OOPS: Exercise 1

Consider the following situation:

> There once was a wise queen, who was a perfect logician. For advice, she relied on three wise men, who were likewise perfect logicians. This was common knowledge among the four of them, as was the fact that none of them would ever lie or cheat.
>
> One day, the queen wanted to demonstrate to her people just how wise her wise men were. She announced that she would place a hat on each of their heads, and that each of the wise men would be able to see the hats of the other two, but not his own.
>
> The queen then announced that she had three red hats and two blue hats total, and that each wise man was to determine the color of his own hat. The queen then placed the hats, and said: "Each man who knows the color of his hat, must now step forward."

a After the queen's first announcement, is it possible that any wise men will step forward? If not, explain why not. Otherwise, explain which color hats the queen must have placed, and how those who step forward have deduced the color(s) of their own hat(s).

b Instead, let us assume that no wise men step forward after the queen's first announcement. She then repeats it: "Each man who knows the color of his hat, must now step forward."

Is it possible that any wise men will step forward after two announcements? If not, explain why not. Otherwise, explain which color hats the queen must have placed, and how those who step forward have deduced the color(s) of their own hat(s).

c Now, we will use `OOPS` to arrive at the same conclusions. First, we must model the background knowledge that the three wise men have: Namely, that (1) there are two blue hats in total and (2) that each wise man has exactly one hat.

Let us define our propositions as follows: Have `r1` mean that wise man 1 has a red hat, `r2` that wise man 2 has a red hat, and `r3` that wise man 3 has a red hat. Let `b1, b2, b3` express the same relationships between wise men and hats, but for blue ones.

Given this set of propositions, how would you model 'there are two blue hats in total' in `OOPS`?

d And what would you need to model 'each wise man has exactly one hat' in `OOPS`?

e First, let us look at the situation after one announcement by the queen, assuming that two wise men are wearing blue hats. For each wise man who steps forward, use `OOPS` to prove that he can derive the color of his own hat.

f Then, let us look at the situation after two announcements by the queen, assuming that no wise men stepped forward after the first announcement, and that one wise man is wearing a blue hat. For each wise man who steps forward, use `OOPS` to prove that he can derive the color of his own hat.