Mathematical Logic in the Netherlands 2011

19–20 May 2011

University of Groningen

LOCATION

Bernoulliborg, Room 165
Nijenborgh 9, Groningen

PROGRAMME

Thursday, 19 May

10.00-11.00	Invited talk: **Henk Barendregt** (University of Nijmegen) ***Obtaining results accidentally***
11.00-11.15	Coffee
11.15-11.40	Contributed talk: **Wim Veldman** (University of Nijmegen) ***Two equivalents of the fan theorem***
11.40-12.05	Contributed talk: **Merlin Carl** (University of Bonn) ***Simplified fine structure for applications of constructibility***
12.05-12.30	Contributed talk: **Yurii Khomskii** (University of Amsterdam) ***Regularity and definability***
12.30-13.30	Lunch
13.30-14.30	Invited talk: **Jaap van Oosten** (University of Utrecht) ***Type theory and homotopy theory***
14.30-14.55	Contributed talk: **Benno van den Berg** (University of Utrecht) ***A functional interpretation for non-standard arithmetic***
14.55-15.20	Contributed talk: **Vieri Benci** (University of Pisa), **Leon Horsten** (University of Bristol), and **Sylvia Wenmackers** (University of Groningen) ***Axioms for non-archimedean probability***
15.20-15.50	Coffee
15.50-16.15	Contributed talk: **Paula Henk** (University of Amsterdam) ***A new perspective on the arithmetical completeness of GL***
16.15-16.40	Contributed talk: **Kohei Kishida** (University of Groningen) ***A forgotten trio: Classical semantics for first-order non-classical languages***
16.40-17.05	Contributed talk: **Frank Roumen** (University of Nijmegen) ***Obtaining the syntactic monoid via duality***

17.15-18.15 **Plenary discussion session about OzSL**

The conference dinner takes place on Thursday, 19 May, from between 19.00 and 19.30, in the following restaurant. *Registration requested*.

NI HAO Kattendiep
Wok restaurant
Gedempte Kattendiep 122
9711 PV Groningen
t. 050-318 14 00 (from 11.00)
f. 050-313 11 37

Friday, 20 May

10.00-11.00 Invited talk:
Alessandra Palmigiano (University of Amsterdam)
Algebraic modal correspondence

11.00-11.15 Coffee

11.15-11.40 Contributed talk:
Johannes Marti (University of Amsterdam)
Relation liftings in coalgebraic modal logic

11.40-12.05 Contributed talk:
Wouter Stekelenburg (University of Utrecht)
Necessary conditions for the construction of a category of assemblies

12.05-12.30 Contributed talk:
Dion Coumans (University of Nijmegen) and **Sam van Gool** (University of Nijmegen)
Free algebras via partial algebras for a functor

12.30-13.30 Lunch

13.30-14.30 Invited talk:
Bas Spitters (University of Nijmegen)
Efficient real computation in constructive type theory

14.30-14.55 Contributed talk:
Lorijn van Rooijen (University of Nijmegen)
Generalised Kripke semantics for the Lambek-Grishin calculus

14.55-15.20 Contributed talk:
Jesse Alama (New University of Lisbon)
Proof analysis using fine-grained dependency information in formal mathematics

15.20-15.50 Coffee

15.50-16.15 Contributed talk:
Matteo Bianchetti (University of Milan)
Logical consequence

16.15-16.40 Contributed talk:
Tonny Hurkens (University of Nijmegen)
Logic in easy form: Four labels explaining inference

16.40-17.05 Contributed talk:
Paniel Reyes Cárdenas (University of Sheffield)
Peirce's mathematical structuralism as pragmatistic realism

Henk Barendregt (University of Nijmegen)
Obtaining results accidentally

In this talk two examples will be given. Alonzo Church was asked to prove for his dissertation that a certain problem was decidable. He could not do it, but came up with the notion of computability. On a more modest level it will be shown how I tried for my dissertation to make a recursion-theoretic model of the lambda calculus. I did not succeed, but came up with the notions of solvability and the omega-rule in lambda calculus. The latter turned out to be related to Cartesian closed categories having enough points. The former turned out to be essential for understanding Scott's models and Boehm-trees, which was an early example of co-induction.

Jaap van Oosten (University of Utrecht)
Type theory and homotopy theory

In recent years, there has been a renewed interest in Martin-Löf's type theory, and most emphatically for the interpretation of Identity types as Path spaces. Around this theme there was a meeting in Oberwolfach featuring Fields medalist Vladimir Voevodsky, who is now championing a homotopy theoretical approach to type theory (and the foundations of mathematics in general). The talk will give a sketch of these developments.

Alessandra Palmigiano (University of Amsterdam)
Algebraic Modal Correspondence

Sahlqvist correspondence theory is among the most celebrated and useful results of the classical theory of modal logic, and one of the hallmarks of its success. Traditionally developed in a model-theoretic setting (cf. [3], [4]), it provides an algorithmic, syntactic identification of a class of modal formulas whose associated normal modal logics are strongly complete with respect to elementary (i.e. first-order definable) classes of frames. Sahlvist result can equivalently be reformulated algebraically, via the well known duality between frames and complete atomic Boolean algebras with operators (BAO's). This perspective immediately suggests generalizations of Sahlqvist's theorem along algebraic lines, e.g. to the cases of distributive [2] or arbitrary lattices with operators. We illustrate, by way of examples, the algebraic mechanisms underlying Sahlqvist correspondence for classical modal logic [1], after having discussed the appropriate duality with the relational semantics. We show how these mechanisms work in much greater generality than the classical setting in which Sahlqvist theory was originally developed. Next we present the newly developed algorithm ALBA [2] which effectively extends the existing most general results on correspondence.

Bibliography:

[1] W. Conradie, A. Palmigiano, S. Sourabh, Algebraic Sahlqvist Correspondence, in preparation.
[2] W. Conradie and A. Palmigiano, Algorithmic Correspondence and Canonicity for Distributive Modal Logic, submitted.
[3] H. Sahlqvist, Correspondence and completeness in the first and second-order semantics for modal logic, in Proceedings of the 3rd Scandinavian Logic Symposium, Uppsala 1973, S. Kanger, ed., 1975, pp. 110-143.
[4] J. van Benthem, Modal logic and classical logic, Bibliopolis, Napoli, 1983.

Bas Spitters (University of Nijmegen)
Efficient real computation in constructive type theory

In 1967 Bishop proposed to use constructive analysis as a language for exact real computations. Martin Löf proposed constructive type theory as an actual programming language. This language now forms the base of both proof assistants and programming languages. I will report on our effort to actually carry out this research program: we provide an efficient computer verified implementation of exact real numbers in the Coq proof assistant.

Jesse Alama (New University of Lisbon)
Proof analysis using fine-grained dependency information in formal mathematics

From a formalization of a mathematical theorem obtained by a theorem prover, one can compute precisely what is needed for the formalization to be successful. By applying this process to not just one theorem but to a large corpus of formalized mathematical knowledge, one obtains a rich database of dependency information on which one can carry out experiments in proof analysis. Since the dependency information comes from particular formalizations of theorems, expressed in certain fixed formal language(s), the dependency information one obtains is intensional. We can learn, in other words, what is minimally sufficient for a particular proof of a theorem to be successful, given that the theorem was formalized and proved in a particular way. Although we are often after extensional dependency information (that is, information about what depends on what, regardless of the language or system used), intensional dependency information can already reveal some interesting results. We apply these conceptual and computational tools toward the MIZAR Mathematical Library [1] (MML), a large corpus of formalized mathematical knowledge. To illustrate the kind of proof analysis one can carry out, we revisit a discussion [2] from the Foundations of Mathematics mailing list about the axiom of choice and strongly inaccessible cardinals in Tarski-Grothendieck set theory and show how the discussion can be aided by dependency information culled from the MML.

[1] `http://mizar.org` .
[2] Solovay, Robert M. "Re: AC and strongly inaccessible cardinals", posted 2008/03/28 to the Foundations of Mathematics mailing list (`http://www.cs.nyu.edu/pipermail/fom/2008-March/012783.html`).

Benno van den Berg (Darmstadt University of Technology)
A functional interpretation for non-standard arithmetic

In this talk I will present an axiomatic system for non-standard arithmetic, extending Heyting arithmetic in all finite types, and discuss how a variant of Gödel's Dialectica interpretation can be used to rewrite non-standard arguments into ordinary standard ones. Also, I will indicate how this rewriting algorithm can be used for term extraction purposes. (This joint work with Eyvind Briseid and Pavol Safarik.)

Matteo Bianchetti (University of Milan)
Logical consequence

LOGICAL CONSEQUENCE (LC)

Etchemendy ([1990], [2008]) claims: Tarski's definition of LC (TLC) ([1936]), based on the notion of formal truth-preservation, is both (1) conceptually and (2) extensionally inadequate.
About (1), Etchemendy reasons: TLC does not explain why truth preservation guarantees that an argument is logically valid and why we are justified to infer a conclusion from certain premises before knowing the truth-value of the sentences in that argument.
About (2), Etchemendy reasons: TLC relies on non-logical assumptions (e.g. in ZF we need the axiom of infinity to show that σ="There are at most n elements" is not logically valid) and, so, TLC can overgenerate.

Reply to (1).
Etchemendy oversimplifies the *status quaestionis*. There are different intuitions about LC: necessity, formality, rationality, analicity, relevance, ... I claim that these different intuitions are bounded to particular metaphysical and epistemological assumptions. Examples: (a) Kant and Brouwer emphasized the role of rationality due to their idealism, (b) Bolzano emphasized necessity and formality due to its belief into per se propositions.
Even in the light of Etchemendy's arguments, then, there is no reason to abandon TLC. TLC relies on the easily acceptable idea that to preserve truth in every logically possible situations is to be logically valid. To give a meta-theory means to make explicit which are the basic metaphysical properties of

the objects taken into account by a certain logic. To define LC trough a meta-theory means to give a workable framework to specify the *logically possible* and *relevant* situations to evaluate an argument. Etchemendy did not recognise that to preserve truth in every situation described by a meta-theory is not to replace LC with material consequence, since the intended meaning of the class of the situations described by the meta-theory is that it is the class of all logically possible and relevant situations.

Reply to (2).
(a) σ is not a logical truth in ZF minus the axiom of infinity even if we cannot falsify it, indeed we can consistently add the axiom of infinity to ZF. We have to distinguish *to be always true* and *to not be falsifiable*. We can extend a theory T to a new consistent theory T' that falsifies a sentence ϕ that was not falsifiable in T. It shows that ϕ was not a logical consequence of T because every model of T' is also a model of T.
(b) Etchemendy misunderstands the role of a meta-theory, that does not need to determine a complete ontology,
(c) the fact that TLC can overgenerate only shows that LC is, here, a relational concept and, by my previous reply, we can consider LC as such.

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Tarski A. [1936c]
On the Concept of Logical Consequence, tr. in Tarski [1983], *Logic, Semantics, Metamathematics*, II ed., tr. J. H. Woodger, intr. J. Corcoran, Hackett Publishing Company, Indianapolis, Indiana, pp. 409-420.

Merlin Carl (University of Bonn)
Simplified fine structure for applications of constructibility

ZFC, Zermelo-Fraenkel set theory with choice, is now widely accepted as "the" formal framework for common mathematics. Independence from ZFC hence means undecidability on the basis of the current view of mathematics. One way to obtain such results is the use of inner models, definable sub-classes of the set-theoretical universe V. The subset-smallest such model is Gödel's constructible universe L. due to its very concrete nature, it can be analyzed very precisely, leading to strong combinatorial principles in L that are hence known to be consistent with ZFC. However, the classical apparatus for such results, Jensen's fine structure theory, is technically rather complex already in the context of L, and even more so when one considers generalizations, in particular core models. Therefore, there have been several attempts do develop simplified fine structures for L and its relatives. I will present one of these, the F-hierarchy, which was invented and exhibited by van Eijmeren, Koepke and myself and give an impression of what can be done with it.

Dion Coumans (University of Nijmegen) and **Sam van Gool** (University of Nijmegen)
Free algebras via partial algebras for a functor

In the algebraic study of a logic $\mathcal{L}$, one assigns a class of algebras $\mathbf{V}_{\mathcal{L}}$ to the logic and uses algebraic methods to obtain properties of this class. The results of this algebraic study can be translated back to properties of $\mathcal{L}$. In particular, if $\mathbf{V}_{\mathcal{L}}$ contains (finitely generated) free algebras, a thorough understanding of these can yield powerful results about the logic $\mathcal{L}$, for example related to questions about term complexity, decidability of logical equivalence, interpolation and normal forms, *i.e.*, problems in which one considers formulas whose variables are drawn from a finite set. If the class of algebras $\mathbf{V}_{\mathcal{L}}$ associated with the logic $\mathcal{L}$ is axiomatized by equations which are rank 1 for an operation f,[1] the algebras for the logic can be represented as algebras for a functor $F_{\mathcal{L}}$ on the category of underlying algebra reducts without the operation f. This functor $F_{\mathcal{L}}$ enables a constructive description of the free $\mathbf{V}_{\mathcal{L}}$ algebras [2].

[1] An equation is of *rank 1 for an operation f* if every variable occurs under the scope of exactly one occurrence of f.

However, rank 1 axioms are rather limited – many interesting logics are not axiomatized by rank 1 axioms. Therefore, one would want to extend the existing techniques to logics which do not have a rank 1 axiomatisation. The first steps on this path are taken by Ghilardi in [3]. Here he describes a method to incrementally build finitely generated free Heyting algebras by constructing a chain of distributive lattices, where, in each step, implications are freely added to the lattice, while keeping a specified set of implications which were already defined in the previous step. In a subsequent paper, Ghilardi extended these techniques to apply to modal logic [4], and used his algebraic methods to derive normal forms for modal logics, notably S4.

Recently, this line of research has been picked up again. In [1] N. Bezhanishvili and Gehrke have re-analysed Ghilardi's incremental construction and have derived it by repeated application of a *functor*, based on the ideas of the coalgebraic approach to rank 1 logics and Birkhoff duality for finite distributive lattices. Shortly after, Ghilardi [5] gave a new construction of the free S4 algebra in the same spirit. However, the methods in [1] and [5] rely on specific properties of Heyting algebras and S4 algebras respectively, and they do not directly apply in a general setting.

We will describe a general functorial method, inspired by the above described earlier work, to construct free algebras, which is applicable outside the setting of pure rank 1 logics. The crucial insight which has led to this result is that *partial algebras* are the natural structures to consider when building free algebras step by step. Our new method encompasses the earlier incremental constructions mentioned above, *i.e.*, the construction of free algebras for exactly rank 1 varieties in [2], Heyting algebras in [1] and S4 algebras in [5] may be viewed as special instances.

References

[1] Nick Bezhanishvili and Mai Gehrke, *Finitely generated free Heyting algebras via Birkhof duality and coalgebra*, to appear in Logical Methods in Computer Science.

[2] Nick Bezhanishvili and Alexander Kurz, *Free modal algebras: a coalgebraic perspective*, in: Algebra and Coalgebra in Computer Science, Lecture Notes in Computer Science, vol. 4624, Springer, 2007, pp. 143-157.

[3] Silvio Ghilardi, *Free Heyting algebras as bi-Heyting algebras*, Math. Rep. Acad. Sci. Canada **XVI** (1992), no. 6, 240-244.

[4] Silvio Ghilardi, *An algebraic theory of normal forms*, Annals of Pure and Applied Logic **71** (1995), 189-245.

[5] Silvio Ghilardi, *Continuity, Freeness, and Filtrations*, Rapporto Interno 331-10, Dipartimento di Scienze dell'Informazione, Università degli Studi di Milano (2010).

[6] Yde Venema, *Algebras and Coalgebras*, Handbook of Modal Logic (Patrick Blackburn, Johan van Benthem, and Frank Wolter, eds.), Elsevier, 2007, pp. 331-426.

Paula Henk (University of Amsterdam)

A new perspective on the arithmetical completeness of GL

GL is the normal modal logic obtained by adding the Löb formula $\Box(\Box\varphi \to \varphi) \to \Box\varphi$ as an extra axiom to K. Interpreting the modal box as the provability predicate Bew (x) of Peano Arithmetic (PA), GL captures exactly what PA is able to prove about provability in itself. In other words, GL is arithmetically sound and complete. The proof of the arithmetical completeness of GL is due to Robert Solovay, and proceeds by simulating a finite Kripke model inside PA.

In this talk, we present a new way of looking at Solovay's proof of the arithmetical completeness of GL. A finite Kripke model for GL will be shown to be bisimilar to an arithmetically defined model whose domain consists of certan nonstandard models of PA. As a prerequisite for defining the arithmetical Kripke model, we will introduce a strengthened version of the relation of interpretability between theories. We will show that the relation of strong interpretability is suited to function as an accessibility relation between models of PA in a Kripke model where Bew (x) has the role of the modal box.

Tonny Hurkens (University of Nijmegen)

Logic in easy form: Four labels explaining inference

In this talk I explain a connection between truth tables, 3-valued Kripke models and natural deductions in terms of four labels (I, L, F and E).

Label the entries of the truth table of a logical constant as follows:

I	truth value 1 in last column (main formula)
L	truth value 1 in other column (subformula)
F	truth value 0 in other column (subformula)
E	truth value 0 in last column (main formula)

The truth value of a formula φ in a world w in a 3-valued Kripke model is either 0, $\frac{1}{2}$ or 1. This leads to four relations between worlds and formulas:

w I φ	φ has positive truth value in w
w L φ	φ has positive truth value in each world w' reachable from w
w F φ	φ has truth value 1 in each world w' reachable from w
w E φ	φ has truth value 1 in w

A natural deduction is either trivial (some formula occurs both as assumption and conclusion) or constructed from subdeductions by applying some inference rule (introduction or elimination). The inference rules can be formulated in such a way that each subdeduction is either a lemma (new conclusion) or a case (extra assumption). The relation between inference rules and truth tables is clear if one labels this extra formula as follows:

I φ	(Intro) assumption φ is the main formula of an introduction step
L φ	(Lemma) conclusion φ is a subformula of the main formula
F φ	(Fall) assumption φ is a subformula of the main formula
E φ	(Elim) conclusion φ is the main formula of an elimination step

The use of label I in a constructive deduction corresponds to adding a trivial case. One gets classical logic by allowing such a case to be non-trivial.

Cancellation in Gentzen's style corresponds to the occurrence of a formula as assumption and last conclusion on a branch (one of these as leaf). In a normal deduction, the pair of labels should be (I, L), (F, L) or (F, E) but not (I, E). This corresponds to the fact that in a world w of a 3-valued Kripke model, w I φ does not imply w E φ: φ may have truth value $\frac{1}{2}$ in w.

Yurii Khomskii (University of Amsterdam)

Regularity and definability

In the study of the real number continuum, *regularity properties* of sets of real numbers play a central role, having many applications in various areas of mathematics. For instance, the concept of a subset of the continuum being *Lebesgue-measurable* is motivated by the attempt to formalize "size" and "volume" of objects in space. The *Baire property* is another concept motivated by topological issues; the *Ramsey property* is based on the extension of the finite Ramsey theorem to infinite dimensions. These are just three examples of a wide array of regularity properties.

Using the well-ordering of the continuum one can always construct sets that are not regular, but one cannot give explicit examples of such sets. In fact, using the methods of descriptive set theory, which classifies sets according to the logical complexity of the formula defining it (the so-called *projective hierarchy*), one can show that the $\mathbf{\Sigma}^1_1$ sets satisfy all regularity properties. Therefore, the first levels of complexity on which we may find irregularities are the $\mathbf{\Sigma}^1_2$-, $\mathbf{\Pi}^1_2$- and $\mathbf{\Delta}^1_2$-levels, and typically, the statement "all $\mathbf{\Delta}^1_2/\mathbf{\Sigma}^1_2$ sets are regular" is independent of ZFC.

Specifically, this statement fails in Gödel's constructible universe L, but holds if we assume sufficient "transcendence" over L, that is, if we assume that the actual set-theoretic universe is far from being L. One can be more specific and prove that the assertion that all sets on the $\mathbf{\Delta}^1_2$-level satisfy a certain regularity property, is equivalent to the statement that the actual universe is larger than L in a certain way. One application of this is that we can control the amount of regularity using the method of iterated forcing over L.

In this survey talk I will present the general theory of regularity and definability, mentioning some general results and, if time permits, paying particular attention to the problems involved in my own work.

Kohei Kishida (University of Groningen)
A forgotten trio: Classical semantics for first-order non-classical languages

This paper revisits the standard definition of classical semantics and extends it to interpret the classical-base part of a first-order non-classical language.

In the case of propositional logic, we have a good insight on what it means for a semantics to be classical even when we interpret non-classical language. Taking the possible-world semantics for propositional modal logic as an example, we can say that each possible world is a classical model in the sense that it interprets the classical, Boolean connectives with the classical, Boolean truth conditions (although, needless to say, truth conditions for modal operators require more machinery). More precisely, given a propositional language $\mathcal{L}$ with non-classical connectives, the class of maps that assign classical truth values to all the sentences of $\mathcal{L}$ while satisfying all the classical truth conditions for the classical connectives provides classical semantics, in the sense that the classical propositional logic in $\mathcal{L}$—the logic in $\mathcal{L}$ given by all (and only) schemes of theorems and rules of classical propositional logic—is sound and complete with respect to it.

This insight on propositional non-classical languages and classical semantics, however, does not extend to the first-order case straightforwardly. Given a first-order non-classical language $\mathcal{L}$, we can define a standard, Tarskian $\mathcal{L}$-structure with a domain of quantification and extensions for primitive vocabulary of $\mathcal{L}$; but valuations over such $\mathcal{L}$-structures need to satisfy, not only the standard, classical truth conditions for primitive vocabulary and classical operators (meaning Boolean ones as well as $\forall$ and $\exists$), but also three more constraints in order to have classical first-order logic sound. The first goal of this paper is to identify these constraints and to prove that they provide, in combination with the standard truth conditions, the right extension of the definition of classical semantics, in which the classical bases of first-order non-classical languages are interpreted so that classical first-order logic is sound and complete. Not only the technical import of these constraints that they are needed for the soundness of classical logic, we also discuss their conceptual import. (These constraints are rarely discussed in literature; as an exception, Belnap [1] discusses them extensively, though not in the context of interpreting non-classical languages.)

In the course of our proof for the observation above regarding when a semantics is said to be classical, we achieve the second goal of this paper, namely, to show how to regard a non-classical first-order language with some non-classical operators as if it were a "purely classical" first-order language with no non-classical operators, by regarding compound sentences whose major operators are non-classical (e.g., $\Box\varphi$ or $\Diamond\psi$) as if atomic. (One can take an analog in a completeness proof for classical first-order logic in which a first-order language is regarded as if it were propositional.) We also mention an application of this "purification" result—we use it to prove a completeness result for first-order modal logic that extends classical first-order logic.

[1] N. Belnap, *Notes on the Science of Logic*, manuscript, 2009.

Johannes Marti (University of Amsterdam)
Relation liftings in coalgebraic modal logic

In the theory of coalgebras one considers structures that are representable as functions $X \to TX$ for a set X and endofunctor T in the category of sets. This allows one to study different types of structures in a uniform way, independent of the concrete choice of T. Examples are different kinds of automata from theoretical computer science and structures used as models for modal logics such as Kripke frames or neighborhood models.

Relation liftings for a functor T are mappings $X \times Y \to TX \times TY$. They are used in the theory of coalgebras to provide a notion of bisimilarity for different types of coalgebras and to define a cover modality (nabla). In my work I have been looking for conditions on relation liftings to yield an adequate notion of bisimilarity and a meaningful cover modality. It turns out that this is the case if the relation lifting is certain type of lax extension for the functor T. Additionally I discuss the connection between relation liftings and the definability of bisimulation quantifiers in the nabla logic for a type of coalgebras.

Paniel Reyes Cárdenas (University of Sheffield)
Peirce's mathematical structuralism as pragmatistic realism

Mathematical objects are things like numbers, sets, functions and the like, mathematics studies the properties of these objects. They are abstract and alien to our direct experience of the senses, i.e., they lack "secondness", yet they are things from which we can "obtain Knowledge" (Hookway 2010, 2).

My purpose here is to see how Peirce's account of mathematics, universals (generals) and his theory of Categories are closely merged in a successful theory for foundations of Mathematics being this a particular version of the Mathematical structuralism achieved by Shapiro. The traditional issues for the assumptions of the subject matter of mathematics are well known as the "Benacerraf problem", and involve at least these three worries:

1. The metaphysical problem: they seem to be different from familiar objects and yet they are truth-makers but in a mysterious way.
2. The reference problem: the objects normally can react in many different ways, but abstract objects are out of what we normally can refer because we can't point them either. (they do not have secondness).
3. How we obtain knowledge of them: the access problem (Hookway 2010, 2)

An answer to this problem is given by Stewart Shapiro's structuralism (Shapiro 1997:5-6), for him "The subject matter of arithmetic is the natural number structure- the property common to any system of objects that has a distinguished initial object and a successor relation that satisfies the induction principle." Indeed, "the essence of a natural number is the relations it has with other natural numbers." It is natural to describe this view as one that holds that mathematics is concerned with "forms of a relation." Peirce gave an answer along these lines. Peirce thinks that the character of mathematics does not attempt to discover a range of truths, or reveal a part of the reality but nonetheless it has a lot of useful applications both in his theorematic and corollarial kinds of deductions.

Mathematical results hold in all possible worlds, not just in the actual ones, but mathematical propositions are themselves thirdness governing over firstness, i.e., real laws of generality governing over real possibles. "The form of a relation" suggests a property more than an individual. His fight against nominalism can testify this: "Peirce is strongly realist about generals, and rejects the nominalist thesis the only things that are real are particulars: there are real thirdness. Thus the idea that structures are general (rather than particulars) would be congenial to his realism" (Hookway 2010, 16). For the pragmatist, it is enough to make it the case that numbers are real: "On pragmatist principles reality can mean nothing except the truth of statements in which the real thing is asserted" (Peirce 1903"N4.162), the numbers, thus, do not have causal properties because they do not react and exist, but they indeed are real, inasmuch they are within all the points of a series in a true proposition addressing a real mathematical structure.

Keywords: Mathematical Structuralism, Pragmatism, Mathematical Truth, Charles Peirce, Stewart Shapiro.

Lorijn van Rooijen (University of Nijmegen)
Generalised Kripke semantics for the Lambek-Grishin calculus

Relational semantics given by Kripke frames play a fundamental role for modal logics. The representation theory for modal algebras naturally gives rise to the traditional Kripke frames. In the setting of substructural logics, there may be no lattice operations or not necessarily distributive ones. Based on the representation theory for such posets and lattices, the papers [DGP05] and [Geh06] suggest an analogous kind of semantics (the generalised Kripke semantics) for a broader setting including that of substructural logics.

The Lambek-Grishin calculus is a substructural logic that captures certain sentence forming mechanisms in linguistics. The generalised Kripke semantics provides the possibility of a modular treatment, based on canonicity and correspondence, of various extensions of the Lambek-Grishin calculus. Additional axioms and connectives modularly slot in as additional first-order properties or relational components on the generalised Kripke frames.

In joint work with Anna Chernilovskaya and Mai Gehrke we took this approach to obtain complete relational semantics for various extensions of the Lambek-Grishin calculus. In addition to the basic Lambek-Grishin calculus, we considered interaction axioms (in particular those identified as pertinent to linguistic phenomena in [Moo09]), associativity, commutativity, weakening and contraction, and additional connectives such as lattice operations and linear-logic type negation.

References

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[Moo09] M. Moortgat. Symmetric categorial grammar. *Journal of Philosophical Logic*, 38(6):681–710, 2009.

Frank Roumen (University of Nijmegen)

Obtaining the syntactic monoid via duality

To each formal language we can assign a monoid, called the syntactic monoid, which can be used to characterize several properties of the language. We will discuss two methods to compute the syntactic monoid of a regular language. The first algorithm is well-known and consists of determining the minimal automaton recognizing the language and finding its transition monoid. By applying duality theory for residuated Boolean algebras, we obtain a completely different algorithm. We will try to indicate why these algorithms yield the same result.

Wouter Stekelenburg (University of Utrecht)

Necessary conditions for the construction of a category of assemblies

I will give a universal property of the category of assemblies, which is a categorical model of the realizability interpretation. The universal property determines the category up to equivalence and helps to define regular functors to other categories. After defining the category of assemblies and the universal property I will sketch a proof and discus generalizations of the category of assemblies that satisfies a similar universal property.

Wim Veldman (University of Nijmegen)

Two equivalents of the fan theorem

Brouwer's Fan Theorem is the following statement:

$$\forall\beta[\forall\alpha\in C\exists n[\beta(\overline{\alpha}n)\neq 0]\to\exists m\forall s\in Bin[length(s)=m\to\exists n\leq m[\beta(\overline{s}n)\neq 0]]]$$

that is:

Every decidable subset of $\mathbb{N}$ *that is a bar in Cantor space* $C=\{0,1\}^{\mathbb{N}}$ *has a finite subset that is a bar in Cantor space.*

For every γ in $\mathcal{N}=\mathbb{N}^{\mathbb{N}}$, for every n in $\mathbb{N}$, we define γ^n, the n-th subsequence of γ by: for all m, $\gamma^n(m):=\gamma(2^n(2m+1)-1)$.

For every γ we let E_γ, *the subset of* $\mathbb{N}$ *enumerated by* γ, be the set of all numbers n such that, for some m, $\gamma(m)=n+1$.

In the formal system BIM for Basic Intuitionistic Mathematics the Fan Theorem is equivalent to the statement:

$$\forall\gamma[\forall\alpha\in C\exists n[n\in E_{\gamma^{\alpha(n)}}]\to\exists n[n\in E_{\gamma^0}\cap E_{\gamma^1}]]$$

that is:

Two enumerable subsets of $\mathbb{N}$ *with the property that every decidable subset of* $\mathbb{N}$ *positively refuses to separate them must have an element in common.*

The Fan Theorem is also equivalent to the statement:

$$\forall\gamma[\forall\alpha\in C\exists i<2\exists n[\gamma^i(\overline{\alpha^i}n)\neq 0]\to\exists i<2\forall\alpha\in C\exists n[\gamma^i(\overline{\alpha}n)\neq 0]]$$

that is:

The logical scheme:

$$\forall\mathsf{x}\forall\mathsf{y}[\mathsf{P}(\mathsf{x})\vee\mathsf{Q}(\mathsf{y})]\to(\forall\mathsf{x}[\mathsf{P}(\mathsf{x})]\vee\forall\mathsf{y}[\mathsf{Q}(\mathsf{y})])$$

is true in every structure (C,P,Q) *where* P,Q *are open subsets of* C.

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Axioms for non-archimedean probability

We propose a new axiomatic basis for probability theory, called Non-Archimedean Probability (NAP). NAP allows infinitesimal probability values (based on non-standard analysis) and is intended to have considerate epistemological advantages over the classical approach of Kolmogorov (1933) in cases with infinite sample spaces. One drawback of classical probability theory is that a fair lottery on $\mathbb{N}$ cannot be represented in it. Wenmackers and Horsten (2010) consider an alternative description of such a lottery in terms of hyperrational probability values. The NAP formalism generalizes this solution to probabilistic problems not only on other countably infinite sample spaces (*e.g.* a fair lottery on $\mathbb{Q}$), but on sample spaces of larger cardinalities as well (*e.g.* a fair lottery on $\mathbb{R}$).

References

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