A Discussion Game for the Grounded Semantics of Abstract Dialectical Frameworks

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Abstract. Abstract dialectical frameworks (ADFs) have been introduced as formalism for the modeling and evaluating argumentation. However, the role of discussion in evaluating of arguments in ADFs has not been clarified well so far. We focus on the grounded semantics of ADFs and provide the grounded discussion game. We show that an argument is acceptable (deniable) in the grounded interpretation of an ADF without any redundant links if and only if the proponent of a claim has a winning strategy in the grounded discussion game.

Keywords. Abstract argumentation frameworks, Abstract dialectical frameworks, Discussion games.

1. Introduction

Argumentation has received increased attention within artificial intelligence, since the remarkable paper of Dung [1], in which abstract argumentation frameworks (AFs) are presented. Abstract dialectical frameworks (ADFs) introduced in [2] are expressive generalizations of AFs in which the logical relations among arguments can be represented. Applications of ADFs have been presented in legal reasoning [3,4] and text exploration [5].

Although dialectical methods have a role in determining semantics of both AFs and ADFs, the roles are not immediately obvious from the definition of semantics. To cover this gap, quite a number of works have been presented to show that semantics of AFs can be interpreted in terms of structural discussion [6,7,8,9,10,11]. Further, in [12] it is shown that the structural discussion method has been used in human-machine interaction.

Because of the special structure of ADFs, existing methods used to interpret semantics of AFs cannot be reused in ADFs. To address this problem, we have presented the first existing game for ADFs [13]. That game characterizes the preferred semantics. In this work we focus on the grounded semantics of ADFs.

A key question is 'How is it possible to evaluate arguments in a given ADF?' Answering this question leads to the introduction of several types of semantics, defined based on three-valued interpretations. Different semantics reflect different types of point of view about the acceptance or denial of arguments.

In ADFs an interpretation is called *admissible* if it does not contain any unjustifiable information. Most of the semantics of ADFs are based on the concept of admissibility. An interpretation is *complete* if it exactly contains justifiable information. In addition, an interpretation is *grounded* if it collects all the information that is beyond any doubt. Each ADF has a unique grounded interpretation, which can be the trivial interpretation. Hence for the grounded semantics the credulous and the skeptical decision problems coincide. Further, in the hierarchy, grounded semantics have the lowest computational complexity [14]. However, by indicating whether an argument is credulously acceptable (deniable) in a given ADF under grounded semantics we have the answer of the skeptical decision problem of the argument in question under complete semantics.

In this work we present a game that can answer the credulous and therefore the skeptical decision problem of a given ADF, called the *grounded discussion game*. In [15] it is shown that each ADF is equivalent with an ADF without any redundant links. Thus, without loss of generality, the current game is presented over the subclass of ADFs that do not have redundant links. This game works locally by considering those ancestors of an argument in question that can affect the evaluation of the argument in the grounded interpretation. In this way, the grounded decision problem can be answered without constructing the full grounded interpretation. Further, the current methodology can be used to answer the decision problems under grounded semantics of formalisms that can be represented as ADFs, such as AFs.

In Section 2, we present the relevant background. Then, in Section 3, we present the *grounded discussion game* that can capture the notion of grounded semantics. In Section 4 we present soundness and completeness of the method.

2. Background

The basic definitions in this section are derived from those given in [2,16,17].

Definition 1. *An abstract dialectical framework (ADF) is a tuple* $F = (A, L, C)$ *where:*

- A *is a finite set of arguments (statements, positions), denoted by letters;*
- $L \subseteq A \times A$ *is a set of links among arguments;*
- $C = \{\varphi_a\}_{a \in A}$ *is a collection of propositional formulas over arguments, called acceptance conditions.*

An ADF can be represented by a graph in which nodes indicate arguments and links show the relation among arguments. Each argument a in an ADF is labelled by a propositional formula, called acceptance condition, φ_a over $par(a)$ such that, $par(a) = \{b \mid (b, a) \in L\}$. The acceptance condition of each argument clarifies under which condition the argument can be accepted [2,16,17]. Further, acceptance conditions indicate the set of links implicitly, thus, there is no need of presenting L in ADFs explicitly.

An argument a is called an *initial argument* if $par(a) = \{\}$. An *interpretation* v (for F) is a function $v : A \mapsto \{\mathbf{t}, \mathbf{f}, \mathbf{u}\}$, that maps arguments to one of the three truth values true (**t**), false (**f**), or undecided (**u**). Truth values can be ordered via the information ordering relation $<_i$ given by $\mathbf{u} <_i \mathbf{t}$ and $\mathbf{u} <_i \mathbf{f}$ and no other pair of truth values are related by $<_i$. Relation $\leq_i$ is the reflexive and transitive closure of $<_i$. Further, v is called *trivial*, and v is denoted by $v_{\mathbf{u}}$, if $v(a) = \mathbf{u}$ for each $a \in A$. Further, v is called a two-valued interpretation if for each $a \in A$ either $v(a) = \mathbf{t}$ or $v(a) = \mathbf{f}$. Interpretations can be ordered via $\leq_i$ with respect to their information content. Let $\mathcal{V}$ be the set of all interpretations for an ADF F. It is said that an interpretation v is an *extension* of another interpretation w, if $w(a) \leq_i v(a)$ for each $a \in A$, denoted by $w \leq_i v$. Further, we denote the update of an interpretation v with a truth value $x \in \{\mathbf{t}, \mathbf{f}, \mathbf{u}\}$ for an argument b by $v|^b_x$, i.e. $v|^b_x(b) = x$ and $v|^b_x(a) = v(a)$ for $a \neq b$.

Semantics for ADFs can be defined via the *characteristic operator* Γ_F which maps interpretations to interpretations. Given an interpretation v (for F), the partial valuation of φ_a by v, is $\varphi^v_a = \varphi_a[b/\top : v(b) = \mathbf{t}][b/\bot : v(b) = \mathbf{f}]$, for $b \in par(a)$. Applying Γ_F on v leads to v' such that for each $a \in A$, v' is as follows:

$$v'(a) = \begin{cases} \mathbf{t} & \text{if } \varphi^v_a \text{ is irrefutable (i.e., } \varphi^v_a \text{ is a tautology)}, \\ \mathbf{f} & \text{if } \varphi^v_a \text{ is unsatisfiable (i.e., } \varphi^v_a \text{ is a contradiction)}, \\ \mathbf{u} & \text{otherwise.} \end{cases}$$

From now on whenever there is no ambiguity, in order to make three-valued interpretations more readable, we rewrite them by the sequence of truth values, by choosing the lexicographic order on arguments. For instance, $v = \{a \mapsto \mathbf{t}, b \mapsto \mathbf{u}, c \mapsto \mathbf{f}\}$ can be represented by the sequence **tuf**. The semantics of ADFs are defined via the characteristic operator as in Definition 2.

Definition 2. *Given an ADF F, an interpretation v is:*

- *admissible in F iff $v \leq_i \Gamma_F(v)$, denoted by adm;*
- *preferred in F iff v is $\leq_i$-maximal admissible, denoted by prf;*
- *complete in F iff $v = \Gamma_F(v)$, denoted by com;*
- *a (two-valued) model of F iff v is two-valued and $\Gamma_F(v) = v$, denoted by mod,*
- *the grounded interpretation of F iff v is the least fixed point of Γ_F, denoted by grd.*

The notion of an argument being accepted and the symmetric notion of an argument being denied in an interpretation are as follows.

Definition 3. *Let $F = (A, L, C)$ be an ADF and let v be an interpretation of F.*

- *An argument $a \in A$ is called acceptable with respect to v if φ^v_a is irrefutable.*
- *An argument $a \in A$ is called deniable with respect to v if φ^v_a is unsatisfiable.*

One of the main decision problems of ADFs is whether an argument is credulously acceptable (deniable) under σ semantics, for $\sigma \in \{adm, prf, com, mod, grd\}$. Given an ADF $F = (A, L, C)$, an argument $a \in A$ and a semantics $\sigma \in \{adm, prf, com, mod, grd\}$, argument a is *credulously acceptable (deniable)* under σ if there exists a σ interpretation v of F in which a is acceptable (a is deniable, respectively).

In ADFs, relations between arguments can be classified into four types, reflecting the relationship of attack and/or support that exists between the arguments. These are listed in the following definition.

Definition 4. *Let $D = (S, L, C)$ be an ADF. A relation $(b, a) \in L$ is called*

- supporting *(in D) if for every two-valued interpretation v, $v(\varphi_a) = \mathbf{t}$ implies $v|^b_{\mathbf{t}}(\varphi_a) = \mathbf{t}$;*
- attacking *(in D) if for every two-valued interpretation v, $v(\varphi_a) = \mathbf{f}$ implies $v|^b_{\mathbf{t}}(\varphi_a) = \mathbf{f}$;*
- redundant *(in D) if it is both attacking and supporting;*
- dependent *(in D) if it is neither attacking nor supporting.*

Note that the grounded discussion game presented in Section 3 is presented on ADFs without redundant links. Further, in the current work we say that the truth value of *a is presented* in v, if $v(a) = \mathbf{t}/\mathbf{f}$. In addition, for each operator f, the nth power of f is defined inductively i.e. $f^n = f(f^{n-1})$.

3. Grounded Discussion Games

In this section we present a discussion game to answer the credulous (skeptical) decision problem under grounded semantics in a given ADF F does not have any redundant relation, without loss of generality, since any ADF has an equivalent of ADF of this kind; see *Theorem* 4.2.13 of [15].

A grounded discussion game (GDG) is a dispute between a proponent (P) and an opponent (O). We now explain how a GDG works. However, for the formal definition of GDG you may skip it an go to Definition 5. A GDG is started by a claim of P about the truth value of argument a in the grounded interpretation of a given ADF. That is, P believes that the trivial interpretation $g_0 = v_{\mathbf{u}}$ can be extended to the grounded interpretation that contains the initial claim. O challenges P by asking whether a is an initial argument. If P finds that a is an initial argument and presents the truth value of a to O, then O has to check whether this value is the same as the initial claim. In this case P wins if the checking of O leads to a positive answer. On the other hand, if P answers that a is not an initial argument, then O asks whether an ancestor of a is an initial argument. If P finds that there is no initial argument in the ancestors of a, then the game is stopped and O wins the game.

However, if a is not an initial argument but P finds that b is an ancestor of a which is also an initial argument, then P updates the information of g_0 with $g = g_0|^b_x$, such that x is the truth value of b in the grounded interpretation F. Further, in this step a set of arguments in the shortest paths, between a and b, are presented by P to O. Note that it is possible that there exists more than one shortest path between two arguments. Actually, by presenting g, P says that g can be extended to the grounded interpretation of F.

Now, O checks a piece of information presented in g and the initial claim. If g contains the initial claim, then the game halts and P wins the games. If the information of g is in contradiction with the initial claim, then O wins the game. Since a is not an initial argument, this checking step by O does not lead to acceptance or rejection of the initial claim. That is, presenting of g by P did not convince O about the initial claim.

Thus, O asks P whether P can extend the information of g to an interpretation that contains the initial claim. To this end, P evaluates the acceptance conditions of the children of the argument presented in g under the information of g and presents g'. Then, O continues the game. If O indicates that g' contains the initial claim, then the game stops. If g and g' contain the same piece of information, O asks P for a new initial ancestor of a. Otherwise, O asks P to extend g' more.

The game continues between P and O alternately. P tries to extend the information of g_0 to an interpretation that contains the initial claim to support the belief. O tries to challenge P by either: 1. checking the information of the interpretation which is presented by P as an answer, or 2. asking whether the argument presented in the initial claim is an initial argument, or 3. requesting P to find an ancestor of a which is an initial argument, or 4. requesting P to extend the information of the answer given by P to an interpretation that contains the initial claim. In Example 1 we show how the game works before presenting the formal definitions. If desire you may skip Example 1 and go to Definition 5.

Example 1. *Let $F = (\{a,b,c,d,e,f\}, \{\varphi_a : \bot, \varphi_b : \neg a \vee \neg e, \varphi_c : b \wedge f, \varphi_d : e \wedge \neg c, \varphi_e : \neg f, \varphi_f : \top\})$ be a given ADF, depicted in Figure 1. We know that $grd(F) = \mathbf{fttfft}$. P claims that d is deniable with respect to the grounded interpretation of F. That is, by the initial claim P believes that $d \mapsto \mathbf{f}$ belongs to the grounded interpretation. In other words, the claim of P says that $g_0 = v_{\mathbf{u}}$ can be extended to the grounded interpretation that contains the initial claim.*

- *P says $g_0 = v_{\mathbf{u}}$ can be extended to the grounded interpretation of F that contains $d \mapsto \mathbf{f}$.*
- *O asks P whether d is an initial argument.*
- *P checks the acceptance condition of d and the answer is 'no, d is not an initial argument'. Thus, the information of g_0 does not change. For technical reasons we let $g_1 = g_0$.*
- *O challenges P by asking whether any of the ancestors of d is an initial argument.*
- *P checks the acceptance conditions of the parents of d, namely c and e; neither of them is an initial argument. Then, P goes one step further and checks the parents of c and e, which are b and f. Here, f is an initial argument. Since P finds an ancestor of a which is an initial argument, P stops searching. By $\varphi_f : \top$, f is acceptable in the grounded interpretation of F. Thus, P presents interpretation $g_2 = g_1|^f_{\mathbf{t}} =$ **uuuuut** and set $Ancestors(d, g_1) = \{d,e,c,f\}$, which contains the arguments on the shortest paths between the initial claim d and the initial argument f, that is presented in g_2 but not in g_1. P claims that g_2 can be extended to the grounded interpretation of F that contains the initial claim.*
- *Then O checks the information that is presented by g_2. Since g_2 does not contain any information about the initial claim, O asks P whether P can extend g_2.*
- *To this end, P evaluates the truth value of the children of f that are in $Ancestors(d, g_1)$ under g_2. The children of f that appear in that set are c and e. Thus, P evaluates $\varphi_c^{g_2} \equiv b \wedge \top \equiv b$ and $\varphi_e^{g_2} \equiv \bot$. That is, e is deniable with respect to the grounded interpretation of F. Thus, P presents $g_3 = g_2|^e_{\mathbf{f}} =$ **uuuuft** to O as an extension of g_2 and P claims that g_3 can be extended to the grounded interpretation of F that contains the initial claim.*

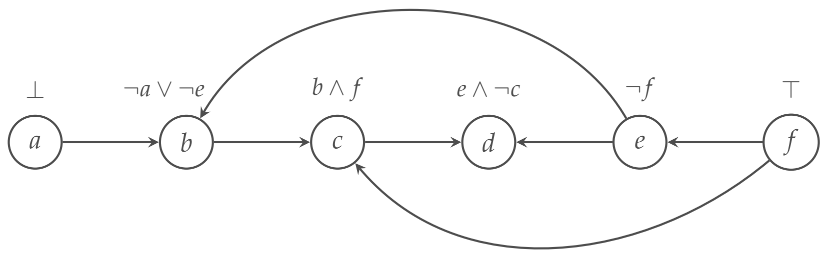

Figure 1. ADF of Examples 1

- *O finds that g_3 extends the information of g_2 and it does not present any information in contrast with the initial claim. However, g_3 does not contain any information about the initial claim. Thus, O asks P whether P can extend g_3 to an interpretation that contains the initial claim.*
- *Again P evaluates the only child of e in set $Ancestors(a, g_1)$, namely d, under g_3. This attempts leads to $g_4 = \mathbf{uuufft}$.*
- *O checks the information given by g_4. Since g_4 contains the initial claim, the discussion between P and O halts here and P wins the game.*

Here, P does not present the grounded interpretation of F, however, P presents a constructive proof for the initial claim. That is, to indicate the initial claim, P works on the truth value of the argument in question locally. Thus, the grounded discussion game can answer the credulous decision problem under the grounded semantics of an ADF without indicating the truth value of all arguments in the grounded interpretation.

Definition 5. *Let $F = (A, R, C)$ be an ADF, let a be an argument and let S be a set of arguments. Function $Par(S)$ shows the set of parents of the elements of S; function $child(a)$ designates the set of children of a; and function $anc(a)$ presents the set all ancestors of a, defined formally in the following.*

- $Par(S) = \bigcup_{a \in S} par(a)$,
- $child(a) = \{b \mid (a, b) \in R\}$,
- $anc(a) = \bigcup_{n=1}^{m} Par^n(a)$ *such that there exists m with* $Par^m(a) \subseteq \bigcup_{i=1}^{m-1} Par^i(a)$.

Note that whenever S contains only one argument a, $Par(S) = par(a)$ and we write $Par(a)$ for $Par(\{a\})$. The aim of $anc(a)$ is to collect a's ancestors and condition $Par^m(a) \subseteq \bigcup_{i=1}^{m-1} Par^i(a)$ is a guarantee that the function does not go into a loop. If $b \in anc(a)$ is an initial argument, then we call it an *initial ancestor of a*.

The grounded discussion game is defined based on the following moves; some of them are functional moves. For instance, $Eval(g)$ is a unary function, defined over interpretations. Some of them are statement moves to present a claim or a request for instance, $IniAnc(a, g)$ is a statement move by which O asks P to find an initial ancestor of a which is not presented in g.

- $IniClaim(a, x)$: with this statement move P presents her/his beliefs that a is assigned to x such that $x \in \{\mathbf{t}, \mathbf{f}\}$ in the grounded interpretation of F.

- $Ini(a)$: with this statement move O asks P whether a is an initial argument.
- $CheckIni(a) : A \to \mathcal{V}$: with this functional move P checks whether a is an initial argument.
- $Check(g_{i-1}, g_i)$: with this move O compares the information presented in g_{i-1} and g_i, i.e. whether $g_{i-1} <_i g_i$ or $g_{i-1} \sim_i g_i$.
- $IniAnc(a, g)$: with this statement move O asks P to present at least one initial ancestor of a which is not presented in g, together with its truth value.
- $NewIniAnc(a, g) : A \times \mathcal{V} \to \mathcal{V}$: with this functional move P presents initial ancestors of a which are requested by O in $IniAnc(a, g)$.
- $Ancestors(a, g) : A \times \mathcal{V} \to 2^A$: with this functional move P presents the set of arguments in the shortest paths between a and the elements of $NewIniAnc(a, g)$.
- $Extend(g)$: with this statement move O requests P to extend g.
- $Eval(g) : \mathcal{V} \to \mathcal{V}$: with this functional move P evaluates the truth value of the children of the arguments presented in g which appears in the last $Ancestors(a, -)$ under g.

In the game, P has the responsibility of constructing a proof for the initial claim. On the other hand, O aims to block the discussion by finding a contradiction or challenging P in such a way that P cannot answer the challenge.

- The game between P and O starts with $IniClaim(a, x)$ by which P presents a belief about the truth value of argument a, namely x in the grounded interpretation of F. In this step, intuitively, P believes that $g_0 = v_{\mathbf{u}}$ can be extended to the grounded interpretation that contains the claim.
- Then, O applies statement $Ini(a)$, asks whether a is an initial argument.
- Now, it is P's turn to apply function $CheckIni(a) : A \to \mathcal{V}$ to check the acceptance condition of a. If a is an initial argument, then the output of $CheckIni(a)$ is $g_1 = g_0|^a_{\mathbf{t}/\mathbf{f}}$. Otherwise, $g_1 = g_0$.
- By $Check(g_{i-1}, g_i)$, O checks whether $g_{i-1} <_i g_i$ or $g_{i-1} \sim_i g_i$.
  * If $g_{i-1} <_i g_i$ and g_i contains the initial claim or the negation of the initial claim, then the game stops.
  * If $g_{i-1} <_i g_i$ and g_i does not contain any information about the initial claim, then O requests P to extend g_i. That is, O applies $Extend(g_i)$.
  * If $g_{i-1} \sim_i g_i$,
    * if g_i is the output of either $CheckIni(a)$ or $Eval(g_{i-1})$, then O asks P to present a new initial ancestor of a. That is, O applies $IniAnc(a, g_{i-1})$,
    * if g_i is the output of $NewIniAnc(a, g_{i-1})$, then the game stops.
- After statement move $IniAnc(a, g_i)$ by O, P applies function $NewIniAnc(a, g_i)$ to find new initial ancestors of a. The output of this function is interpretation g_{i+1} with $g_{i+1} = g_i|^b_{\mathbf{t}/\mathbf{f}}$ such that b is an initial ancestor of a, that was not presented in g_i. This function will be defined precisely in the following.

- Further, after move $IniAnc(a,g_i)$ presented by O, P presents a set of arguments between the initial claim and the elements of $NewIniAnc(a,g_i)$, with the shortest distance, by applying function $Ancestors(a,g_i) : A \times \mathcal{V} \rightarrow 2^A$. If there are more than one shortest path between the initial claim and an element of $NewIniAnc(a,g_i)$, then $Ancestors(a,g_i)$ presents the arguments of all paths with the shortest length.
- After statement move $Extend(g_i)$ presented by O, P applies function $Eval(g_i) : \mathcal{V} \rightarrow \mathcal{V}$. The output of this function is interpretation g_{i+1} with $g_{i+1} = g_i|^{b}_{\varphi_b^{g_i}}$ such that b is a child of an argument that is presented in g_i that also appears in the last output of $Ancestors(a,-)$.

The only function that needs more explanation is $NewIniAnc(a,g)$, by which P tries to find the truth values of the initial ancestors of a that are not presented in g. To this end, P uses the modification of the function *anc*, defined in Definition 5, which is called $NewAnc(a,g) : A \times \mathcal{V} \rightarrow 2^A$. This function is a binary function that takes the argument a and interpretation g, and returns the set of ancestors of a. However, if there exists an initial ancestor of a, the truth value of which is not indicated in g, then the function stops. This is the reason why this function is called the new ancestors of a with respect to g.

$NewAnc(a,g) = \bigcup_{n=1}^{m} Par^n(a)$ such that there exists m such that $(Par^m(a) \subseteq Par^{m-1}(a)) \vee (\exists p \in Par^m(a)$ such that $\varphi_p \equiv \top / \bot \wedge p$ was not presented in $g)$

Then among the elements of $NewAnc(a,g)$, P looks for the initial arguments. Function $NewIniAnc(a,g) : A \times \mathcal{V} \rightarrow \mathcal{V}$, presented in the following, takes a and g, and updates g by adding the truth values of the initial ancestors of a that appear in $NewAnc(a,g)$.

$NewIniAnc(a,g) = g|^{b}_{\varphi_b^{g}}$ such that $b \in NewAnc(a,g)$ and b is an initial argument.

Definition 6. *Let $F = (A, R, C)$ be an ADF. A grounded discussion game for credulous acceptance (denial) of $a \in A$ is a sequence $[g_0, \ldots, g_n]$ such that the following conditions hold:*

- $g_0 = v_{\mathbf{u}}$;
- $g_1 = CheckIni(a)$;
- *for* $0 \leq i < n$, $g_i \leq_i g_{i+1}$;
- g_n *contains either*
  - ∗ *the initial claim, or*
  - ∗ *the negation of the initial claim, or*
  - ∗ g_{n-1} *is the output of* $NewIniAnc(a, g_{n-2})$ *and* $g_{n-1} \sim g_n$.
- *for* $1 < i < n$, *if* $g_{i-1} <_i g_i$, *then* g_{i+1} *is the output of* $Eval(g_i)$;
- *for* $0 < i < n$, *if* $g_{i-1} \sim g_i$, *then* g_{i+1} *is the output of* $NewIniAnc(a, g_i)$.

Definition 7. *Let F be a given ADF. Let $[g_0, \ldots, g_n]$ be a grounded discussion game for credulous acceptance (denial) of an argument.*

- *P wins the game if g_n satisfies the initial claim,*
- *O wins the game if g_n satisfies the negation of the initial claim, or $g_{n-1} = NewIniAnc(a, g_{n-2})$ and $g_{n-1} \sim g_n$.*

Example 2 is an instance of a game in which O wins.

Example 2. *Let $F = (\{a, b, c\}, \{\varphi_a : \neg b, \varphi_b : \neg c, \varphi_c : \neg a\})$ be an ADF. We know that $grd(F) = v_{\mathbf{u}}$. P claims that b is acceptable in the grounded interpretation of F.*

- *$IniClaim(b, \mathbf{t})$: P believes that g_0 can be extended to the grounded interpretation of F in which b is acceptable.*
- *O asks $Ini(b)$.*
- *P applies $CheckIni(b)$ to answer the challenge. The output is $g_1 = g_0$.*
- *O applies $Check(g_0, g_1)$. Since $g_0 \sim g_1$ and g_1 is the output of $CheckIni(b)$, O requests $IniAnc(b, g_1)$.*
- *To answer $IniAnc(b, g_1)$, P applies $NewIniAnc(b, g_1)$. To this end, first P computes $NewAnc(b, g_1) = \{a, b, c\}$. Since none of them is an initial argument, then the output of $NewIniAnc(b, g_1)$ is $g_2 = g_1$.*
- *O applies $Check(g_1, g_2)$, which leads to $g_1 \sim g_2$. Since g_2 is an output of function $NewIniAnc(b, g_1)$, the game stops and by Definition 7, O wins the game.*

That is, the initial claim of P that b is acceptable with respect to the grounded interpretation of F is false. This corresponds with the fact that the grounded interpretation $v_{\mathbf{u}}$ of F does not satisfy the belief of P.

4. Soundness and Completeness

In this section we show that the presented method is sound and complete. To show the completeness, first we show that in an ADF without any redundant links, the grounded interpretation assigns the truth value of an argument to $\mathbf{t}/\mathbf{f}$ if it is either an initial argument or its truth value is affected by the initial ancestors. This corollary is the direct result of Lemma 1.

Lemma 1. *Let F be an ADF without any redundant link, that does not have any initial argument. Then the grounded interpretation of F is $v_{\mathbf{u}}$.*

Proof. Toward a contradiction, assume that F does not contain an initial argument and $grd(F) \neq v_{\mathbf{u}}$. Let a be an arbitrary argument. We show that $\varphi_a^{v_{\mathbf{u}}}$ is neither irrefutable nor unsatisfiable. Since F does not have any initial argument, a has a parent.

- Consider that a has a parent b such that (b, a) is a dependent link. By the definition of dependent link, there are two-valued interpretations v, w such that $v(\varphi_a) = \mathbf{t}$ and $v|_{\mathbf{t}}^{b}(\varphi_a) \neq \mathbf{t}$, and $w(\varphi_a) = \mathbf{f}$ and $w|_{\mathbf{t}}^{b}(\varphi_a) \neq \mathbf{f}$. Thus, $v, w \in [v_{\mathbf{u}}]_2$ and $v(\varphi_a) \neq w(\varphi_a)$. Therefore, $\varphi_a^{v_{\mathbf{u}}}$ is neither irrefutable nor unsatisfiable.

- Consider that none of the parents of a is dependent. Construct the two-valued interpretation v in which 1. $b \mapsto \mathbf{f}$ if (b,a) is an attacker, and 2. $b \mapsto \mathbf{t}$ if (b,a) is a supporter. Construct the two-valued interpretation w in which 1. $b \mapsto \mathbf{t}$ if (b,a) is an attacker, and 2. $b \mapsto \mathbf{f}$ if (b,a) is a supporter. That is, $v,w \in [v_{\mathbf{u}}]_2$. If either $a \notin par(a)$ or (a,a) is a supporter, then $v(\varphi_a) \equiv \mathbf{f}$ and $w(\varphi_a) \equiv \mathbf{t}$. Thus, $\varphi_a^{v_{\mathbf{u}}}$ is neither irrefutable nor unsatisfiable. If $a \in par(a)$ and (a,a) is an attacker, then $v(\varphi_a) = w(\varphi) = \mathbf{u}$. Thus, $\varphi_a^{v_{\mathbf{u}}}$ is neither irrefutable nor unsatisfiable.

Thus, the assumption that $a \mapsto \mathbf{t}/\mathbf{f} \in grd(F)$ is wrong. Hence, the unique grounded interpretation of F is $v_{\mathbf{u}}$. □

Corollary 2. *Every argument that is acceptable (deniable) with respect to the grounded interpretation of ADF F, without any redundant links, either is an initial argument or has at least one initial ancestor.*

Proof. Let F be an ADF without redundant links. Assume that a is an argument that is not an initial one and does not have any initial ancestor. By the proof method of Lemma 1, $\varphi_a^{v_{\mathbf{u}}}$ is neither irrefutable nor unsatisfiable. Thus, a is neither acceptable nor deniable with respect to the grounded interpretation of F. □

Theorem 3. *(Soundness) Let F be a given ADF. If there is a grounded discussion game for an initial claim of P in which P wins, then the grounded interpretation of F satisfies the initial claim of P.*

Proof. Suppose that the initial claim of P is that 'a is acceptable (deniable) in the grounded interpretation'. Let $[g_0, \ldots, g_n]$ be a grounded discussion game for the initial claim of P, that is, g_n satisfies the initial claim. We show that the grounded interpretation v of F satisfies the initial claim. By the definition the grounded interpretation of F is the least fixed point of the characteristic operator. That is, there exists m such that $\Gamma_F^m(v_{\mathbf{u}}) = v$. We show that $g_n \leq_i v$.

In the grounded discussion game if $n = 1$, that is $[g_0, g_1]$, then a is an initial argument. Thus, clearly $g_1 \leq_i \Gamma_F(v_{\mathbf{u}})$. Since Γ is a monotonic operator, $g_1 \leq_i v$. Consider that in the grounded discussion game $n > 1$. By induction on n it is easy to show that for each m with $0 \leq m \leq n$, $g_m \leq_i v$ holds.

Therefore, in the grounded discussion game $[g_0, \ldots, g_n]$ for any i with $0 \leq i \leq n$, $g_i \leq_i v$ holds. In specific, $g_n \leq_i v$. Thus, the initial claim of P is satisfied in the grounded interpretation of F.

□

Definition 8. *Let F be an ADF. The distance from argument a to b in F is the distance from a to b in the associated directed graph of F, denoted by $d(a,b)$. That is, $d(a,b)$ is the length of a shortest directed path from a to b in the directed graph associated to F.*

Theorem 4. *(Completeness) Let F be a given ADF without any redundant links. If a is acceptable (deniable) in the grounded interpretation of F, then there is the grounded discussion game for the initial claim of accepting (denying) of a.*

Proof. Let F be an ADF and let v be the grounded interpretation of F. Further, let a be an argument which is accepted (denied) with respect to v. Since F does not have any redundant links, by Corollary 2, either a is an initial argument or a has at least one initial ancestor. We construct a grounded discussion game for the initial claim of $a \mapsto \mathbf{t}/\mathbf{f}$ in which P wins. Let $g_0 = v_{\mathbf{u}}$. If a is an initial argument, then $g_1 = g_0|^a_{\mathbf{t}/\mathbf{f}}$. Thus, $[g_0, g_1]$ is the grounded discussion game, in which $g_1 = \mathit{CheckIni}(a)$, that satisfies the initial claim.

If a is not an initial claim, then let $g_{1_1} = g_0$ and list the set of initial ancestors of a, for instance $L = [a_1, \ldots, a_k]$. Assume that L is ordered based on the distance to a, increasingly. That is, $d(a_i, a) \leq d(a_{i+1}, a)$, for i with $1 \leq i < k$. Let us categorize L based on the distance of arguments to a. For instance, let $L_1 = \{a_1\} \cup B$ such that $B = \{a_i \mid d(a_i, a) = d(a_1, a)\}$. If $B \neq \{\}$, then m is an integer such that $d(a_m, a) = d(a_1, a)$ and $m > i$ for $a_i \in B$, otherwise, $m = 1$. Let $L_2 = \{a_i \mid d(a_i, a) = d(a_{m+1}, a)\}$. Continue this process. Since L is finite, there exists p such that $L = \bigcup_{i=1}^{p} L_i$.

Let $g_{2_1} = g_{1_1}|^b_{v(b)}$ such that $b \in L_1$. For $j \geq 1$, for $i \geq 2$, 1. if $g_{i_j} > g_{i-1_j}$, then let $g_{i+1_j} = g_{i_j}|^b_{v(b)}$ such that b is a child of an argument in L_j that is on a path between a and an element of L_j. 2. If $g_{i_j} \sim g_{i-1_j}$, then let $g_{i+1_j} = g_{i_j}|^b_{v(b)}$ such that $b \in L_{j+1}$. If any of the g_{i_j} satisfies the initial claim, then stop the above loop.

Because the number of arguments on the paths between a and elements of L is finite, then the above loop will stop. Consider that the above loop halts in g_{i_j}. We claim that $D = [g_0, \ldots, g_{i_j}]$ is the GDG that satisfies the initial claim. To show that D is a GDG it is enough to show that D satisfies the fourth item of Definition 6. Assume that $a \mapsto \mathbf{t} \in v$. Toward a contradiction, assume that $a \mapsto \mathbf{t} \notin g_{i_j}$. Since each element of D is the update of the previous interpretation in D by updating the truth value of a b with $v(b)$, it is not possible that $a \mapsto \mathbf{f} \in g_{i_j}$. On the other hand, $a \mapsto \mathbf{u} \in g_{i_j}$ means that there is c initial ancestor of a that $v(c) = \mathbf{u}$. It is a contradiction that v is the grounded interpretation of F. □

5. Conclusion

Grounded discussion games between two agents are presented in this work to answer the credulous decision problem of ADFs under grounded semantics. Since each ADF is equivalent with an ADF without any redundant links, we present the game over this subclass of ADFs. If the graph associated to a given ADF is disconnected, then the current method only checks the ancestors of the argument in question to answer the decision problem and not the whole graph. Thus, in general, even in the worst case, the presented method does not coincide with the least-fixed-point algorithm of grounded interpretation. Further, the method is sound and complete. In each move, P tries to show that the initial claim can be in an extension of the trivial interpretation, and O tries to challenge P by checking the content of the interpretation presented by P and either finding the initial claim or requesting P to extend the interpretation or find a new initial ancestor. As future work, we are investigating a game for infinite ADFs and for ADFs for which the acceptance conditions are not restricted to propositional formulas.

Acknowledgements. Supported by the Center of Data Science & Systems Complexity (DSSC) Doctoral Programme, at the University of Groningen.

References

[1] P. M. Dung, "On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games," *Artificial Intelligence*, vol. 77, pp. 321–357, 1995.

[2] G. Brewka and S. Woltran, "Abstract dialectical frameworks," in *Proceedings of the Twelfth International Conference on the Principles of Knowledge Representation and Reasoning (KR 2010)*, pp. 102–111, 2010.

[3] L. Al-Abdulkarim, K. Atkinson, and T. J. M. Bench-Capon, "A methodology for designing systems to reason with legal cases using abstract dialectical frameworks," *Artificial Intelligence and Law*, vol. 24, no. 1, pp. 1–49, 2016.

[4] L. Al-Abdulkarim, K. Atkinson, and T. J. M. Bench-Capon, "Abstract dialectical frameworks for legal reasoning," in *Legal Knowledge and Information Systems JURIX*, vol. 271 of *Frontiers in Artificial Intelligence and Applications*, pp. 61–70, IOS Press, Amsterdam, 2014.

[5] E. Cabrio and S. Villata, "Abstract dialectical frameworks for text exploration," in *Proceedings of the 8th International Conference on Agents and Artificial Intelligence (ICAART (2)*, pp. 85–95, SCITEPRESS-Science and Technology Publications, Lda, 2016.

[6] H. Jakobovits and D. Vermeir, "Dialectic semantics for argumentation frameworks," in *Proceedings of the 7th International Conference on Artificial Intelligence and Law*, pp. 53–62, ACM Press, New York, 1999.

[7] H. Prakken and G. Sartor, "Argument-based extended logic programming with defeasible priorities," *Journal of Applied Non-classical Logics*, vol. 7, no. 1-2, pp. 25–75, 1997.

[8] S. Modgil and M. Caminada, "Proof theories and algorithms for abstract argumentation frameworks," in *Argumentation in Artificial Intelligence* (G. R. Simari and I. Rahwan, eds.), pp. 105–129, Springer, 2009.

[9] M. Caminada, "Argumentation semantics as formal discussion," in *Handbook of Formal Argumentation* (P. Baroni, D. Gabbay, M. Giacomin, and L. van der Torre, eds.), pp. 487–518, 2018.

[10] P. M. Dung and P. M. Thang, "A sound and complete dialectical proof procedure for sceptical preferred argumentation," in *Proceedings of the LPNMR-Workshop on Argumentation and Nonmonotonic Reasoning (ArgNMR07)*, pp. 49–63, 2007.

[11] F. H. van Eemeren, B. Garssen, E. C. W. Krabbe, A. F. S. Henkemans, B. Verheij, and J. H. M. Wagemans, eds., *Handbook of Argumentation Theory*. New York: Springer, 2014.

[12] R. Booth, M. Caminada, and B. Marshall, "DISCO: A web-based implementation of discussion games for grounded and preferred semantics," in *Proceedings of Computational Models of Argument COMMA* (S. Modgil, K. Budzynska, and J. Lawrence, eds.), pp. 453–454, IOS Press, Amsterdam, 2018.

[13] A. Keshavarzi Zafarghandi, R. Verbrugge, and B. Verheij, "Discussion games for preferred semantics of abstract dialectical frameworks," in *European Conference on Symbolic and Quantitative Approaches with Uncertainty* (G. Kern-Isberner and Z. Ognjanovic, eds.), pp. 62–73, Springer, Berlin, 2019.

[14] H. Strass and J. P. Wallner, "Analyzing the computational complexity of abstract dialectical frameworks via approximation fixpoint theory," *Artificial Intelligence*, vol. 226, pp. 34–74, 2015.

[15] S. Polberg, *Developing the abstract dialectical framework*. PhD thesis, PhD thesis, TU Wien, Institute of Information Systems, 2017.

[16] G. Brewka, H. Strass, S. Ellmauthaler, J. P. Wallner, and S. Woltran, "Abstract dialectical frameworks revisited," in *Proceedings of the Twenty-Third International Joint Conference on Artificial Intelligence (IJCAI 2013)*, pp. 803–809, 2013.

[17] G. Brewka, S. Ellmauthaler, H. Strass, J. P. Wallner, and S. Woltran, "Abstract dialectical frameworks. An overview," *IFCoLog Journal of Logics and their Applications (FLAP)*, vol. 4, no. 8, 2017.