A formal approach to case comparison in case-based reasoning: research abstract

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1 Introduction

In this abstract, we introduce an approach about the comparison of cases in case-based reasoning with a formal theory that described in a series of research [2,3,5,6].

As we discussed in [6], our approach provides a new generalization and a new refinement of comparisons in case-based reasoning. We illustrate these contributions with an example (shown in Figure 1) from the domain of trade secret law of the United States, which has been discussed in [1,3,6]. As shown in Figure 1, in this example, the *American Precision* case [2] and the *Yokana* case [3] are considered as precedents, and the *Mason* case [4] is considered as a current situation, of which the outcome needs to be decided.

2 Method

We use a propositional logic language L generated from a set of propositional constants. We write $\neg$ for negation, $\wedge$ for conjunction, $\vee$ for disjunction, $\leftrightarrow$

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[1] This paper is a research abstract of [5,6].
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[2] American Precision Vibrator Co. v. National Air Vibrator Co., 764 S.W.2d 274 (Tex.App.-Houston [1st Dist.] 1988)

[3] Midland-Ross Corp. v. Yokana, 293 F.2d 411 (3rd Cir.1961)

[4] Mason v. Jack Daniel Distillery, 518 So.2d 130 (Ala.Civ.App.1987)

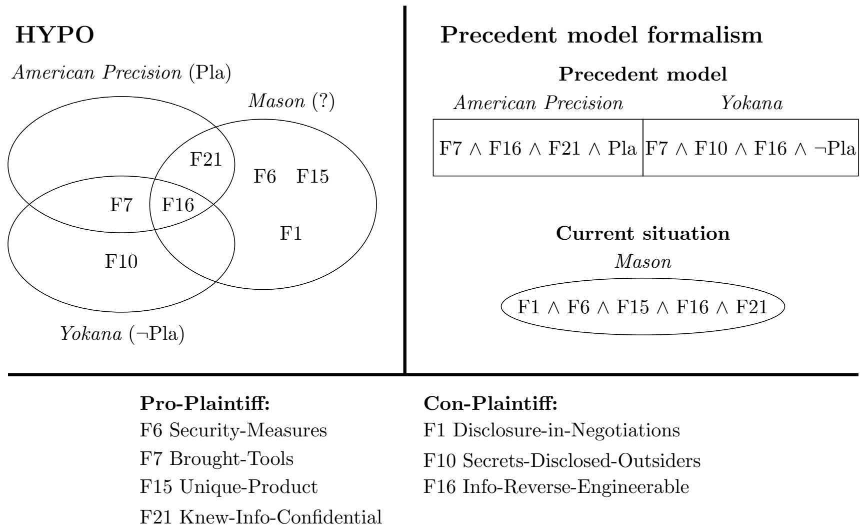

Fig. 1. A Venn diagram [1] and a precedent model [3] about the *Mason* problem

for equivalence, $\top$ for a tautology, and $\bot$ for a contradiction. The associated classical, deductive, monotonic consequence relation is denoted $\models$.

Precedents consist of factors and outcomes. We consider both *factors* and *outcomes* are literals. A literal is either a propositional constant or its negation. We use $F \subseteq L$ to represent a set of factors, $O \subseteq L$ to represent a set of outcomes. The sets F and O are disjoint and consist only of literals. If a propositional constant p is in F (or O), then $\neg p$ is also in F (respectively in O). A factor represents an element of a case, namely a factual circumstance. Its negation describes the opposite fact. An outcome always favors a side in the precedent, its negation favors the opposite side.

Definition 2.1 [Precedents] A *precedent* is a logically consistent conjunction of distinct factors and outcomes $\pi = \varphi_0 \land \varphi_1 \land \ldots \land \varphi_m \land \omega_0 \land \omega_1 \land \ldots \land \omega_{n-1}$, where m and n are non-negative integers. We say that $\varphi_0, \varphi_1, ..., \varphi_m$ are the *factors* of π, $\omega_0, \omega_1, ..., \omega_{n-1}$ are the *outcomes* of π. If $n = 0$, then we say that π is a *situation* with no outcomes, otherwise π is a *proper precedent*.

Notice that both m and n can be equal to 0. When $m = 0$, there is one single factor. When $n = 0$, the precedent has no outcome and the empty conjunction $\omega_0 \land \ldots \land \omega_{n-1}$ is equivalent to $\top$. We do not assume precedents are complete descriptions. That is, factors may exist which do not occur in the precedent. Furthermore, we do not assume that the negation of a factor holds when the factor does not occur in the precedent.

Example 2.2 As shown in Figure 1, the precedents in the formalism are represented as follows:
(i) *American Precision*: F7 $\land$ F16 $\land$ F21 $\land$ Pla;
(ii) *Yokana*: F7 $\land$ F10 $\land$ F16 $\land$ $\neg$Pla;

(iii) *Mason*: F1 $\wedge$ F6 $\wedge$ F15 $\wedge$ F16 $\wedge$ F21.

A *precedent model* is a set of logically incompatible precedents forming a total preorder representing a preference relation among the precedents.

Definition 2.3 [Precedent models] A *precedent model* is a pair $(P, \geq)$ where P is a set of precedents such that for all $\pi, \pi' \in P$ with $\pi \neq \pi'$, $\pi \wedge \pi' \vDash \bot$; and $\geq$ is a total preorder over P.

As customary, the asymmetric part of $\geq$ is denoted $>$. The symmetric part of $\geq$ is denoted $\sim$.

Example 2.4 Figure 1 shows a precedent model with precedents *American Precision* and *Yokana*. As suggested by the size of the boxes, these two precedents are as preferred as each other.

Notions of comparing precedents in case-based reasoning include analogies, distinctions and relevances, they are related to general formulas, not only the factors or outcomes.

Definition 2.5 [Analogies, distinctions and relevances] Let $\pi, \pi' \in L$ be two precedents, we define:

(i) a sentence $\alpha \in L$ is an *analogy between π and π'* if and only if $\pi \vDash \alpha$ and $\pi' \vDash \alpha$.

(ii) a sentence $\delta \in L$ is a *distinction in π with respect to π' (π-π' distinction)* if and only if $\pi \vDash \delta$ and $\pi' \vDash \neg\delta$.

(iii) a sentence $\rho \in L$ is a *relevance in π with respect to π' (π-π' relevance)* if and only if $\pi \vDash \rho$, $\pi' \nvDash \rho$ and $\pi' \nvDash \neg\rho$.

Example 2.6 When comparing *Mason* with *Yokana* through the precedent model formalism:

(i) Analogies between *Yokana* and *Mason*: *e.g.*, F16, F16 $\vee$ F21, (F7 $\wedge$ F10 $\wedge$ F16 $\wedge$ $\neg$Pla) $\vee$ (F1 $\wedge$ F6 $\wedge$ F15 $\wedge$ F16 $\wedge$ F21);

(ii) *Mason*-*Yokana* relevances: *e.g.*, F6 $\wedge$ F15 $\wedge$ F21, F1 $\wedge$ F21;

(iii) *Yokana*-*Mason* relevances: *e.g.*, F10, F16 $\wedge$ $\neg$Pla;

(iv) There is no distinction between *Mason* and *Yokana*.

3 Discussion and conclusion

The formalism we use for constructing precedent models is different from HYPO and CATO, as they describe cases as sets of factors. For instance, the *Yokana* case is represented by set {F7, F10, F16} in HYPO/CATO. While in our formalism, it is represented by a logical conjunction of factors and outcomes. Therefore, the comparison of cases in HYPO is by the notions related to sets, such as the relevant similarity (the set of shared factors by two cases, which is used for the reason that the two cases should have the same outcome) and the relevant difference (the set of unshared factors by two cases, which can be used for pointing out the two cases should be decided differently).

For instance, in the example shown in Figure 1, HYPO uses set {F16}, as the relevant similarity between *Mason* and *Yokana*, for the reason that *Mason* should have the same outcome as *Yokana*, and uses set {F6, F15, F21, F10} as the relevant difference between *Mason* and *Yokana* that can be used for arguing the two cases should have different outcome.

Comparing with our formalism, where we represent the relevant similarity and difference between *Mason* and *Yokana* as an analogy and a relevance respectively (Example 2.6), we thereby show that our approach provides a new generalization and a new refinement of the comparison in case-based reasoning. For the new generalization, our approach is able to not only compare cases by the factors themselves, but also compare them with the compound formulas that based on the factors, as shown in Example 2.6.

For the new refinement, our approach distinguishes the unshared formulas between cases as distinctions and relevances. As in Example 2.6, we can refine the relevant difference between *Mason* and *Yokana* with the relevances, and treat the different outcomes between *American Precision* (Pla) and *Yokana* (¬Pla) as distinctions.

The research abstract we present here shows a new generalization and a new refinement of case comparison in case-based reasoning with a formal theory. In recent publications, we further apply the approach to general case models [4], and discuss hard cases in law with the formalism by connecting the hardness with the involved arguments' validities [7]. The formal theory has the potential to further model case-based reasoning in the future.

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