Agent-based models for higher-order theory of mind

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Abstract. Agent-based models are a powerful tool for explaining the emergence of social phenomena in a society. In such models, individual agents typically have little cognitive ability. In this paper, we model agents with the cognitive ability to make use of theory of mind. People use this ability to reason explicitly about the beliefs, desires, and goals of others. They also take this ability further, and expect other people to have access to theory of mind as well. To explain the emergence of this higher-order theory of mind, we place agents capable of theory of mind in a particular negotiation game known as Colored Trails, and determine to what extent theory of mind is beneficial to computational agents. Our results show that the use of first-order theory of mind helps agents to offer better trades. We also find that second-order theory of mind allows agents to perform better than first-order colleagues, by taking into account competing offers that other agents may make. Our results suggest that agents experience diminishing returns on orders of theory of mind higher than level two, similar to what is seen in people. These findings corroborate those in more abstract settings.

1 Introduction

In everyday life, we regularly interpret and predict the behaviour of other people by reasoning about what they know or believe. This *theory of mind* [1] allows us to understand why people behave a certain way, to predict future behaviour, and to distinguish between intentional or accidental behaviour. People also take this ability one step further, and consider that others have a theory of mind as well. This second-order theory of mind allows us to understand sentences such as "Alice doesn't know that Bob knows that she is throwing him a surprise party", by attributing to Alice the ability to have beliefs about Bob's knowledge. In this paper, we make use of agent-based computational models to explain why our ability to reason about mental content of others may have evolved.

The human ability to make use of higher-order (i.e. at least second-order) theory of mind is well-established, both through tasks that require explicit reasoning about second-order belief attributions [2, 3], as well as in strategic games [4, 5]. However, the use of any kind of theory of mind by non-human species is a controversial matter [6–8]. These differences in the ability to make use of theory of mind raise the issue of the reason for the evolution of a system that allows humans to use higher-order theory of mind to reason about what other people understand about mental content, while other animals, including chimpanzees and other primates, do not appear to have this ability.

A possible explanation for the emergence of higher-order theory of mind is that higher-order theory of mind is needed in situations that involve mixed-motive interactions such as negotiations or crisis management [9, 10]. In these situations, interactions are partially cooperative in the sense that an interaction can lead to a mutually beneficial outcome, but also partially competitive when there is no outcome that is optimal for everyone involved. For example, both the buyer and the seller of a house benefit from a successful sale. However, the buyer prefers a low sales price, while the seller prefers a high sales price.

In this paper, we consider agent-based computational models to investigate the advantages of making use of higher-order theory of mind in mixed-motive settings. We therefore model cognitively more sophisticated agents, in which there has been increasing interest in recent years [8, 11–13]. These agents perform actions based on their own desires and goals, but also take into account that the actions of other agents can influence their situation. By controlling the cognitive abilities of agents and monitoring their performance, we determine the extent to which higher-order theory of mind provides agents with an advantage over agents that are more restricted in their use of theory of mind. We have selected to study the interaction of cognitive agents in the Colored Trails setting, introduced by Grosz, Kraus and colleagues [14, 15], which provides a useful test-bed to study mixed-motive situations. Section 2 describes this setting in more detail.

We compare simulation results of agents of two different types. Agents of the first type base their beliefs on the iterated best-response. We also consider agents that use utility-proportional beliefs, which is more consistent with the behaviour of real life agents [16]. The results from the latter agents should provide insight in the effectiveness of higher-order theory of mind in mixed teams of agents and humans, which occur in an increasing number of domains [17–19]. Section 3 describes the two agent types and how these agents make use of theory of mind.

Section 5 presents the results of the simulations. These results are discussed in Section 6, in which we draw conclusions about whether or not mixed-motive situations may have contributed to the emergence of higher-order theory of mind in humans, as well as the extent to which higher-order theory of mind may be useful for computational agents that interact with people.

2 Colored Trails

To determine the effectiveness of higher-order theory of mind in mixed-motive settings, we have selected the Colored Trails (CT) setting. Colored Trails is a board game designed as a research test-bed for investigating decision-making in groups of people and computer agents [14]. The game is played by two or more players on a board of colored tiles. Each player starts the game at a given initial tile with a set of colored chips. The colors of the chips match those on the tiles of the board. A player can move to a tile adjacent to his current location by handing in a chip of the same color as the destination tile. Each player is also

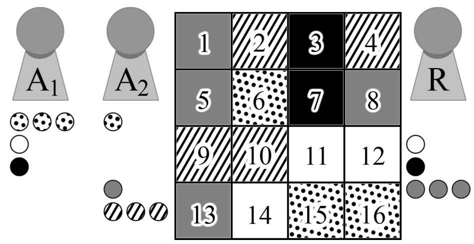


Fig. 1: An example of a Colored Trails game played by three agents. Agents A_1 and A_2 on the left are allocators. They both start at the tile marked 1, and aim to get as close to tile 16 as possible. Agent R on the right is a responder. She starts at tile 4 and tries to get as close as possible to tile 13.

assigned a goal location, which the player has to approach as closely as possible. To achieve this goal, players are allowed to trade chips among each other.

Figure 1 shows an example of a Colored Trails setting, in which there are three players, each with their own set of chips. Agent A_2 on the left, for example, has one dotted chip, one gray chip, and three striped chips. If agent A_2 is at the tile marked as 3, he can therefore move to tile 2 if he hands in one of his striped chips. However, if agent A_2 wishes to move to tile 7, he will have to make a trade with either agent A_1 or agent R to obtain a black chip.

Depending on the aspect of negotiation that is being investigated, scoring rules vary. Following [14], a player that reaches his goal tile is awarded 50 points. If a player is unable to reach his goal tile, he pays a penalty of 10 points for each tile in the shortest path from his current location to his goal location. To focus our research on the effectiveness of higher-order theory of mind in mixed-motive settings, players do not receive any points for unused chips. This way, players have to compete to obtain the chips they need to reach their goal location, and cooperate to find a mutually beneficial trade.

We consider a standard Colored Trails setup in which players are put into either the role of *allocator* or the role of *responder* [14]. An allocator can offer to trade some of his chips against some of the responder's chips. The responder does not make trades of her own. Instead, she chooses whether to accept an offer made to her by an allocator. We focus our attention on the scenario that includes two allocators and one responder. Here, allocators may benefit from considering the goal of the responder, as well as possible offers of the competing allocator.

The Colored Trails game is an example of a mixed-motive situation, in which players can generally improve their score by trading chips with another player. Since mutually beneficial trades may exist, an allocator may benefit from using theory of mind, and explicitly consider the goals of his trading partner. We expect that allocator agents capable of using theory of mind will outperform agents that are unable to consider the goals of other agents. Furthermore, we also expect that in cases where there are multiple allocators, allocator agents perform better when they are of a higher order of theory of mind.

3 Agents playing Colored Trails

In our simulations, we consider repeated single-shot Colored Trails games, in which the set of players is divided into distinct sets of allocators and responders. Each allocator can offer to trade any subset of his own chips for any subset of chips belonging to one of the responders. For example, an allocator can give all his chips to the responder, or ask that the responder give all her chips to the allocator. The responder chooses whether or not to accept any of these offers.

3.1 Agent types

We consider two types of theory of mind agents. Both types of theory of mind agents play the best-response given their beliefs about the behaviour of others, but they differ in the way they form these beliefs. Agents with iterated best-response beliefs (IBR) maximize their own expected payoff under the assumption that other agents do the same. This behaviour is similar to the iterated best-response models such as cognitive hierarchy models [20] and level-n theory [21]. IBR agents believe that other players will only choose an action that maximizes their expected score, and assign probability zero to the event that a co-player will perform any other action. This approach guarantees the best outcome when the agent's beliefs are correct. However, this approach ignores that other players may have different beliefs or a different understanding of the situation.

The assumption of iterated best-response models can be weakened by assuming that players choose better actions with higher probabilities, such as in t-solutions [22], quantal response equilibria [23], or utility proportional beliefs [16]. In addition to the iterated best-response agents described above, we also consider utility-proportional beliefs (UPB) agents in the setting of Colored Trails. The UPB agent believes that other allocators may choose any offer that would increase the allocator's score, but that the probability that he will make a certain offer is proportional to the expected utility of that offer. As a result, a UPB agent may perform better than an IBR agent when his beliefs are incorrect.

The following subsections illustrate the different orders of theory of mind reasoning involved in the game of Colored Trails. To avoid confusion, we will refer to allocators as if they were male, and responders as if they were female.

3.2 Responders

In the Colored Trails game, a responder is a player that does not offer to trade chips herself. Instead, she receives offers from other players, and decides whether to accept any of these offers. We assume that a responder refuses any offer that strictly decreases her score. If a responder is offered more than one acceptable trade, we assume that she chooses in a utility-maximizing way. That is, the responder selects the offer that allows her to reach her goal location as closely as possible without considering the score of the allocator. If multiple offers satisfy this condition, she selects one of these offers at random.

Once the responder has made a choice, a Colored Trails game ends. We do not consider learning across games, which may allow a responder to influence the behaviour of an allocator in future games. This means that for a responder, there is no additional benefit of predicting the offers an allocator is likely to make. The responders described here therefore do not consider the beliefs, desires, and goals of other agents, and as a result, do not make use of theory of mind.

3.3 Zero-order theory of mind allocator

A zero-order theory of mind (ToM_0) allocator understands the game, but is unable to attribute any mental content such as beliefs, desires, or goals to a responder. That is, although a ToM_0 allocator can determine what chips a responder would need to reach her goal location, he is unable to consider the possibility that she wants to reach her goal location. Instead, the zero-order theory of mind allocator considers the total set of chips that are owned by himself and the responder. He then determines the subset of chips C that will allow him to move to a tile as close as possible to his goal location. If there are multiple subsets of chips that satisfy this condition, the ToM_0 allocator selects one of these subsets at random. The allocator then offers to trade in such a way that he receives all the chips in the subset C, while leaving the remaining chips for the responder.

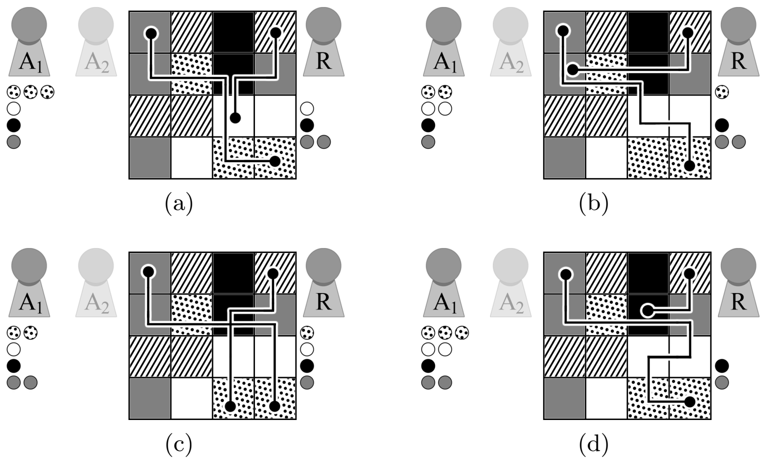
Since the ToM_0 allocator cannot attribute goals or beliefs to other agents, we assume that he does not make any predictions about the offers made by the competing allocator. Example 1 shows the behaviour of a ToM_0 allocator in a game with two allocators and one responder. In this example, agents form beliefs based on iterated best-response.

Example 1. Consider the setup illustrated by Figure 1. In this situation, the game is played on a 4 by 4 board with five different colors. There are two allocators, indicated by A_1 and A_2 to the left of the board, each with their own set of chips. Allocators are initially placed on the top left tile (tile 1) and aim to get as close as possible to the bottom right tile (tile 16). There is a single responder R, depicted to the right of the board. Unlike the allocators, she is initially placed at the top right tile (tile 4) and aims to reach the bottom left tile (tile 13).

Suppose that the agent A_1 is a ToM_0 allocator. Allocator A_1 cannot move with his own chips, but with the combined set of chips of agent A_1 and agent R, there are four possible paths for agent A_1 to reach his target, as depicted in Figure 2. The agent randomly selects one of these paths and makes corresponding offer that would yield him the chips that he needs to reach his goal.

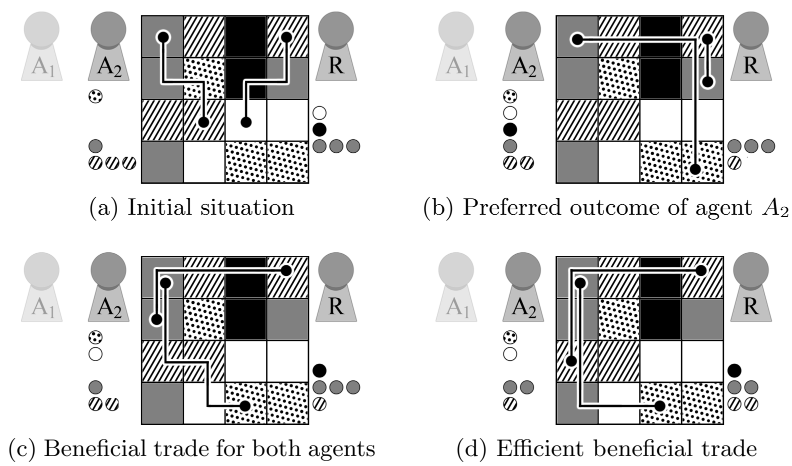
3.4 First-order theory of mind allocator

The first-order theory of mind (ToM_1) allocator considers that responders and other allocators have beliefs and goals. While deciding to offer a trade to a responder, the ToM_1 allocator considers the viewpoint of the responder to determine whether he would accept if he were in her place. Concretely, the ToM_1 allocator does not make any offers that would decrease the score of the responder.



(a) (b) (c) (d)

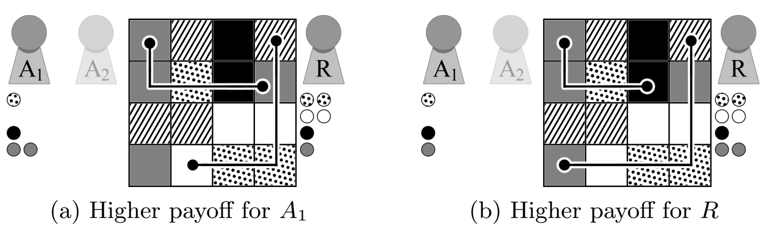
Fig. 2: If agent A_1 is a ToM_0 allocator as in Example 1, he is unable to consider the goals of responder R when making a trade offer. Instead, he offers to make a trade that would maximize his own score.



(a) Initial situation (b) Preferred outcome of agent A_2

(c) Beneficial trade for both agents (d) Efficient beneficial trade

Fig. 3: If agent A_2 is a ToM_1 allocator as in Example 2, he considers both his own goals, as well as the goals of the responder R and competing allocator A_1.



(a) Higher payoff for A_1 (b) Higher payoff for R

Fig. 4: If agent A_1 is a ToM_2 allocator as in Example 3, he believes that agent A_2 also considers the responder's goals when making an offer to her. In this case, the ToM_2 allocator A_1 chooses between two alternatives.

The ToM_1 allocator also considers the offers that he believes the competing allocator to make. However, the ToM_1 allocator does so without considering the possibility that the competing allocator is trying to predict the offer he is going to make himself. Instead, the ToM_1 allocator assumes that the competing allocator is a ToM_0 allocator. Using the procedure outlined in the previous subsection, the ToM_1 agent determines which offers the competing allocator is likely to make. The ToM_1 allocator then chooses to offer the trade that he expects will yield him the highest score. Example 2 illustrates the behaviour of a ToM_1 allocator.

Example 2. Consider the setup illustrated by Figure 1, and suppose that agent A_2 is a ToM_1 allocator. Initially, agents A_2 and R can each move three steps towards their own goal location (see Figure 3a). There is a trade that would allow agent A_2 to reach his goal (see Figure 3b), but responder R would only be able to move one step towards her goal in this case. Using his first-order theory of mind, agent A_2 concludes that responder R would not accept this trade.

It is not possible for agent A_2 to offer a trade that will allow him to reach his goal, and also increase the responder's score. However, the ToM_1 allocator can compromise by offering either the trade shown in Figure 3c or the one shown in Figure 3d. Although the ToM_1 agent is indifferent between these outcomes, he knows that the responder prefers the outcome of Figure 3d. Moreover, he knows that if agent A_1 makes an offer that allows the responder to move exactly four tiles closer to her goal, the offer shown in Figure 3c could be rejected by the responder, while the offer shown in Figure 3d would still be accepted. The ToM_1 allocator A_2 therefore chooses to make the offer as shown in Figure 3d.

3.5 Higher-order theory of mind allocator

Similar to the first-order theory of mind allocator discussed above, the second-order theory of mind (ToM_2) allocator forms beliefs about the trades that other allocators will offer, as well as the likelihood that a responder will accept a given offer. Note that since the responder does not make use of theory of mind, allocators do not benefit from considering the beliefs, goals, and intentions the responder may be attributing to others. As a result, both the ToM_1 allocator and the ToM_2 allocator believe that the responder will accept the trade that will yield her the highest score. The difference in performance of ToM_1 agents and ToM_2 agents is determined only by their ability to compete with other agents.

While the ToM_1 allocator believes that competing allocators offer a trade that maximizes their personal score, the ToM_2 allocator believes that competing allocators also take the point of view of the responder into account. That is, the ToM_2 allocator believes that competing allocators know that the goal of a responder is to approach her goal location as closely as possible. The ToM_2 allocator also believes that competing allocators try to predict the trade he is going to offer himself, and takes this into account when making his offer.

For increasingly higher orders of theory of mind, theory of mind allocators continue this pattern of forming increasingly deeper nested beliefs, and assuming that other agents are more sophisticated. In this paper, we restrict our investigation to ToM_i agents for $i = 0, 1, 2, 3, 4$.

Example 3. Consider the setup illustrated by Figure 1, and suppose that agent A_1 is a ToM_2 allocator. Following the process described in Example 2, agent A_1 concludes that agent A_2 is going to make the offer depicted in Figure 3d. This trade would allow the responder to move five tiles towards her goal location.

The ToM_2 allocator A_1 can choose to match this offer by making the offer shown in Figure 4a. If responder R were to accept this offer, allocator A_1 can move four tiles closer to his goal location, increasing his score by 40. However, allocator A_1 believes that allocator A_2 will make an offer that allows responder R to move five tiles towards her goal as well. In this case, responder R will randomly select which offer to accept, which means that there is a 50% probability that the responder will not accept the offer of allocator A_1. The ToM_2 allocator A_1 therefore assigns an expected gain of 20 to the offer shown in Figure 4a.

Alternatively, the ToM_2 allocator can make a better offer to responser R by allowing her to reach her goal location (Figure 4b). Allocator A_1 expects that responder R will accept this offer, allowing him to move three tiles to his goal location and increase his score by 30. Since this is the higher expected gain, ToM_2 allocator A_1 decides to make the offer shown in Figure 4b.

4 Simulation

We performed simulations of single-shot Colored Trails games, designed after the games in [14]. Games were played on a 4 by 4 board of square tiles. Each tile on the board was randomly colored with one of five possible colors. Players were allowed to move horizontally and vertically, but diagonal movements were not allowed. Each game involved two allocators and one responder. To make individual game settings more comparable, the responder was always initially located on the top right tile, while her goal was to reach the bottom left tile. As a result, the responder has 20 different possible paths to reach her goal, each using six chips. Both allocators were initially placed on the top left tile, while their goal location was the bottom right tile. In this setup, the goal of the responder overlaps partially with the goals of the allocators, but not completely.

At the start of the game, each player received an initial set of six randomly colored chips. Since each player needs at least six chips to reach his or her goal location, it is sometimes possible that after a trade, both the allocator and the responder can reach their respective goals. However, this is not always the case. To ensure that each allocator has an incentive to negotiate to increase his score, game settings in which some player can reach his or her goal with the initially assigned set of chips without trading were excluded from analysis.

To determine the effectiveness of theory of mind, we generated 10,000 random game settings. In each of these settings, we determined the score of a focal ToM_i allocator in the presence of a competing ToM_j allocator, for each combination of $i, j = 0, 1, 2, 3, 4$. The average score was measured for the same 10,000 game settings in each condition. This was both done for agents that base beliefs on iterated best-response, as well as for agents that hold utility-proportional beliefs.

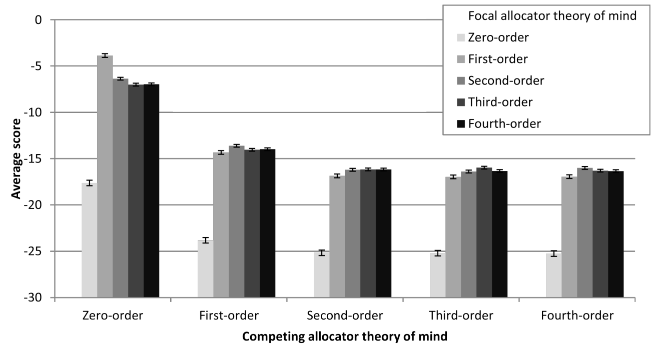


Fig. 5: Average scores of iterated best-response agents that differ in their order of theory of mind over 10,000 initial situations. Brackets indicate standard error.

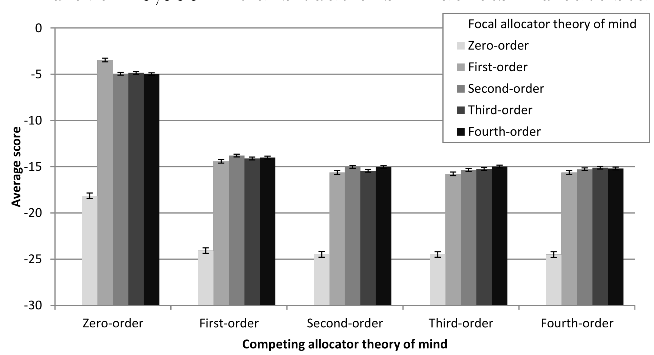


Fig. 6: Average scores of agents with utility-proportional beliefs that differ in their order of theory of mind over 10,000 initial situations. Brackets indicate standard error.

5 Results

We ran simulations of agents playing the Colored Trails for the two types of agents described in Section 3.1. The results for the agents who base their beliefs on iterative best-response are shown in Figure 5, while Figure 6 shows the results for agents that make use of utility proportional beliefs. Both figures summarize the average score of a focal allocator in the Colored Trails game as a function of his order of theory of mind and the order of theory of mind of the competing allocator. The figures show that, irrespective of the theory of mind ability of the competing allocator, focal ToM_1 allocators always score higher than focal ToM_0 allocators. When the competing allocator is a ToM_0 agent, the focal ToM_1 allocator also outperforms focal higher-order theory of mind allocators. Note that in this case, the focal ToM_1 allocator's assumption about the theory of mind abilities of the competing allocator are correct, while higher-order theory of mind allocators overestimate the competing allocator.

The focal allocator benefits from higher-order theory of mind when he competes with an allocator that can reason about the mental states of others. Paired t tests show that for any order of theory of mind of the competing allocator, performance of a focal ToM_2 allocator differs significantly from the performance of a focal ToM_1 allocator. However, this difference is low compared to the advantage the focal ToM_1 allocator has over the focal ToM_0 allocator. Orders of theory of mind higher than the second do not seem to benefit an allocator significantly. As a result, the average score of the focal ToM_3 allocator is not consistently higher than the average score of the focal ToM_2 allocator.

Figure 5 and Figure 6 summarize the performance of agents that differ in the way they form beliefs about other agents. The distributions differ significantly (K-S, $p < 0.01$), but one needs to look carefully to see the differences. For agents that form beliefs based on iterated best-response, a ToM_i allocator is correct in his beliefs when the competing allocator is a ToM_{i-1} agent. However, these agents do not consider the possibility that their beliefs may be wrong. In Figure 5, this results in a stronger advantage for having correct beliefs. In particular, when facing a competing ToM_0 allocator, the focal ToM_1 allocator performs best, while the average score of the focal ToM_2 allocator is the highest when the competing allocator is a ToM_1 agent.

Agents that form utility-proportional beliefs are never completely correct in their beliefs, since their beliefs reflect the possibility of mistakes. Figure 6 shows that as a result, the focal allocator has less of an advantage for being exactly one order of theory of mind higher than the competing allocator. Interestingly, this does not appear to cause lower performance of agents with utility-proportional beliefs compared to agents with iterated best-response beliefs.

6 Discussion and conclusion

Many of the interactions that people engage in on a daily basis involve mixed motives, which are not fully competitive or fully cooperative. When the goals of interacting individuals overlap, there may be an advantage to considering the goals and beliefs of others explicitly, through a theory of mind. In this paper, we investigated whether agents benefit from the ability to reason about higher orders of theory of mind in the particular mixed-motive setting Colored Trails.

In the setting of Colored Trails we used, agents were put into the role of either allocator or responder [14]. The agents engaged in single-shot negotiations, where allocators made an offer which the responder could either reject or accept. Allocator agents were found to benefit greatly from first-order theory of mind, allowing them to consider the goals of other agents when making an offer.

Allocators could also benefit from higher-order theory of mind through competition with another allocator. Our results showed that second-order theory of mind benefits allocators whenever the competing allocator also has a theory of mind. By recognizing that competitors may also consider the point of view of the responder, second-order theory of mind allowed allocators to offer trades that the responder accepted more often than the offers of a first-order theory of mind allocator. These results are compatible with earlier research into the advantage of higher-order theory of mind in competitive settings [24, 25]. However, we did not find any benefit for the use of third-order theory of mind here. A possible explanation is that the settings in [24, 25] are zero-sum games, while allocators in Colored Trails compete for the opportunity to trade with a responder. In Colored Trails, an allocator of a higher order of theory of mind generally makes an offer that is more beneficial to the responder at the expense of his own score.

We compared the performance of allocators that made offers based on iterated best-response models with allocators that form utility proportional beliefs. Allocators that form beliefs based on iterated best-response models believe that every agent is a utility-maximizing agent, while an agent with utility-proportional beliefs takes into account that competing allocators may make mistakes. Interestingly, although our model did not include mistakes, iterated best-response agents did not outperform agents with utility-proportional beliefs.

Our results suggest that in mixed teams of humans and agents, agents that make use of theory of mind will perform better. Based on the experiments in Colored Trails, we expect that cognitive agents will suffer diminishing returns on higher orders of theory of mind. Interestingly, similar results are found for human participants [5], who do well on first-order theory of mind tasks, and have increasingly more difficulty with higher-order theory of mind tasks. In future work, we intend to compare the performance of human participants and theory of mind agents by letting them play directly against each other.

In future research, we aim to increase the emphasis on the mixed-motive nature of Colored Trails by allowing multiple rounds of negotiations. A responder that is allowed to make offers would benefit from considering the beliefs of others. This may give an allocator incentive to consider these beliefs of the responder in his initial offer. Higher orders of theory of mind may also become more effective when the game setting is not fully observable. In our setup, agents know the initial location, the goal location, and the chips in possession of every player. However, in everyday negotiation situations, the goals of the participants are usually not fully known [26]. Higher orders of theory of mind may be beneficial in determining the information available to each agent, as well as the information that agents may be revealing or trying to hide by making a specific offer.

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