Quantum Information Processing Theory

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The cognitive revolution that occurred in the 1960's was based on classical computational logic, and the connectionist/neural network movements of the 1970's were based on classical dynamical systems. These classical assumptions remain at the heart of both cognitive architecture and neural network theories, and they are so commonly and widely applied that we take them for granted and presume them to be obviously true. What are these critical but hidden assumptions upon which all traditional theories rely? Quantum information processing theory provides a fundamentally different approach to logic, reasoning, probabilistic inference, and dynamical systems. For example, quantum logic does not follow the distributive axiom of Boolean logic; quantum probabilities do not obey the disjunctive axiom of Kolmogorov probability; quantum reasoning does not obey the principle of monotonic reasoning. Nevertheless Mother Nature seems to rely quite heavily on quantum computing principles in many domains of science. This tutorial will provide an exposition of the basic assumptions of classic versus quantum information processing theories. These basic assumptions will be examined, side by side, in a parallel and elementary manner. For example, classical systems assume that measurement merely observes a pre existing property of a system; in contrast, quantum systems assume that measurement actively creates the existence of a property in a system. The logic and mathematical foundation of classic and quantum theory will be laid out in a simple and elementary manner that uncovers the mysteries of both theories. Classic theory will emerge to be seen as a possibly overly restrictive case of the more general quantum theory. The fundamental implications of these contrasting assumptions will be examined closely with concrete examples and applications to cognition. New research programs in cognition based on quantum information processing theory will be reviewed.

Dr. Busemeyer is a professor of Psychological and Brain Science and Cognitive Science at Indiana University. He is also the chief editor of the *Journal of Mathematical Psychology*. He recently ended his position as manager of the Cognition and Decision Program at the Air Force Office of Scientific Research. His research interests include decision making and dynamic modeling. Dr. Zheng Wang is an assistant professor of Communication at Ohio State University. Her main research interests are affective and cognitive responses to mediated information.

Participant Background: This tutorial will introduce participants to an entirely new area and no previous experience or background with quantum theory will be assumed. No background in physics is required. In fact, except for two simple examples to motivate the introduction, little or no reference to physics will be made during main part of the tutorial. What is required is an elementary background in classic logic (e.g., conjunction, disjunction), classic probability (e.g., AND/OR rules), and linear algebra (e.g. matrix multiplication). Furthermore, as much as possible within the time frame, the tutorial will attempt to review matrix algebra concepts (e.g., eigenvectors, projection matrices).

Material to Be Covered:
1. The tutorial will begin with two simple but dramatic examples of quantum findings from physics, along with interpretations of these into two cognitive science examples.
2. Examination of the differences between classic versus quantum theories of logic, showing how classic logic is a special and restrictive case of quantum logic. The key to this section will be a concrete example illustrating the violation of the distributive axiom of Boolean logic by quantum logical systems.
3. Presentation of the differences between classic versus quantum theories of probability. This section will show how classic probability theory is a special and restrictive case of quantum probability. The concept of superposition is introduced and distinguished for classic and quantum systems, and an example violation of the OR rule by quantum theory is provided.
4. Identifying differences between simple classical dynamical systems and quantum dynamical systems. A parallel development of Markov and quantum processes will be shown. The concept of a state will be distinguished for Markov and quantum systems. A key feature of this

section is to show when and how quantum processes depart from Markov processes.

5. Introduction to quantum computing ideas. This section contrasts: a bit with a qubit, conjunctive properties with tensor products, production rules with controlled U-gates, connectionist networks with quantum transformations. The concept of fuzzy representation will be compared for fuzzy set theory, Bayesian theory, and quantum theory. The idea of an entangled state will be described. Bell's inequality will be introduced, and the dramatic implications of violations of this inequality for classical theories will be discussed.

6. The final section will review three ongoing research programs in cognitive science that are making use of quantum information processing theory. The first is quantum game theory, the second is a quantum theory of concepts, and the third is a quantum theory of decision making. In addition, the last section will present the details of MATLAB programs used to perform quantum computing for some complex information processing tasks. This includes pattern recognition and planning event dependent action sequences under uncertainty. Basic tools of quantum computing will be used including Kronecker products to perform U-gate operations, and partial traces for measurement of components of a complex system.

Further Information can be found at:
http://mypage.iu.edu/~jbusemey/quantum/Quantum Cognition Notes.htm

Major references:

Nielsen, M. A., & Chuang, I. L. (2000). *Quantum Computation and Quantum Information.* Cambridge, UK: Cambridge University Press.

Hughes, R. I. G. (1989). *The Structure and Interpretation of Quantum Mechanics.* Cambridge, MA: Harvard University Press.

You can download the following related articles:

Busemeyer, J. R., Wang, Z., & Townsend, J. T. (2006). Quantum dynamics of human decision making. *Journal of Mathematical Psychology*, *50*, 220-241. Available at http://mypage.iu.edu/~jbusemey/QD.pdf

Pothos, E. M., & Busemeyer, J. R. (in press). A quantum probability explanation for violations of "rational" decision theory. *Proceedings of the Royal Society B.* Available at http://mypage.iu.edu/~jbusemey/quantum/QDT.pdf

Busemeyer, J., Wang, Z., & Mogiliansky, A. L. (in press). Empirical comparison of Markov and quantum models of decision making. *Journal of Mathematical Psychology*. Available at http://mypage.iu.edu/~jbusemey/QvsM.pdf