Complexity of Contextual Reasoning[1]

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It has been widely argued that the establishment of a solid paradigm for contextual knowledge representation and reasoning is of paramount importance for the development of sophisticated theory and applications in Artificial Intelligence.

McCarthy [7] pleaded for a formalization of context as a possible solution to the problem of *generality*, whereas Giunchiglia [4] emphasized the principle of *locality*: reasoning based on large (common sense) knowledge bases can only be effectively pursued if confined to a manageable subset (context) of that knowledge base. Indeed, in contemporary settings like those of the *Grid* [2] and the *Semantic Web* [1] the notion of local, distributed knowledge has become an indispensable requisite. In general, modern architectures impose highly scattered, heterogeneous knowledge fragments, which a central reasoner is not able to deal with. This engenders a high demand for distributed, contextual reasoning procedures.

Contextual knowledge representation has been formalized in various ways. Most notable are the propositional logic of context (PLC) devised by McCarthy, Buvač and Mason [8, 9], and the multi-context systems (MCS) developed by Giunchiglia and Serafini [5], which have later become associated with the local model semantics (LMS) introduced by Giunchiglia and Ghidini [3]. MCS has been proven to be both conceptually and technically more general than PLC [12].

We seek to characterize the computational complexity of contextual reasoning. Our most significant results are an equivalence theorem with bounded modal logic, and the so-called bounded model property for propositional multi-context systems. This property yields a rather general insight into the inherent difficulty of contextual reasoning. It allows us to prove NP-completeness for MCS and to obtain new complexity bounds for both MCS and PLC, in the latter case improving earlier results due to Massacci [6]. Moreover, it provides for an encoding of contextual satisfiability into purely propositional satisfiability, which paves the way for the implementation of contextual reasoners based on already existing SAT solvers.

[1] A full-fledged version of this paper appears in the proceedings of the *National Conference on Artificial Intelligence* [10]. A complimentary account of concrete decision procedures for contextual reasoning has been presented during the *European Conference on Artificial Intelligence* [11]. All material is available from `http://home.student.uva.nl/f.roelofsen/`.

References

[1] G. Antoniou and F. van Harmelen. *A Semantic Web Primer*. MIT Press, 2004.

[2] I. Foster. *The Grid : Blueprint for a New Computing Infrastructure*. Morgan-Kaufmann, 2^{nd} edition, 2004.

[3] C. Ghidini and F. Giunchiglia. Local models semantics, or contextual reasoning = locality + compatibility. *Artificial Intelligence*, 127(2):221–259, 2001.

[4] F. Giunchiglia. Contextual reasoning. *Epistemologia*, XVI:345–364, 1993.

[5] F. Giunchiglia and L. Serafini. Multilanguage hierarchical logics, or: how we can do without modal logics. *Artificial Intelligence*, 65(1):29–70, 1994.

[6] F. Massacci. Contextual reasoning is NP-complete. In *National Conference on Artificial Intelligence (AAAI 96)*, pages 621–626, 1996.

[7] J. McCarthy. Generality in artificial intelligence. *Communications of ACM*, 30(12):1030–1035, 1987.

[8] J. McCarthy. Notes on formalizing context. In *International Joint Conference on Artificial Intelligence (IJCAI 93)*, pages 555–560, 1993.

[9] J. McCarthy and S. Buvač. Formalizing context (expanded notes). In *Computing Natural Language*, volume 81 of *CSLI Lecture Notes*, pages 13–50. 1998.

[10] F. Roelofsen and L. Serafini. Complexity of contextual reasoning. In *National Conference on Artificial Intelligence (AAAI 04)*, pages 124–129, 2004.

[11] F. Roelofsen, L. Serafini, and A. Cimatti. Many hands make light work: Localized satisfiability for multi-context systems. In *European Conference on Artificial Intelligence (ECAI 04)*, pages 58–62, 2004.

[12] L. Serafini and P. Bouquet. Comparing formal theories of context in AI. *Artificial Intelligence*, 155:41–67, 2004.