

## **Invariant properties between stroke features in handwriting**

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### **Abstract**

A handwriting pattern is considered as a sequence of ballistic strokes. Replications of a pattern may be generated from a single, higher-level memory representation, acting as a motor program. Therefore, those stroke features which show the most invariant pattern are probably related to the parameters of the higher-level representation, whereas the more noisy features are probably related to the parameters derived at the lower levels (top-down hierarchy). This hierarchy of invariances can be revealed by the signal-to-noise ratio (SNR), the between-parameter correlations, and the between-condition correlations. Similarly, at the higher level a sequence of strokes may act as a unit from which individual strokes are derived (sequence hierarchy). This hierarchy of invariances can be revealed by the between-stroke correlation, which forms a weaker criterion than rescalability, which has been rejected mostly. Previous research showed that vertical stroke size has higher SNRs and higher between-condition correlations than stroke duration or peak force, whereas the latter two features were also negatively correlated. This suggested that vertical stroke size is a higher-level parameter than the other two. The present research largely confirmed this top-down-hierarchy and even for upstrokes and downstrokes separately. Downstrokes were more invariant than upstrokes in terms of vertical stroke size. However, contrary to the vertical stroke size, the horizontal stroke size was not invariant. Both vertical and horizontal sizes showed substantial between-stroke correlations. In contrast, the stroke durations did not show any between-stroke correlations. This suggests that stroke segmentation is reliable in spite of the discrete sampling of the handwriting movements.

## Introduction

Cursive handwriting is rather irregular, which forms the major difficulty in segmenting and recognizing on-line cursive script (Schomaker & Teulings, 1990; Teulings & Schomaker, 1991). Not only do writers have different styles of handwriting, but also a single writer produces the letters in many different ways. Differing contexts or conditions may cause systematic variations in terms of stroke sizes and durations (Thomassen & Schomaker, 1986) or allograph choices (Wing et al., 1983; Van der Plaats & Van Galen, 1991). But even letters produced by a single writer under identical conditions vary slightly. This paper deals with this type of motor noise. It is supposed that replications of a pattern in various conditions may be generated from a single, higher-level memory representation, acting as a motor program. A motor program should be understood here as a prescription of a series of abstract movement parameters which sufficiently describe the movement (e.g., Schmidt, 1975; Keele, 1981). The higher-level, abstract movement representation, which can be executed in different conditions and with different limbs, e.g., at different speeds or sizes, is referred to as the notion of "motor equivalence" (e.g., Bernstein, 1967). Therefore, those stroke features which show the most invariant pattern are probably related to the parameters of the higher-level representation, whereas the more noisy features are probably related to the parameters derived at the lower levels. Notice the terminology that a feature can be measured from the writing movement, whereas a parameter refers to a feature's possible internal representation at some level of the motor system.

The final aim of the present research is to identify the parameters stored in the motor program. We suppose that movements have been stored in terms of a parsimonious set of parameters, whereas most other parameters, which may need to be adapted for each replication, are derived at the lower levels of the motor system. Therefore, each time when the same writing pattern is executed, a similar pattern of stroke features is generated. Conversely, if the pattern of a specific stroke feature appears to be highly invariant between replications, it is reasonable to suppose that this stroke feature is related to a parameter stored in the motor program. The reversal of this conclusion is allowed as the other two thinkable reasons for invariance of the pattern of features may be ruled out. Firstly, invariances may be caused when pushing the motor system to hard limits. For example, invariances in joint kinematics may appear when making maximally large movements (Van Emmerik & Newell, 1990). However, normal handwriting does not challenge any hard limits as it can always be performed slightly larger or smaller, faster or slower. Secondly, feedback might allow that some visual

representation is copied, which has nothing to do with the reproducing a motor representation. However, handwriting strokes with durations of 100 ms (e.g., Smyth & Silvers, 1987) cannot be produced under constant visual-feedback with much longer correction times (in arm movements) of the order of 190 - 260 ms (Keele & Posner, 1968). Also proprioceptive feedback is probably too slow to become effective during fast handwriting. The slowness of any feedback can be illustrated by the observation that when the pen-to-paper friction is suddenly increased or decreased, the correction of the distorted stroke sizes is realized only after producing several strokes (Denier van der Gon & Thuring, 1965). Therefore, the rationale seems valid that invariant features observed in replications of a handwriting pattern originate from the movement parameters stored in the motor program, whereas the more varying ones are related to parameters derived at a lower level.

Invariant features in handwriting have frequently been mentioned by movement researchers. Most notable are the observations that handwriting is personal (e.g., Stockholm, 1979), and that handwriting is remarkably invariant when performed by different effectors (e.g., Wright, 1990). Personal movement patterns have also been found in gait and posture (e.g., Cutting & Kozlowski, 1977). Schmidt et al. (1979) state that the temporal relationships among various muscle contractions as well as the relative force amplitudes may be fundamental invariants. According to their impulse-variability model both variability of duration and peak force contribute to the impulse-variability of movement distance. Similarly, Ivry (1986) suggested that timing and force and also force activation and force deactivation may form parameters of the motor program.

In order to compare the extent of invariance of stroke size, timing and force, Teulings et al. (1986) employed a dimensionless measure of invariance, the signal-to-noise ratio (SNR). The SNR is the ratio of the standard deviation of the average pattern (i.e., the signal) and the standard deviation of the noisy deviations. Each replication can be described as the average (and normalized) pattern plus noisy deviations per stroke (See Appendix A). Although a rate parameter for the normalization may not be reliable (e.g., Gentner, 1987; Heuer, 1988) we expect little problems if the corrections are small. In Teulings et al. (1986) it appeared that the pattern of vertical stroke sizes was more invariant than the pattern of stroke durations or peak forces. Also other criteria, such as the high between-condition correlations and the negative between-parameter correlations (between duration and peak force) supported that the vertical stroke sizes form the highest-level movement information, whereas the patterns of durations and peak forces are probably derived at the lower levels from stroke size (and probably shape)

(e.g., Lacquaniti et al., 1983; Thomassen & Teulings, 1985). Moore & Marteniuk (1986) came to a similar conclusion on the basis of kinematic and EMG data of single-phasic fore-arm extensions: the movement planning hierarchy was concerned with the spatial domain, whereas the amplitude and timing of muscular activity were planned at a lower level.

Apart from verifying the previous result that vertical size is more invariant than duration or vertical peak force, this study intends to investigate this also for the horizontal dimension and for the up and the downstrokes separately. The hierarchy of the higher-level, invariant parameters over lower-level, more varying parameters can be called top-down hierarchy. However, there exists also something that can be called a sequence hierarchy (e.g., Rosenbaum et al., 1984; Sternberg et al., 1990; Povel & Collard, 1982; Van Galen, 1991). A sequence hierarchy implies that a sequence of strokes may act as a unit at the higher level from which individual strokes are derived. The strokes, belonging to the same hierarchical unit, show significant correlations (e.g., Benecke et al., 1986), or at least a smaller variability between land marks (e.g., key taps) within a hierarchical unit (Vorberg & Hambuch, 1978). Wing (1978) found in zig-zag writing patterns "v", "u", "w", or "m", in a simple reaction-time paradigm, indeed positive correlations between up-down stroke pairs, indicating that stroke pairs rather than single strokes form action units.

Lacquaniti (1989) suggested that a sequence of independent, curved strokes or loops may be sufficient to construct all writing patterns. This may seem reasonable if one realizes that loops belonging to different letters of someone's handwriting, e.g., "h" and "l", are similar. However, Edelman et al. (1990) pointed out that individual stroke pairs are not scaled independently as otherwise atypical letters may result. In fact, there is evidence that complete letters, consisting of several strokes, form units. Teulings et al. (1983) found that letter pairs consisting of identical letters (e.g., "ee" or "nn") seemed to be retrieved faster than letter pairs, consisting of different letters. It did not play a role whether the letter pairs consisted of similarly curved strokes (e.g., "eu" or "jn") or of differently curved strokes (e.g., "en", "ju"). Furthermore, only the sequences of identical letter units show the peculiar slowing of execution speed (Wing et al., 1979; Sternberg et al., 1978). Therefore, the experiment also examines whether each stroke varies independently, or whether the spontaneous variation of one stroke is compensated in the next stroke, e.g., because the sequence of strokes belonging to a complete allograph varies as a unit.

## Experiment

An experiment was conducted to verify that stroke length is an invariant, higher-level parameter, whereas stroke duration, and peak force are more varying, lower-level parameters (top-down hierarchy). Previous research supported this for the vertical dimension (Teulings et al., 1986). The present experiment intends to verify this for the horizontal dimension and for the up and downstrokes separately. Furthermore, this experiment was conducted to examine whether these parameters are controlled independently stroke-by-stroke or hierarchically for a sequence of strokes (sequence hierarchy). In the experiment, subjects wrote a single handwriting pattern several times under normal, faster, and slower than normal conditions. Four criteria for identifying the hierarchy of invariant movement parameters were used:

### (1) Signal-to-noise ratios per feature and condition

The signal-to-noise ratio (SNR) of a feature can be estimated in a series of replications of a single handwriting pattern (Teulings et al., 1986). The SNR of a feature is defined here as the standard deviation across the pattern (i.e., the signal) divided by the standard deviation of the additional noise per replication (See Appendix A). Note that the SNR does not depend upon the movement amplitude.

### (2) Between-parameter correlations

In order to execute a movement, described by a parsimonious set of higher-level parameters, several lower-level parameters need to be derived by the motor system. Many combinations of the lower-level parameters may satisfy the same higher-level parameters, which appear as degrees of freedom. For example, two lower-level parameters (e.g., A, and B) may be derived from a single, higher-level parameter (e.g., C), satisfying one of the mechanical equations  $A + B = C$  or  $A \times B = C$ . If A and B have covariance  $\text{cov}(A,B)$  then

$$\text{var}(A) + \text{var}(B) + \text{cov}(A,B) = \text{var}(C)$$

or

$$\text{var}(A) / \text{mean}(A)^2 + \text{var}(B) / \text{mean}(B)^2 + \text{cov}(A,B) / \text{mean}(A) \times \text{mean}(B) \approx \text{var}(C) / \text{mean}(C)^2,$$

respectively. Therefore, if  $\text{cov}(A,B)$  is sufficiently negative, the variance of  $C$  can be even significantly smaller than those of  $A$  and  $B$ . At any rate, a negative correlation between parameters  $A$  and  $B$  indicates that  $C$  is a higher-level parameter, controlling the lower-level parameters  $A$  and  $B$  (Teulings et al., 1986). Note that the between-parameter correlation is distinct from any empirical relation between  $A$  and  $B$  when they are varied across a wide range. Here we focus on virtually exact replications.

### (3) Between-condition correlations

It is supposed that a single higher-level movement representation is used to perform a movement pattern under a variety of execution conditions (e.g., different speed, size, limb, orientation, or slant). As speed-induced sloppiness is a major problem in cursive-script recognition, instructed writing speed will be manipulated in the present experiment. Similarly as proposed before, the stroke pattern in terms of features related to motor program parameters will be highly invariant. Conversely, if the stroke pattern of a feature appears to correlate accurately between conditions, this stroke feature may be related to a higher-level movement parameter (Teulings et al., 1986). In fact, the correlation is a less strict criterion than rescalability of the whole pattern or even of a few strokes only.

### (4) Between-stroke correlations

The previous criteria referred to the top-down hierarchy. The between-stroke correlation (i.e., between successive strokes) of a particular movement feature allows to quantify also its sequence hierarchy. For example, is stroke size varying independently per stroke or are the strokes of a whole allograph varying as a higher-level unit? The between-stroke correlation of duration allows also a check whether handwriting has been sufficiently accurately segmented into discrete strokes at the absolute-velocity minima, which normally coincide with the moments of the sharp changes of trajectory direction (e.g., Lacquaniti et al., 1983) (See Figure 1). However, at usual sampling rates of 100 Hz, it may seem that time resolution (i.e., 10 ms or 1/10th of a fast stroke) is worse than the digitizer's position accuracy which is optimally 0.025 mm or 1/100th of a short stroke. This might cause that the stroke durations seem more varying than the stroke sizes, solely due to the greater measuring error of stroke duration. In order to minimize this artifact, segmentation points will be estimated more precisely by quadratic interpolation of the absolute velocity curve. Nevertheless, the extent of this effect will be tested post hoc, using Wing & Kristofferson's (1973) model, where an (isochronous)

time keeper generates the motor commands for the finger taps. They assumed that each tap is produced after a transmission delay which varied at random with a certain variance. This variance became manifest as a negative correlation between successive inter-tap intervals, equal to minus twice the motor-delay variance. Therefore, any substantial measuring variability should cause negative correlations also between successive, nonequal stroke durations. The statistical absence of a negative lag-one correlation would indicate that the error of stroke segmentation is small relative to the timing variability of the movement pattern.

## **Methods**

### Subjects

Four male, right-handed subjects of ages between 20 and 40 years took part of the experiment. They were used to writing cursive-script or handprint.

### Apparatus

The handwriting movements were captured by sampling  $(x(i), y(i))$  of the pen-tip isochronously and simultaneously, at a frequency of 105 Hz using a CalComp 9600 digitizer with a worst-case RMS accuracy of 0.2 mm.

### Conditions

Subjects wrote the pattern "elementary" according to their preferred handwriting style, position, and speed after hearing the start beep. The orientation of the writing was about horizontal. The first 4 to 8 seconds of handwriting were recorded. Incorrect trials were repeated. 16 Trails were done. Then the subject was asked to perform another 16 trials but at a higher speed than normal. Finally, the subject was asked to perform another 16 trials but now at a lower speed. These are common conditions in cursive-script recognition.

### Analysis

The writing patterns were lowpass filtered, with a sinusoidal transition band from 10 to 37 Hz, and differentiated. The movement was segmented into ballistic strokes by the local minima of the absolute-velocity time curve, using quadratic interpolation through the velocity

samples before and after the minimum sample in order to simulate a higher time resolution than the sampling frequency. See Figure 1 for examples of two patterns. In fact, for the segmentation, the vertical velocity component was weighted twice relative to the horizontal one so that segmentation occurs primarily at the tops and at the bottoms in the writing pattern. We do not expect this would significantly favor the vertical dimension. Furthermore, short strokes of less than 40 ms or 0.5 mm were concatenated in order not to segment into more strokes at hesitations. Then the pattern was rotated such that the least-squares line through the bottom segmentation points of the pattern was horizontal. The first 2 strokes and the last few strokes of the pattern were omitted so that all patterns had 28 strokes. The following features were estimated per stroke: The horizontal and vertical stroke sizes ( $dx$ ,  $dy$ ), stroke duration ( $dt$ ), peak forces in horizontal and vertical directions (assumed proportional to peak accelerations,  $ax$ ,  $ay$ ), and their force efficiencies ( $effx$ ,  $effy$ ). The force efficiency  $effx$  is the parameter which fits  $dx = effx \times ax \times dt^2$ , and similarly for  $effy$ . The mechanical equations to be examined are:

$$\begin{aligned}
 dy &= effy \times ay \times dt^2 \\
 dx &= effx \times ax \times dt^2 \\
 dy_{12} &= dy_1 + dy_2 \\
 dx_{12} &= dx_1 + dx_2 \\
 dt_{12} &= dt_1 + dt_2
 \end{aligned}$$

Here  $dy_1$  and  $dy_2$  refer to the vertical displacements of two successive strokes, and analogously for  $dx$  and  $dt$ . Upstrokes and downstrokes (or up-down or down-up pairs) were analysed separately in order to prevent confounding with the trivial difference between them.

————— Insert Figure 1 —————

The trials were "homogenized" by aligning and removing outliers. First, the most representative, normal trial was identified by the trial having the smallest average rank distance from the median pattern of horizontal and vertical stroke sizes and stroke directions. Aligning was necessary as some trials had an additional or missing first stroke. The required shift per pattern could easily be estimated by the location of the peak cross correlation between the patterns of vertical stroke sizes and that of the most representative pattern. The peak cross correlation should be at least 0.8. If less than a third of the trials per condition remained, the next representative trial was taken. Trials with an outlying stroke height, width,



or direction relative to the most representative trial were removed. Outlying strokes have a z score larger than 3, relative to the most representative normal-condition trial. If less than a third of the trials per condition satisfied these criteria, both the outlier and the correlation criterion were weakened by factors 1.4 and 0.95, respectively.

## Results

Table 1 shows an example of the SNRs and the between-parameter correlations, in the normal condition and the correlations between the normal and the fast conditions based upon the downstrokes in one subject for the features of the mechanical equation  $dy = \text{effy} \times ay \times dt^2$ . In order to appreciate the effects one should realize that the differences between subjects are large and that the data are non-normally distributed. As we are only interested in differences between data within subjects, two-tailed sign tests were performed on the 24 independent comparisons of SNRs and correlations. Namely, each of the three conditions (normal, fast, slow), each of the four subjects, and the up and the downstrokes, represent 24 independent, paired statistics per feature. However, only the 16 between-condition correlations with the normal condition are independent and the 12 comparisons between up and downstrokes.

In nearly all comparisons it appears that the vertical stroke size  $dy$ , shows a higher SNR than any of the other features  $\text{effy}$ ,  $ay$ , or  $dt$  ( $N=24$ ,  $k \leq 1$ ,  $p < 0.001$ ). Furthermore, the between-condition correlations between the normal condition and the slow or the fast conditions were higher for  $dy$  than for the other features ( $N=16$ ,  $k \leq 3$ ,  $p < 0.05$ ). The between-parameter correlations in the right-hand term, show the consistent pattern of negative mutual correlations ( $N=24$ ,  $k \leq 2$ ,  $p < 0.001$ ), although the correlation between,  $ay$  and  $dt^2$  failed to be significantly negative ( $N=24$ ,  $k=8$ ,  $p > 0.05$ ). Apart from this, the results largely replicate the previous findings (Teulings et al., 1986) and suggest that vertical stroke size  $dy$  is probably related to a higher-level parameter, whereas  $ay$  and  $dt$  are probably related to the parameters derived at the lower levels (top-down hierarchy).

————— Insert Table 1 —————

The data for the horizontal component,  $dx = \text{effx} \times ax \times dt^2$ , do not show such a clear picture as most differences do not reach significance ( $N=24$ ,  $k \geq 7$ ,  $p > 0.05$ ). Only the SNR of  $dx$  is higher than the SNR of  $\text{effx}$  ( $N=24$ ,  $k=6$ ,  $p < 0.05$ ). It seems that horizontal stroke size  $dx$

is not a predominant higher-level movement parameter.

Table 2 shows a similar representation as Table 1 but now with the parameters of the equation of the net vertical displacement of two successive strokes, i.e.,  $dy_{12} = dy_1 + dy_2$ , where  $dy_1$  and  $dy_2$  are the vertical displacements of the downstroke and the successive upstroke, having opposed signs. Successive vertical stroke displacements are negatively correlated ( $N=24, k \leq 2, p < 0.001$ ), and the SNRs of  $dy_{12}$  are always smaller than the greatest of the SNRs of  $dy_1$  and  $dy_2$  ( $N=24, k \leq 3, p < 0.001$ ). Therefore, it seems that if  $dy_1$  happens to overshoot upward,  $dy_2$  will also overshoot but then downward (i.e., in opposite direction), such that the baseline position and the x-height are relatively invariant. The down-up pairs showed similar results as the up-down pairs in terms of  $dy$  and also in terms of  $dx$  and  $dt$ , suggesting that the effects are not restricted within letters as the letter boundaries are most often at the down-up pairs. Interestingly,  $dy$  of the downstrokes were more invariant than  $dy$  of the upstrokes ( $N=12, k=2, p < 0.05$ ). The between-condition correlations failed to be higher in  $dy_{12}$  than in  $dy_1$  or  $dy_2$  ( $N=16, k \geq 6, p > 0.1$ ), which is not really what we would have expected. In summary,  $dy_{12}$  seems certainly not cumulating the motor-noise errors of  $dy_1$  and  $dy_2$ , which suggests that the vertical stroke sizes do not vary independently (sequence hierarchy).

————— Insert Table 2 —————

In the equation of the net horizontal displacement of a stroke pair, i.e.,  $dx_{12} = dx_1 + dx_2$ , the correlation between  $dx_1$  and  $dx_2$  is negative ( $N=24, k \leq 5, p < 0.01$ ), similarly to vertical sizes. Therefore, it seems that if  $dx_1$  happens to overshoot to the right,  $dx_2$  will also overshoot but then to the left (i.e., in opposite direction), such that the net horizontal displacement after two strokes is relatively invariant. The negative correlation does not seem to be strong, however, such that the SNR of  $dx_{12}$  was not greater, but even smaller than that of  $dx_1$  or  $dx_2$  ( $N=24, k \leq 3, p < 0.001$ ). The between-condition correlations of  $dx_{12}$  are not higher than that of  $dx_1$  or  $dx_2$  ( $N=16, k \geq 5, p > 0.1$ ), similarly to the vertical sizes. In summary, there are only weak indications that there is a sequence hierarchy for the horizontal stroke sizes.

Finally, in the equation of the stroke-pair duration, i.e.,  $dt_{12} = dt_1 + dt_2$ , there is no sign of a negative correlation between  $dt_1$  and  $dt_2$  ( $N=24, k=11, p > 0.1$ ). The SNR of  $dt_{12}$  is always smaller than the SNR of  $dt_1$  or  $dt_2$  ( $N=24, k=0, p < 0.001$ ). Between-condition correlations of  $dt_{12}$  are also not higher than those of  $dt_1$  or  $dt_2$  ( $N=16, k \geq 6, p > 0.1$ ). This indicates that there

are probably no substantial stroke segmentation errors, which systematically might have caused overestimated stroke durations followed by underestimated stroke durations and vice versa.

## **Summary and Discussion**

The rationale in this paper is that relatively invariant movement features are closely related to the highest-level movement parameters of the motor program, whereas more varying features are derived at a lower level as they inherit noise from the higher-level parameters and as they may have some more degrees of freedom. The experimental data were collected by having subjects produce a single writing pattern several times under normal, faster and slower execution conditions. The writing patterns were segmented into strokes at the absolute velocity minima. Per stroke we estimated duration, horizontal and vertical stroke size, and horizontal and vertical peak force (assumed proportional to peak acceleration). The extent of invariance was expressed by the signal-to-noise ratios (SNRs), between-parameter correlations, between-condition correlations (Teulings et al., 1986), and between-stroke correlations. The first three criteria, which discriminate between the higher-level, stored parameters and the lower-level, derived parameters, express a top-down hierarchy. The latter criterion, which discriminates between strokes and multi-stroke sequences, expresses a sequence hierarchy. There may still be another mechanism introducing motor noise, apart from the mechanism of the invariant memory representation and the varying, derived parameters. Namely, each time a different linear combination of compatible but slightly different abstract motor-programs may be selected. Unfortunately, this may yield similar data.

The results support a top-down hierarchy, where the spatial feature (vertical stroke size) is a more invariant, higher-level parameter than the dynamical features (stroke duration and vertical peak force) which are probably derived at a lower level (Teulings et al., 1986). This conclusion was based on the observations that the signal-to-noise ratios for patterns of the vertical stroke sizes were higher than those for the stroke durations or the vertical peak forces. Furthermore, the between condition correlations were highest for the patterns of the vertical stroke sizes. This may seem trivial as total size was not at all manipulated between conditions but only total time. However, a similar result was obtained earlier, where instructed size variation was included (Teulings et al., 1986). The suggestion that force and duration parameters are derived at a lower level corresponds with theories saying that first muscle-independent and then muscle-dependent parameters are specified (Van Galen & Teulings,

1983).

The high invariance of the vertical stroke size, in spite of the varying stroke duration and peak vertical force, is compatible with the more often observed negative correlations between duration and peak force (Newell et al., 1982). Of course, handwriting is a graphical task, where the spatial structure is the aim. In tasks where timing is the final purpose, it still may be timing which constitutes the high-level movement information. However, then only timing patterns can be stored which satisfy the rules of timing memory, which say that only simple duration ratios are stored (Povel, 1981). Similar statements have been made by Semjen and Aiguier (1991).

Interestingly, downstrokes were more invariant than upstrokes in terms of the vertical size. Therefore, it may indeed be appropriate that the vertical component of the down strokes plays an important role in some cursive-script recognizers (e.g., Ph. Wright, 1988). The reason was that upstrokes are often connection strokes between letters, but this seems another reason. The finding that the downstroke is more invariant than the upstroke fits nicely with findings that the upstroke is more sensitive to instructions to write wider or narrower (Maarse & Thomassen, 1983). Furthermore, the horizontal sliding from left to right during writing is superimposed upon the horizontal component of the movements producing the letters. Upstrokes may vary even more because it seems that this sliding is discontinuous (e.g., Thomassen & Teulings, 1983), although some handwriting models assume a continuous sliding (e.g., Dooijes, 1983). More noisy connection strokes have also been found by Meulenbroek et al. (1989). Finally, upstrokes are often produced by the wrist joint, which is fast (Teulings et al., 1989) but has relatively low amplitude-accuracy versus speed tradeoff (Langolf, et al., 1976). Indeed, there are many reasons why downstrokes seem the information carriers in handwriting.

The results also support a sequence hierarchy, where multi-stroke sequences provide the frame for the individual strokes, as the motor-noise error does not seem to cumulate across strokes so that baseline and x-height are rather invariant. In order to express the sequence hierarchy, between-stroke correlations of movement features were estimated. The sequence hierarchy reveals that stroke pairs show more accurate net displacements than explained on the basis of the cumulated accuracies of the individual strokes. The negative correlations between successive stroke sizes reveal a hierarchy of at least two successive strokes. Importantly, the negative between-stroke correlations were not found for stroke durations. This confirms that

the artificial stroke segmentation of the continuous movement is not introducing the varying stroke durations observed. Furthermore, this provides additional evidence that handwriting movements are not controlled by a time keeper plus a varying transmission delay (e.g., Wing & Kristofferson, 1973). For the purpose of automatic handwriting recognition, the sequence hierarchy implies that it is useful to characterize handwriting in larger segments than a single stroke, e.g., a stroke including half of the previous and the next stroke (Schomaker & Teulings, 1991), or pairs of strokes (Edelman et al., 1990; Guberman & Rozentsveig, 1976). Furthermore, there were no differences between down-up and up-down stroke sequences. As the allograph boundaries are mostly at the down-up sequences, it seems that the sequence hierarchy is not restricted to within allographs but also across allographs.

The between-condition and the between-stroke correlations are less strict criteria than the often-used, but mostly rejected, criterion of rescalability. Rescalability is the proportional parameter change for all strokes in the pattern (e.g., macro context; Thomassen & Teulings, 1985). A useful property of rescalability is that it can be tested statistically (e.g., Gentner, 1987), which is not the case with invariances. Much attention has been paid to the rescalability of time, where all durations of a pattern are supposed to be controlled by a single rate parameter. Initially, the rescalability of time was observed in various skills such as handwriting and typewriting (Viviani & Terzuolo, 1980), or gait (Shapiro et al., 1981). This has been intuitively taken as an indication that durations are stored in a "generalized motor program" (Shapiro et al., 1991; Schmidt, 1985). In a similar way as in the present paper, Heuer (1984) investigated the spontaneous variability of force-time curves (estimated by the acceleration) in aiming movements. However, he rejected the rescalability of time, but suggested that it still could hold between different conditions. Peak forces seemed rescalable, though. Also Zelaznik et al. (1986) rejected the rescalability of time in aiming movements as the time to peak acceleration did not vary at all when total movement time was varied. Gentner (1987) statistically rejected time rescalability in typewriting. Using the same method, Wann & Nimmo-Smith (1990) found that rescalability is not reliable in handwriting as well, and rejected the idea that time is represented in the motor program. However, Heuer (1988a) noted that the rescalability of time may have been masked: A perfectly rescalable, central timer with varying motor delays, could still yield nonrescalable time intervals.

What could be the source of the rescalability of time, which often seems approximately the case (e.g., Viviani & Terzuolo, 1980; Shapiro et al., 1981)? Graphical tasks are probably not generated by an underlying timing pattern, but rather by an underlying spatial pattern which is

translated into a specific timing pattern according to fixed rules, depending upon size and limb (e.g., C. Wright, 1991). Various approximated relations between time, size and shape exist such as the 2/3-power law (Lacquaniti et al., 1983; Thomassen & Teulings, 1985; Wann et al., 1988) and the minimum-snap model (Edelman et al., 1990; Wann et al., 1988). These relations seem rather robust, even between different execution conditions. Therefore, the rescalability of time is a byproduct rather than a result of the motor program.

If the same movement pattern is executed at another size or speed, some parameters need to be intentionally rescaled. Heuer and Schmidt (1988) showed that time rescaling was easier between multi-phasic movement patterns of elbow flexions with equal relative timing, than between patterns with clearly different relative timing. On the other hand, it seems that well-practised rhythmic tapping patterns (e.g., long, short, short) cannot even be suppressed when repeating this pattern at maximal speed (Keele & Summers, 1976). Heuer (1988b) found that transfer of an unnatural timing pattern (i.e., where spatial and temporal parameters do not fit together naturally) to a more natural timing pattern is much better than vice versa, because extra skills are learned for the unnatural timing pattern. Heuer and Schmidt (1988) and Heuer (1988b) hypothesized that timing is not stored in motor memory. Instead other underlying parameters are stored, from which the relative durations are derived according to some rules.

In summary the data provide evidence that at the highest level vertical stroke sizes are stored, especially the downstrokes, whereas peak force and stroke durations are parameters derived at the lower levels (top-down hierarchy). Motor-noise error of the horizontal and vertical stroke sizes are largely corrected in the subsequent stroke (sequence hierarchy). The upstrokes, which have mostly a large horizontal component, are varying more because the horizontal component is more noisy at all. Importantly, stroke duration showed no time correction in the subsequent stroke confirming that the artificial stroke segmentation was optimal. The search of invariant parameters may provide criteria to evaluate the plausibility of handwriting-generation models. Invariant features are also of practical relevance for handwriting instruction, writer identification in forensic handwriting expertise, and automatic recognition of online and offline cursive script.