R.B. (1986). Force and timing components of the motor program. Journal of Motor Behavior, 18, 449-474.

Keele, S.W. (1981). Behavioral analysis of movement. In V.B. Brooks (Ed.), Handbook of Physiology (Vol. 2, pp. 1391-1414). Bethesda, MD: American Physiological Society.

Keele, S.W., Ivry, R.I., & Pokorny, R.A. (1987). Force control and its relation to timing. Journal of Motor Behavior, 19, 96-114.

Keele, S.W., & Posner, M.I. (1968). Processing of visual feedback in rapid movements. Journal of Experimental Psychology, 77, 155-158.

Keele, S.W., & Summers, J.J. (1976). The structure of motor programs. In G.E. Stelmach (Ed.), Motor control: Issues and trends (pp. 109-142). New York: Academic.

Lacquaniti, F. (1989). Central representations of human limb movement as revealed by studies of drawing and handwriting. Trends in Neuroscience, 12, 287-291.

Lacquaniti, F., Terzuolo, C., & Viviani, P. (1983). The law relating kinematic and figural aspects of drawing movements. Acta Psychologica, 54, 115-130.

Langolf, G.D., Chaffin, D.B., & Foulke, J.A. (1976). An investigation of Fitts' law using a wide range of movement amplitudes. Journal of Motor Behavior, 8, 113-128.

Maarse, F.J., Thomassen, A.J.W.M. (1983). Produced and perceived writing slant: difference between up and down strokes. Acta Psychologica, 54, 131-147.

Meulenbroek, R.G.J., & Van Galen, G.P. (1989). The production of connecting strokes in cursive writing: Developing co-articulation in 8 to 12 year-old children. In Plamondon, R., Suen, C.Y., & Simner, M. (Eds.), Computer recognition and human production of handwriting (pp. 273-286). Singapore: World Scientific.

Moore, S.P., & Marteniuk, R.G. (1986). Kinematic and electromyographic changes that occur as a function of learning a time-constrained aiming task. Journal of Motor Behavior, 18, 397-426.

Newell, K.M., Carlton, L.G., & Carlton, M.J. (1982). The relationship of impulse to response timing error. Journal of Motor Behavior, 14, 24-45.

Pick, H.L., Jr., & Teulings, H.L. (1983). Geometric transformations of handwriting as a function of instruction and feedback. Acta Psychologica, 54, 327-340.

Povel, D.J. (1981). The internal representation of simple internal patterns. Journal of Experimental Psychology, 7, 3-18.

Povel, D.J., & Collard, R. (1982). Structural factors in patterned finger tapping. Acta Psychologica, 52, 107-123.

Rosenbaum, D.A., Inhof. A.W., & Gordon, A.M. (1984). Choosing between movement sequences: A hierarchical editor model. Journal of experimental Psychology, 113, 373-393.

Schmidt, R.A. (1975). A schema theory of discrete motor learning. Psychological Review, 82, 225-260.

Schmidt, R.A. (1985). The search for invariance in skilled movement behavior. Research Quarterly

for Exercise and Sport, 52, 188-200.

Schmidt, R.A., Zelaznik, H., Hawkins, B., Frank, J.S., & Quinn, J.T. (1979). Motor-output variability: A theory for the accuracy of rapid motor acts. Psychological Review, 86, 415-451.

Schomaker, L.R.B., & Teulings, H.L. (1990). A handwriting recognition system based on properties of the human motor system. In C.Y. Suen (Ed.), *Frontiers in handwriting recognition* (pp. 195-209). Montreal: CENPARMI. ISBN: 1-895193-00-1.

Schomaker, L.R.B., Thomassen, A.J.W.M., & Teulings, H.L. (1989). A computational model of cursive handwriting. In R. Plamondon, C.Y. Suen, & M. Simner (Eds.), *Computer recognition and human production of handwriting* (pp. 153-177). Singapore: World Scientific.

Schomaker, L.R.B., & Teulings, H.L. (1991). Stroke- versus character-based recognition of on-line, connected cursive script. 2nd International Workshop on Frontiers in Handwriting Recognition, Bonas, France, September 24-27, 1991.

Semjen, A., & Aiguier, J. (1991). Timing of serial movements. Presented at Time, action and cognition, Saint-Malo: Oct 22-25.

Shapiro, D.C., Zernicke, R.F., Gregor, R.J., & Distel, J.D. (1981). Evidence for generalized motor programs using gait pattern analysis. Journal of Motor Behavior, 13, 33-47.

Smyth, M.M., & Silvers, G. (1987). Functions of vision in the control of handwriting. Acta Psychologica, 65, 47-64.

Sternberg, S., Knoll, R.L., & Turock, D.L. (1990). Hierarchical control in the execution of action sequences: tests of two invariance properties. In M. Jeannerod (Ed.) Attention and Performance XIII: Motor representation and control. Hillsdale, NJ: Lawrence Erlbaum, 3-55.

Sternberg, S., Knoll, R.L., Monsell, S., & Wright, C.E. (1978). The latency and duration of rapid movement sequences: Comparison of speech and typewriting. In G.E. Stelmach (Ed.), Information processing in motor control and learning (pp. 117-152). New York: Academic.

Stockholm, E. (1979). Recognition of a writer as a function of his method of writing. Perception and Motor Skills, 483-488.

Teulings, H.L., & Schomaker, L.R.B. (1991). Unsupervised learning of prototype allographs in cursive-script recognition using invariant handwriting features. 2nd International Workshop on Frontiers in Handwriting Recognition, Bonas, France, September 24-27, 1991.

Teulings, H.L., Thomassen, A.J.W.M., & Van Galen, G.P. (1983). Preparation of partly precued handwriting. Acta Psychologica, 54, 165-177.

Teulings, H.L., Thomassen, A.J.W.M., & Maarse, F.J. (1989). A description of handwriting in terms of main axes. In R. Plamondon, C.Y. Suen, & M. Simner (Eds.), Computer recognition and human production of handwriting (pp. 193-211). Singapore: World Scientific.

Teulings, H.L., Thomassen, A.J.W.M., & Van Galen, G.P. (1986). Invariants in handwriting: The information contained in a motor program. In H.S.R. Kao, G.P. Van Galen, & R. Hoosain (Eds.), *Graphonomics*: *Contemporary research in handwriting* (pp. 305-315). Amsterdam: North-Holland.

Thomassen, A.J.W.M., & Teulings, H.L. (1985). Time, size and shape in handwriting: Exploring spatio-temporal relationships at different levels. In J.A. Michon & J. Jackson (Eds.), Time, Mind and Behavior (pp. 253-263). Berlin: Springer.

Van Der Plaats, R.E., & Galen, G.P. van (1991). Allographic variability in adult handwriting. Human Movement Science, 10, 291-300.

Van Emmerik, R.E., Newell, K.M. (1990), The influence of task and organismic constraints on intralimb and pen-point kinematics in a drawing task. Acta Psychologica, 73, 171-190.

Van Galen, G.P. (1991). Handwriting: Issues for a psychomotor theory. Human Movement Science, 10, 165-191.

Van Galen, G.P., & Teulings, H.L. (1983). The independent monitoring of form and scale parameters in handwriting. Acta Psychologica, 54, 9-22.

Viviani, C.A., Terzuolo, V. (1980). Space-time invariance in learned motor skills. In G.E. Stelmach & J. Requin, Tutorials in motor behavior (pp. 525-533). Amsterdam: North-Holland.

Vorberg, D., & Hambuch, R. (1978). On the temporal control of rhythmic performance. In J. Requin (Ed.), Attention and Performance VII (pp. 535-555). Hillsdale, NJ: Lawrence Erlbaum.

Wann J.P., & Jones J.G. (1986). Space-time invariance in handwriting: Contrasts between primary-school children displaying advanced or retarded handwriting acquisition Human Movement Science, 5, 275-296.

Wann, J.P., & Nimmo-Smith, I. (1990). Evidence against the relative invariance of timing in handwriting. Quarterly Journal of Experimental Psychology Human Experimental Psychology, 42, 105-119.

Wann, J.P., Nimmo-Smith, I., & Wing, A.M. (1988). Relation between velocity and curvature in movement: Equivalence and divergence between a power law and a minimum-jerk model. Journal of Experimental Psychology: Human Perception and Performance, 14, 622-637.

Wing, A.M. (1978). Response timing in handwriting. In G.E. Stelmach (Ed.), Information processing in motor control and learning (pp. 469-486). New York: Academic.

Wing, A.M., & Kristofferson, A.B. (1973). The timing of interresponse intervals. Perception and Psychophysics, 13, 455-460.

Wing, A.M., Lewis, V.J., & Baddeley, A.D. (1979). The slowing of handwriting by letter repetition. Journal of Human Movement Studies, 5, 182-188.

Wing, A.M., Nimmo-Smith, I., & Eldridge, M.A. (1983). The consistency of cursive letter formation as a function of position in the word. Acta Psychologica, 54, 197-204.

Wright, C.E. (1990). Generalized motor programs: reexamining claims of effector independence in writing. In M. Jeannerod (Ed.), Attention and Performance XIII (pp. 294-320). Hillsdale, NJ: Erlbaum.

Wright, C.E. (1991). Temporal invariance in handwriting. 5th Handwriting conference of the IGS (pp. 86-89), Oct, 1991, Tempe, USA. ISBN: 0-9630246-0-4.

Wright, Ph.T. (1988). Design and implementation of a handwriting recognizer. Plessey New Technology, 2, 1988.

Zelaznik, H.N., Schmidt, R.A., & Gielen, C.C.A.M. (1986). Kinematic properties of rapid aimed hand movements. Journal of Motor Behavior, 18, 353-372.

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Table 1

An example of the SNRs and the between-parameter correlations r, in the normal condition and the correlations between the normal and the fast condition, all based upon the down strokes in one subject for the parameters of the mechanical equation of vertical stroke size dy, peak force ay, duration dt, and force efficiency effy.

Mechanical Equation	dy =	effy ×	ay ×	dt^2
SNR	5.0	2.6	2.1	4.2
Between Parameter-Correlations				
r(dy,effy), r(effy,ay), r(ay,dt^2)	+0.13	-0.56	-0.04	
r(dy,ay), r(effy,dt^2)		+0.43	-0.36	
r(dy,dt^2)			+0.46	
Between-Condition Correlations				
r(normal,fast)	0.99	0.96	0.95	0.97

Table 2

An example of the SNRs and the between-parameter correlations r, in the normal condition and the correlations between the normal and the fast condition, in one subject for the parameters of the mechanical equation of the net vertical displacement of a stroke pair dy_{12}, the vertical displacement of a downstroke dy_1, and of its successive upstroke dy_2.

Mechanical Equation	dy_{12} =	dy_1 +	dy_2
SNR	4.2	5.0	3.8
Between Parameter-Correlations			
$r(dy_{12},dy_1)$, $r(dy_1,dy_2)$	+0.46	-0.47	
$r(dy_{12},dy_2)$		+0.10	
Between Condition-Correlations			
r(normal,fast)	0.98	0.99	0.99

Figure Captions

Figure 1a and 1b. Two replications of a handwriting pattern together with several time functions in calibrated scales: x, y (horizontal and vertical coordinates), vx, vy (horizontal and vertical velocities), v (absolute velocity), ax and ay (horizontal and vertical accelerations). Circles indicate the stroke segmentations on the basis of relative minima of v. Dotted traces refer to movements above the paper. The top-down hierarchy says that (vertical) stroke sizes are more invariant than stroke durations or vertical-force peaks (derived from ay). The sequence hierarchy says that the motor noise of stroke size is immediately corrected in the next stroke but the noise of stroke duration is not.