Formalism and interpretation in the logic of law

Book review
Henry Prakken (1997). *Logical Tools for Modelling Legal Argument. A Study of Defeasible Reasoning in Law*. Kluwer Academic Publishers, Dordrecht.

1   The logic of law

In the field of the formal modeling of legal reasoning, the *logic of law*, for short, a great deal of research has been performed over the last decade.[1] It is ever wider recognized that logic has more to offer than just the correct use of the classical logical connectives, such as 'if ..., *then* ...' and '... *or* ...'. Especially, reasoning with exceptions, conflicts and applicability, all common in law, turn out to be formally analyzable using modern logical techniques.

In the late 80s and early 90s, the research on the logic of law got a strong impulse by developments in AI and law, as exemplified by Prakken's (1993) dissertation 'Logical Tools for Modelling Legal Argument'. At the end of 1997, a revised version of Prakken's dissertation was published in Kluwer's Law and Philosophy Library. In the book, the logical aspects of legal reasoning are investigated. Special attention is paid to the defeasibility of legal reasoning. The non-monotonic logics, as they have been developed in artificial intelligence, are an important source of inspiration. The new version of the book contains an adapted formalism (that has been developed by Prakken in cooperation with Giovanni Sartor in the last couple of years) and an updated and extended discussion of related research.[2]

The following review of Prakken's book is divided into two parts. Part I (sections 2 to 4) contains the book review proper, and is addressed at the reader with a general interest in artificial intelligence and law. Part II (sections 5 to 9) is mainly aimed at the readers more specifically interested in the logic of law, and consists of comments on Prakken's theory of defeasible argumentation. The comments are meant as a contribution to the ongoing discussion on the nature of defeasible argumentation in the law, and its formalization.

Part I: Prakken's book on the formalization of legal argumentation

2   Logic and law

The central message of Prakken's book, with which I wholeheartedly agree, is that the role of logic in law can be strengthened.

The fact that a connection between logic and law exists does not come as a surprise: logic has reasoning as its topic, and in law, reasoning is one of the main activities. There is a difference in focus: a central question of logic is what *valid* reasoning is, while the law is primarily interested in *persuasive* reasoning. An important bridge between logic and law is the fact that both in logic and in law *formal* aspects of reasoning receive special attention. 'Formal' is meant in a literal sense here: determined by form, according to a pattern. The logician thinks of following *Modus ponens* ($P, P \rightarrow Q / Q$), the lawyer of applying a rule of law to the case facts. Of course the role of the formal aspects of reasoning differs in logic and in law: in logic formal aspects of reasoning are the central *topic* of consideration; in the law, they are a *means* to serve general goals, such as legal security and limitation of state power. For instance, a logician is interested in the completeness of a proof theory with respect to a semantics, while a lawyer is interested in the fact that a judge's decision space is limited by formal constraints.

For long, the connection between logic and law remained quite restricted. In legal education, mostly some attention is paid to logic, such as to the relations of logical connectives '... *and* ...' and '... *or* ...'. However, the ambiguous meaning of '*P and Q or R*' and De Morgan's Laws - both of immediate practical use when reading legal regulations, are already advanced topics. One ground for the limited connection between logic and law is the fact that common patterns of legal reasoning have properties that for long were logical taboos. Two examples of such patterns are reasoning involving rules with exceptions and reasoning involving rules with conflicting conclusions. The corresponding taboo properties are non-monotonicity and inconsistency.

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[1]   For instance, Freeman and Farley (1996), Gordon (1995), Hage (1996, 1997), Lodder (1998), Loui and Norman (1995), Prakken and Sartor (1996), Verheij (1996) and Yoshino (1995) have worked on the logic of law.
[2]   For a review of the dissertation version of the book, the reader is referred to Loui (1995).

2.1 Rules with exceptions and non-monotonicity

The first example of a pattern of legal reasoning with a taboo property is reasoning involving rules with exceptions, very common in legal reasoning. Many legal articles contain an 'unless'-construction, or a section in which an exception to the main rule is formulated.[3] It is also often the case that *another* article contains an exception. These cases are relatively simple, since the exceptions are explicitly made in the law. However, a lawyer does not hesitate to argue that some *unexpected*, uncodified exception exists. Certain crucial legal decisions can be seen as the recognition of such new exceptions. A famous example in Dutch law is the Supreme Court's decision in the 'Milk and water'-case (HR February 14, 1916, NJ 1916, 681), in which *absence of all culpability* (in Dutch: 'afwezigheid van alle schuld', or 'avas') was recognized as a new, uncodified ground of immunity for punishment.

The logical taboo property related to reasoning involving rules with exceptions is *non-monotonicity*: a conclusion that at first is drawn, is withdrawn in the light of additional information. Classical logic is monotonic: once a conclusion is drawn it is never withdrawn in the light of new information.

Let me give an example of non-monotonicity in the law. Last week John grabbed Anne's bike out of her hands and quickly rode off on it. According to the public prosecutor, this is a clear example of the unlawful appropriation of someone else's good (roughly the formulation of article 310 of the Dutch penal code on theft). The prosecutor concludes that John is punishable because of theft. In court, it turns out that John was acting in *force majeure*: he hastened away on Anne's bike to be able to save the five year old Peter, who just fell into the canal. Accordingly, the judge concludes that John cannot be punished and acquits John of theft. (Clearly, a case like this will not easily come to trial.) It is important to note that there was nothing wrong with the reasoning of the public prosecutor. It was just based on *insufficient information*. As the example shows, in defeasible reasoning, this can *with hindsight* lead to the wrong conclusions.

2.2 Rules with conflicting conclusions and inconsistency

The second example is reasoning involving rules with conflicting conclusions. In the law, it is not uncommon that legal rules have conflicting conclusions. Sometimes such a situation is taken care of in the legal code itself by specifying the relation of the rules, e.g., by an exception construction. The law also has formal conflict rules, such as *Lex superior derogat legi inferiori*: a legal rule of a higher authority takes precedence over a rule of a lower authority. The conflict resolving capacities of lawyers go much further than the codified means at their disposal. Again and again a lawyer is confronted with conflicting interests, which have to be weighed. Such weighing often boils down to political or social choices. Many legal regulations can be seen as the explication of such a choice. An example is the question of euthanasia. Strongly simplified: what weighs more, the patient's wish, or the intrinsic value of human life? For now, the social choice in the Netherlands is that taking someone's life on request is punishable (article 293 of the Dutch penal code). A judicial decision is even based on individual choices, namely of the judge (or the judges). The decision space of a judge is somewhat delimited if the factors that have to be weighed, have been made explicit. Some judicial decisions can be seen as the explication of such factors. A well-known example in Dutch law is the Supreme Court's decision in the 'Trapdoor'-case (HR November 5, 1965, NJ 1966, 136), in which it was decided that, in order to determine the liability for damages in endangerment cases, among others the chance and the severity of the damages to be expected must be weighed against the difficulty of taking precautions.

The logical taboo property of reasoning involving rules with conflicting conclusions is *inconsistency*: cases might occur in which conflicting conclusions can be drawn. The taboo of inconsistency is, if possible, even stronger than the taboo of non-monotonicity. For, in classical logic, it is the case that *anything* follows from an inconsistency.

Here is an example of inconsistency in the law. Again we consider John's case of 'theft'. John's *force majeure* consisted of a conflict of duties. He is not allowed to steal Anne's bike, but has to be in time to save Peter.[4] In this case, the duty not to steal is outweighed. Clearly, there is a connection with non-monotonicity: the prima facie inconsistency is solved by additional information, in this case the weighing.

When logicians (re-)focused on commonsense reasoning - especially because of developments in artificial intelligence -, the taboos of non-monotonicity and inconsistency were abandoned, and a lot of new research started. Especially in the eighties, much progress was made. Prakken's (1993) dissertation gave an overview of this research with an eye to the law, and became a starting point for research in the

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[3] I focus on the continental, statute-based legal tradition.

[4] Sometimes a conflict of duties is not considered to be a *logical* inconsistency. This debate in deontic logic is irrelevant here.

logic of law. The logic of law is still a lively domain of research, as is demonstrated by the amount of new material in the second printing of Prakken's dissertation.

3   Overview of Prakken's book

Prakken's book is divided into 11 chapters. In the first three chapters, Prakken explicates his view on the role of logic in legal reasoning, and argues that new logical tools are required. Central in his argument are reasoning on the basis of rules with exceptions and with conflicting conclusions, as discussed above.

Then follows the technical part. He first treats a number of non-monotonic logics (chapter 4). Subsequently, he discusses in four separate chapters a number of themes that are especially relevant for the law: exceptions, specificity, inconsistency, and priorities. In chapter 5, he gives a classification of kinds of exceptions to rules. Prakken first distinguishes exceptions that make the general rule inapplicable, and exceptions that lead to the negation of the rule, corresponding to Pollock's (1987) well-known distinction between *undercutting* and *rebutting* defeaters.[5] Prakken's second distinction is that between *soft* and *hard* exceptions: soft exceptions can themselves be subject to exceptions, hard exceptions can't. According to Prakken, rebutter-type exceptions can be further distinguished: they either *reinstate the general rule* or they don't.[6] Prakken thoroughly analyzes how the different kinds of exceptions can be formalized (if possible) in different logical systems

In the chapters 6, 7 and 8, on specificity, inconsistency, and priorities, respectively, Prakken explicates the argument-based logical formalism that he has developed with Sartor (see, e.g., Prakken and Sartor, 1996). The explication is in parallel with the discussion of existing techniques. In the formalism, a special type of rule is introduced (as a conditional in the logical language), in order to represent *defeasible* rules. Defeasible rules can be used to construct *arguments*, by means of a dedicated rule of inference called *Default Modus Ponens*: for a defeasible rule $P \Rightarrow Q$, *Default Modus Ponens* is the rule of inference $P, P \Rightarrow Q / Q$. Prakken introduces a second form of defeasible rule, and an adapted form of *Default Modus Ponens*, inspired by logic programming. A typical example is $P \wedge \sim Q \Rightarrow R$. Here $\sim Q$, called the *justification* of the rule, stands for 'Assuming that not Q', and should be distinguished form the ordinary negation of Q, denoted $\neg Q$.

In Prakken's formalism, arguments can *defeat* each other. In chapter 6, two types of defeat are defined. The first is defined in terms of rule specificity: roughly, an argument that conflicts with another argument, but is based on more specific rules, defeats that argument. The second type of defeat is defined in terms of the justifications of rules: intuitively, an argument with justification $\sim Q$ is defeated by an argument with conclusion Q. In chapter 7, a priority relation on the arguments in terms of an ordering on rules (defined outside the logical object language) is incorporated in the formalism. In chapter 8, the definitions are adapted in order to allow reasoning about priorities.

Which arguments *justify their conclusions* is neatly defined in terms of *dialogues*. A dialogue is a sequence of moves by two players, the proponent and the opponent. If the proponent of an argument can *win* all dialogues starting from the argument, the argument justifies its conclusion. It turns out that some arguments justify their conclusion, and that some are *overruled*. Prakken calls arguments that neither justify their conclusion, nor are overruled, *defensible*.

In chapter 9, Prakken discusses several other formalisms modeling defeasible argumentation. Chapter 10 shows possible uses of the formalism. In this chapter, Prakken also explains his view on (legal) argumentation. According to him, argumentation is hierarchically structured in *four layers*. According to Prakken, the first layer is the *logical layer*. This is the layer in which it is specified what contradiction is and which arguments support a conclusion at all. The second layer is the *dialectical layer*, in which for instance the defeat and attack of arguments is defined. Whether an argument justifies its conclusion is also determined in the second layer. The third layer is the *procedural layer*. This layer specifies how a dispute is conducted. The fourth and final layer is the *strategic layer*. This is the layer, where argumentation tactics and heuristics belong. Prakken's book concludes with a summary of the main results and suggestions for further research.

The overview already shows that Prakken pays a lot of attention to the scientific context of his research. Moreover, he does this in an exemplary way: he is very conscientious in the technical description of related, sometimes competitive formalisms, and gives a well-balanced opinion by pointing to technical similarities and differences.

Two topics will receive special attention in Part II of this review: Prakken's formalization criteria, and his philosophical views on logic, logical tools and reasoning.

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[5]   As will be seen below (in section 7), Prakken also speaks of undercutters in a sense, different from Pollock's.
[6]   Prakken also mentions rules with conflicting conclusions, which as he acknowledges is strictly speaking a topic different from rules with exceptions.

3.1 Prakken's formalization criteria

In the context of the formal representation of rules and exceptions, Prakken gives a useful overview of formalization criteria (p. 103ff.): resemblance to natural language, structural resemblance, modularity, implementability, and expressiveness. Clearly, these criteria can be generalized to the wider context of the logical modeling of legal argument.[7] I recount Prakken's criteria.

Resemblance to natural language is the criterion that the formalization of an individual 'source unit', i.e., the smallest identifiable unit of information to be represented (cf. p. 35)[8], should not contain elements that have no counterpart in the source unit. For instance, if an article in a code of law contains a general rule, while there is an exception to it in another article, the representation of the rule should not contain a representation of the exception.

Structural resemblance is the criterion that the formal representation should preserve the separation of natural-language source units. For instance, if a rule and its exception occur in separate articles in a code of law, the formalization should represent the rule and the exception separately. (Prakken's criterion of structural resemblance is for the *relation* of separate source units what his criterion of resemblance to natural language is for *individual* source units.)

Modularity is the criterion that a natural-language expression is formalized without having to consider the rest of the domain. For instance, in a code of law, exceptions to rules sometimes occur at distant places in the code, or even in another code. This is a non-modular representation because of the interaction of separated information.

Implementability[9] is the criterion that the formalization should give prospects for implementation, e.g., in legal information systems.

Expressiveness is the criterion that rival formalizations should be able to express intuitively distinct notions. For instance, different forms of expressing exceptions in articles in codes of law, e.g., by exception clauses in the article itself, or by explicit references in another article, should have different formalizations.

In the second part of the present review (starting from section 5), I will at occasions refer to these formalization criteria. It should be noted that I will do this in the wider context of the logical modeling of legal argument, while Prakken's explicit intention is, as said, to restrict the criteria to the formalization of rules and exceptions.

3.2 Prakken's philosophical view on logic, logical tools and reasoning

At the end of the book, Prakken gives his general view on legal argumentation, when he discusses the four layers of argumentation. In the beginning of the book, Prakken explains his views on logic, logical tools and reasoning. What is, according to Prakken, the relation of logic, logical tools and reasoning?

Prakken rather consistently speaks of the role of logic *in* legal reasoning. For him, 'logic should not be seen as a model of, but rather as a tool in legal reasoning' (p. 8). It is however hard to grasp Prakken's intuitions in this connection completely clear, since he introduces several philosophical notions related to logic, logical tools and reasoning, used in a specific and apparently personal way, but does not give many sufficiently explicit examples of what he considers these notions to be. I attempt to give a systematic overview of some of Prakken's notions.

If Prakken speaks of reasoning, he distinguishes three modes of reasoning: *deductive*, *analogical* and *inductive* reasoning (p. 8). On p. 59/60, he contrasts these three modes of reasoning with a fourth: *nonmonotonic* reasoning. He also distinguishes *inferential* and *non-inferential* reasoning, and *justifying decisions* and *suggesting premises* (p. 26/27). To him, deductive reasoning is (apparently) a way of justifying a decision, since Prakken states that analogical reasoning is 'an essentially different kind of activity than justifying a decision, for which reason it should be called a *non-inferential* rather than a *non-deductive* mode of reasoning' (p. 26). The essentially different activity of analogical reasoning is suggesting premises: Prakken argues that analogical reasoning is a heuristic for suggesting premises (p. 27). A final important distinction made by Prakken is that between *justifying force as a matter of form* and *as a matter of content*. Prakken argues that 'the justifying force of an analogy is entirely a matter of content' (p. 27). From the context, it appears that Prakken considers the justifying force of a deduction as a matter of form.

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[7] Prakken's criterion of the exclusiveness of specificity (in fact not a criterion, but a research question) has been omitted, as it is especially focused on the representation of exceptions, and cannot be directly generalized to the wider context of modeling legal argument.

[8] All references to page numbers are to pages of the book on review.

[9] Prakken speaks of *implementation* (p. 105).

In part II of the present review below, I will occasionally refer to this recounting of Prakken's views on logic, logical tools and reasoning.

4 Evaluation of Prakken's book

Prakken has convincingly argued that logic has a lot to offer for the modeling of legal argument. He shows that reasoning with exceptions, specificity, inconsistency, and priorities can be analyzed using logical tools. As a result, he has turned the logic of law into a respectable field of research. Moreover, Prakken gives an accurate presentation of existing technical work. His knowledge of the technical state of the art is exemplary. Prakken's book is indispensable for all researchers interested in the logic of law, and belongs to the best work in the field. Also those who already own the dissertation version of the book, can benefit from the revised version. The formalism has been completely revised. The discussion of related research is updated and extended with a lot of relevant work that appeared after the earlier version of the book. Prakken's four-layered model of argumentation, which was not described in the dissertation, is an interesting contribution to the foundations of the logic of law. The book has been made more accessible by the addition of an index. As a result, Prakken's book remains a profitable starting point for research in the logic of law.

Part II: Comments on Prakken's theory of defeasible argumentation

In the following, I more systematically discuss Prakken's theory of defeasible argumentation, and its formalization. It has been developed in cooperation with Giovanni Sartor in the last couple of years. I start with the logical language (section 5). Then follows a discussion of arguments and their validity (section 6). I continue with Prakken's definitions of argument attack and defeat (section 7). Subsequently, the process of argumentation is considered (section 8).

5 The logical language

The first ingredient of a logical system is its language. It determines the expressiveness of the logical system. Prakken's logical language is that of first-order predicate logic, extended with an expression for defeasible rules. Prakken introduces two forms of defeasible rules, the first on p. 153, the second on p. 172. A typical example of Prakken's first form of defeasible rule is:

$$P \Rightarrow Q$$

It represents a defeasible rule with antecedens P and consequens Q. Because of their structural resemblance, a comparison of Prakken's defeasible rule with the material conditional with antecedens P and consequens Q (of classical logic, e.g., first-order predicate logic) is in place:

$$P \rightarrow Q$$

Taking rules as the directed connection between an antecedens and a consequens, Prakken's defeasible rule and the material conditional are both well-suited to represent rules.

Let me give some other possible notations for the expression of rules (as conditionals in the object language):

1. There is a rule with antecedens P and consequens Q.
2. There is a defeasible rule with antecedens P and consequens Q.
3. The rule with antecedens P and consequens Q is valid.
4. Er is een regel met antecedens P en consequens Q.
5. Valid(rule(p, q))[10]
6. $\{\{P\}\} \rightarrow Q$
7. Abc P xyz Q.
8. Abc Q xyz P.

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[10]   Here the terms p and q are translations of the sentences P and Q. In this review, the subtle distinction between the use of uppercase and lowercase characters, as it is used by Hage and myself in order to distinguish terms and sentences that express the same state of affairs, is ignored. That would take us too far. See, e.g., Hage's book (1997) or my dissertation (Verheij, 1996).

Each notation can express the same formal relation between an antecedens and a consequens.[11] In all notations, the schematic, directed connection of *P* and *Q* can be expressed. Set-theoretically, any notation that can express an ordered pair (*P*, *Q*) will do.[12] How can all these different notations for the same form make a difference? The difference can only become apparent *when different notations are interpreted differently*, i.e., in the semantics. Certainly the different notations suggest different interpretations, but these interpretations are not 'in' the notations themselves. For the notation of rules (taken as directed connection between an antecedens and a consequens), it does not matter whether the word 'defeasible' occurs in the notation (as in the second example above), or not (as in the first). It might be thought that the notations '*P* ⇒ *Q*' and 'There is a rule with antecedens *P* and consequens *Q*' are fundamentally different since the latter seems to be *about* rules, whereas the former seems to be the rule itself. At this point, however, where the two notations are *uninterpreted*, there is no fundamental difference and both notations can serve equal purposes.

Prakken chooses to use a *dedicated* notation for defeasible rules to distinguish them from non-defeasible rules, such as the material implication. This puts a strong restriction on the expressiveness of Prakken's system: whether a rule is defeasible or not is determined by the linguistic form, and does not depend on contingent information. Another choice is to express rules, whether they are defeasible or not, *in the same way*, and make their defeasibility expressible in the logical language, as in Reason-Based Logic (see, e.g., Hage 1996, 1997; Verheij, 1996). As we saw in the example notations above, rules can be expressed by the same notation, whether they are defeasible or not.

But how can the difference in interpretation between strict and defeasible rules be made explicit? In logic, this can be done in terms of *semantic constraints*, e.g., in terms of truth values. Let's take strict rules as rules that cannot have exceptions, and defeasible rules as rules that can. (Clearly other interpretations distinguishing strict and defeasible rules are possible. E.g., in Reason-Based Logic, experimental interpretations of rule application and the weighing of reasons have been proposed.)

Which semantic constraints hold for strict rules? If we take strict rules as rules that cannot have exceptions, the semantic constraints (here in terms of truth values) would include the following:

Strict rules are followed
> If 'There is a rule with antecedens *P* and consequens *Q*' and '*P*' are both true, then '*Q*' is true.[13]

Strict rules have no exceptions
> 'There is no exception to the rule with antecedens *P* and consequens *Q*' is true.

We can look for semantic constraints on the *same* types of sentences, but this time interpreted for defeasible rules (taken as rules that can have exceptions). Obviously, the two constraints for strict rules do not hold for defeasible rules. The second semantic constraint clearly does not have a counterpart for defeasible rules, but the counterpart of the first could be as follows:

Defeasible rules without exceptions are followed
> If 'There is a rule with antecedens *P* and consequens *Q*', '*P*' and 'There is no exception to the rule with antecedens *P* and consequens *Q*' are all three true, then '*Q*' is true.

Experiments with constraints like these can be found in the work on Reason-Based Logic. In Prakken's system, such explicit constraints on the interpretation of rules have no direct counterpart. (At the end of his book, Prakken suggests otherwise, viz. when he speaks of the role of a classical model-theoretic semantics for his argumentation formalism. See section 8 below.)

In the meantime, our object language had to be extended slightly, since not only rules, but also exceptions to rules[14] have been introduced. We have encountered the following (non-elementary) types of facts:

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[11]   The first and the second are put together in the list in order to recall that the formal occurrence of the word 'defeasible' does not imply that the notation is indeed interpreted for defeasible rules. The third is included to suggest that the existence and validity of a rule coincide, or at least are closely related. The fourth is in Dutch. The fifth and sixth occur in my dissertation version of Reason-Based Logic and CumulA, respectively (Verheij, 1996). The seventh and eighth are included to recall that for their formal relation not even the relative position of *P* and *Q* matters.

[12]   Prakken speaks of a 'one-directional' conditional, which does not have to do with the directed structure of the conditional, but with the way it behaves in reasoning. Roughly, a one-directional conditional, in Prakken's sense, is a conditional that does not allow *Modus tollens*.

[13]   Obviously, the semantic constraint is meant to hold for the instances of *P* and *Q* allowed by the language. In this case, *P* and *Q* will express states of affairs. For present purposes, this is no longer stated explicitly.

There is a rule with antecedens P and consequens Q.
There is no exception to the rule with antecedens P and consequens Q

Prakken uses another way to introduce exceptions to rules, by his second form of defeasible rule (p. 172). A typical example of this form of defeasible rule is:

$$P \wedge \sim Q \Rightarrow R$$

It represents a rule with antecedens P, consequens R, and justification $\sim Q$. Here $\sim Q$ stands for 'Assuming that not Q', while Q can be seen as the expression of an exception. The justification $\sim Q$ must be distinguished from the ordinary negation $\neg Q$. Prakken calls $\neg Q$ the assumption corresponding to the justification $\sim Q$.

   This representation of a defeasible rule is strange in the light of Prakken's central formalization criteria of resemblance to natural language and structural resemblance. For, in the law and in general, an exception to a rule is often expressed separate from the rule itself. This is a serious problem for Prakken's formalism, since (for exceptions of the undercutting type; cf. Pollock, 1987) it is *impossible* to express an exception to the rule $P \Rightarrow R$ by adding contingent information. To add the information that there is an (undercutter-type) exception to the rule $P \Rightarrow R$, the rule must be *replaced* by a rule of the form $P \wedge \sim Q \Rightarrow R$ (where Q can, e.g., be an exception clause). This is in disagreement with the common technique in the law to express an exception to the general rule in an article separately from the rule, simply by adding a new section to the article (or an entirely new article). In Prakken's system, this technique has no counterpart obeying the criteria of resemblance to natural language and structural resemblance, since the information that there is an exception to the rule $P \Rightarrow R$ cannot be expressed in the logical language.

   Prakken is aware of this problem and attempts to solve it by using a *naming technique* (in combination with, e.g., exception clauses). Naming techniques have been used extensively in the literature on defeasible reasoning (as is meticulously shown by Prakken, especially in the context of exceptions to rules), and also in the law: it is not unusual that an article contains an explicit reference to another article by mentioning its name (e.g., Art. 6:102 of the Dutch civil code contains a reference to Art. 6:101 using its name 'artikel 101'). An example of Prakken's use of the naming technique is the following (cf. the section on the validity of rules on p. 176/177):

$$P \wedge \sim\neg \mathrm{Valid}(d) \Rightarrow Q$$

Prakken's intended interpretation of this formula is that, if P obtains and the rule named d is not assumed not to be valid, then Q obtains. (Prakken's intricate use of a double negation here is fundamental for his treatment of defeasible arguments, that is irrelevant for present purposes.) In Prakken's system, the attachment of the name d *in* the logical language to the rule $P \wedge \sim\neg\mathrm{Valid}(d) \Rightarrow Q$ occurs *outside* the logical language. Prakken does this by writing:

$$d\colon\ P \wedge \sim\neg \mathrm{Valid}(d) \Rightarrow Q$$

Note that the part 'd: ' does *not* belong to the logical language of Prakken's system, whereas 'd' as it occurs in '$\sim\neg\mathrm{Valid}(d)$' is part of the logical language.

   The obvious question to ask is (Verheij, 1996, p. 91): to what does the name d refer? It is in agreement with Prakken's use of the naming technique that the name d refers to the rule $P \wedge \sim\neg\mathrm{Valid}(d) \Rightarrow Q$. So the rule named d has $P \wedge \sim\neg\mathrm{Valid}(d)$ as its antecedens and Q as its consequens. The sentence $\mathrm{Valid}(d)$ expresses that the rule with $P \wedge \sim\neg\mathrm{Valid}(d)$ as its antecedens and Q as its consequence is valid. The next question is then: how do we express the validity of the rule with antecedens P and consequens Q? I.e., in Prakken's notation for rules: how do we express the validity of the rule $P \Rightarrow Q$? Let's give this rule a name (again *outside* the logical language):

$$d'\colon\ P \Rightarrow Q$$

Apparently, the validity of *this* rule should be denoted as $\mathrm{Valid}(d')$.

   For four reasons, Prakken's method seems problematic. First, in Prakken's formalism the rule $P \Rightarrow Q$ with name d' cannot be blocked by expressions containing the rule's name, such as $\neg\mathrm{Valid}(d')$.

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[14]   In the sense of Pollock's (1987) undercutters. Cf. also Prakken's distinction of kinds of exceptions, as summarized in section 3.

Information containing d' can have no (direct) effect on the consequences of the rule $P \Rightarrow Q$: drawing the conclusion Q from P and the rule $P \Rightarrow Q$ is not in general prevented by the assumption of $\neg\text{Valid}(d')$. A rule's name can only have effects if it occurs in an expression in the rule's antecedens, as in the case of the rule $P \wedge \sim\neg\text{Valid}(d) \Rightarrow Q$ with name d.

The second reason why Prakken's method seems problematic, is that it is in disagreement with the formalization criteria of resemblance to natural language and structural resemblance: stating a rule's validity always involves a name, as in $P \wedge \sim\neg\text{Valid}(d) \Rightarrow Q$. In other words, there is no closed form that states the rule's validity corresponding to natural language sentences of the following form (in which no name occurs):

The rule with antecedens P and consequens Q is valid.

As a result, the formalization criteria of resemblance to natural language, structural resemblance, and expressiveness are not met for Prakken's formalization of rules.

Third, the possibility that there is a connection between the expression of a rule and its validity, is obscured. If d is the name of the rule with antecedens P and consequens Q and $\text{Valid}(d)$ expresses the rule's validity, then it seems that $P \Rightarrow Q$ and $\text{Valid}(d)$ are somehow related. E.g., if $\text{Valid}(d)$ expresses legal validity, the truth of $\text{Valid}(d)$ might imply the truth of $P \Rightarrow Q$. If $\text{Valid}(d)$ expresses logical validity, the truth of $\text{Valid}(d)$ might even be considered to be equivalent to the truth of $P \Rightarrow Q$. In the latter case, $\text{Valid}(d)$ would be just $P \Rightarrow Q$ 'in disguise'. In Prakken's formalism, no connection between the expression of a rule and its (legal or logical) validity is made.

Fourth, the primary use of the naming technique, as in $P \wedge \sim\neg\text{Valid}(d) \Rightarrow Q$, is self-referential: the name d refers to the rule in which it occurs. Self-reference is not a problem in itself, but can lead to strange phenomena. It is tempting (especially if a connection between the expression of a rule and its validity is recognized) to repeatedly replace d by the rule it stands for:

$P \wedge \sim\neg\text{Valid}(d) \Rightarrow Q$
$P \wedge \sim\neg\text{Valid}(P \wedge \sim\neg\text{Valid}(d) \Rightarrow Q) \Rightarrow Q$
$P \wedge \sim\neg\text{Valid}(P \wedge \sim\neg\text{Valid}(P \wedge \sim\neg\text{Valid}(d) \Rightarrow Q) \Rightarrow Q) \Rightarrow Q$
$P \wedge \sim\neg\text{Valid}(P \wedge \sim\neg\text{Valid}(P \wedge \sim\neg\text{Valid}(P \wedge \sim\neg\text{Valid}(d) \Rightarrow Q) \Rightarrow Q) \Rightarrow Q) \Rightarrow Q$
...

The sequence never ends. Wisely, Prakken's logical language does not allow this.

A final point on Prakken's logical language concerns the 'nesting' of rules. For the material implication, $R \rightarrow (P \rightarrow Q)$ is an example of nesting: the rule $P \rightarrow Q$ occurs as the consequence of the rule $R \rightarrow (P \rightarrow Q)$. By such nesting, it can be expressed that a rule (in the example: $P \rightarrow Q$) depends on contingent information (in the example: R). In Prakken's formalism, such nesting is not allowed for *defeasible* rules. E.g., $R \Rightarrow (P \Rightarrow Q)$ is not an element of his logical language. It is unclear why this distinction between strict and defeasible rules is made: the exclusion of the nesting of defeasible rules seems to be an *ad hoc* decision.

Moreover, by the naming technique, Prakken's formalism allows the expression of nested defeasible rules 'in disguise': if d is the name of the rule with antecedens P and consequens Q, then an expression, such as $\text{Exists}(d)$ (or $\text{Valid}(d)$!) is naturally interpreted as $P \Rightarrow Q$ 'in disguise'. As a result, $R \Rightarrow \text{Exists}(d)$ can even be regarded as $R \Rightarrow (P \Rightarrow Q)$ 'in disguise'. Note that this point only concerns the *expressiveness* of Prakken's logical language. Whereas it is possible to express nested defeasible rules (be it in disguise) in analogy to nested strict rules, nested defeasible rules do not automatically have higher level effects, analogous to nested strict rules. We return to this in section 6.6 below, on reasoning about rules.

The language of Reason-Based Logic (e.g., Hage 1996, 1997; Verheij, 1996) differs significantly from Prakken's. Rules are represented as specific terms in the logical language, e.g., rule(p, q), and their validity as sentences, Valid(rule(p, q)). Technically, the terms rule(p, q) play a role that is analogous to Prakken's names. The important difference is that the terms rule(p, q) encode the antecedens and consequens of the rule. In this way, the validity of a rule with antecedens P and consequens Q[15] is expressed directly and without the mentioned problems of Prakken's naming technique. Exceptions to rules are expressed in separate sentences. E.g., Excludes(r, rule(p, q)) can express that R is an exclusionary reason to the application of the rule with antecedens P and consequens Q. An example of the expression of a nested rule is Valid(rule(r, valid(rule(p, q)))).

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[15]   See note 10.

Prakken fears that the latter technique can lead to paradoxes and inconsistencies, since it involves 'metalevel reasoning' (p. 218). It should be noted, however, that he raises merely a suspicion, and does not give an example of such a paradox or contradiction. Apparently, he refers to paradoxes, such as the liar's paradox.[16] It should be noted however that 'paradoxes and inconsistencies' are only to be feared if the whole system collapses, i.e., if *any* theory becomes inconsistent. A single consistent example suffices to show that this is not the case (and such examples are readily available for Reason-Based Logic). The fact that inconsistencies can arise at all is obviously not in itself problematic. For instance, it is not a problem for classical logic that P and $\neg P$ cannot both be satisfied. Of course there are *new* causes for inconsistencies if new interpretations are allowed. For instance, if rules are interpreted defeasibly (recall the constraint that defeasible rules without exceptions are followed), the following four sentences cannot be all satisfied:

There is a rule with antecedens P and consequens Q.
P.
There is no exception to the rule with antecedens P and consequens Q.
$\neg Q$.

Such 'new' inconsistencies are not a problem in themselves; they are intended consequences of the interpretation of the types of facts involved. Just as it is an intended consequence of the interpretation of $\neg P$ that P and $\neg P$ cannot both be satisfied, it is an intended consequence of the interpretation of 'There is a rule with antecedens P and consequens Q' and 'There is no exception to the rule with antecedens P and consequens Q' that they cannot be satisfied together with P and $\neg Q$. One way to avoid such harmless inconsistencies is to leave out explicit semantic constraints (e.g., in terms of truth values) on the interpretation of rules, as Prakken does.

6 Arguments

In the formal modeling of legal argument, the notion of an argument is especially important. I discuss the following topics with regards to Prakken's views on the formal modeling of legal argument: argument structure (section 6.1), formal patterns of reasoning, in the sense of rules of inference (section 6.2), their relation with semantic constraints (section 6.3), analogical reasoning (section 6.4), the validity of formal patterns of reasoning (section 6.5), and reasoning about rules (section 6.6).

6.1 Argument structure

Prakken's formal arguments are sequences of sentences (p. 154, 155), e.g.,

$[P \quad P' \quad P \wedge P' \Rightarrow Q \quad P \wedge P' \quad Q]^{17}$

Each sentence in the sequence is either a premise or a conclusion. Conclusions are the result of applying a rule of inference using sentences preceding it. In the example, the sentences P, P', and $P \wedge P' \Rightarrow Q$ are premises and $P \wedge P'$ and Q are conclusions, since they are the result of applying a classical rule of inference (sometimes called $\wedge$-*Introduction*) and Prakken's rule of inference *Default modus ponens* on the rule $P \wedge P' \Rightarrow Q$.

I have not found an explicit defense of Prakken's choice to represent arguments in this way. An advantage is that it is in agreement with the formalization criteria of structural resemblance and resemblance to natural language, in the sense that arguments (in writing) often seem to have the form of a sequence of sentences. It should be noted however that Prakken gives no natural language examples. A disadvantage is that this formalization tends to obscure the tree-like structure of arguments, that is the

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[16]   The liar's paradox in natural language is exemplified by sentences such as 'This sentence is false' and 'I lie'. It has a logical counterpart, as follows. If a logical system has a truth predicate True, such that True(P) if and only if P, then the possibility to construct a liar's sentence L for which L if and only if $\neg$True(L), makes the whole system contradictory (assuming the ordinary logical laws). This led Tarski to propose a hierarchy of languages. (Cf. Haack, 1978, p. 135ff.)
[17]   Prakken often includes the meta-logical name labels of rules in the arguments (in disagreement with his formal definitions). An example is the occurrence of *d:* in $[P \quad P' \quad d{:}P \wedge P' \Rightarrow Q \quad Q]$.

result of the 'reason-conclusion'-relations in it.[18] (Below, in section 7 on attack and defeat, I argue that this actually leads to problems in Prakken's formalism.)

6.2   Formal patterns of reasoning for defeasible rules

One of the central questions of logic is: what are the *formal patterns* of reasoning? In logic, such formal patterns of reasoning often go by the name 'rule of inference'. In the following, I intentionally use the term 'formal pattern' in order to avoid confusion with the term 'rule'.

   A typical example of a formal pattern of reasoning, well-known from classical logic, is *Modus ponens*. It can be formulated as follows (cf. the remarks on the notation of rules in section 5):

Modus ponens
    Premises:
        There is a rule with antecedens *P* and consequens *Q*.
        P.
    Conclusion:
        Q.

In connection with his separate class of defeasible rules, Prakken considers one new, accompanying formal pattern of reasoning:

Default modus ponens[19]
    Premises:
        There is a defeasible rule with antecedens *P* and consequens *Q*.
        P.
    Conclusion:
        Q.

I have two problems with Prakken's view. First, though I consider *Default modus ponens* an important formal pattern of reasoning in the context of rules with exceptions, I do not consider it to be *new*. Second, I consider *other* formal patterns of reasoning to be interesting with regards to defeasible rules. I will elaborate on both points.

   My first problem is that I do not consider *Default modus ponens* to be a *new* formal pattern of reasoning. My point is simply that it has the *same form* as classical *Modus ponens*. It is a misunderstanding[20] to think that their forms differ because another notation is used for rules.[21] We already encountered my reason for this in the section on language (section 5): another notation for rules does not imply another interpretation of rules. (The next section contains a discussion of the relation of formal patterns of reasoning to semantic constraints, in order to elaborate on this point.)

   My second problem is that I consider formal patterns of reasoning other than *Modus ponens* to be interesting. An example is the unusual *Modus non excipiens*:

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[18]   The tree-like reason-conclusion structure of arguments is for instance at the heart of the well-known argumentation theory of Van Eemeren and Grootendorst (1981, 1987). E.g., Vreeswijk (1993, 1997) and Verheij (1996) use similar argument representations in their formal models of defeasible reasoning.

[19]   Definition 6.4.2, p. 154. Prakken uses different notation. On p. 173, Prakken uses an adapted version of *Default modus ponens* inspired by logic programming. Prakken chooses a form very different from his earlier *Default modus ponens* (e.g., with a structured antecedent, and surprisingly with the meta-logical rule's name in it). The justifications of the form ~*Q* that occur in it, have a limited role, and should be distinguished from ordinary premises of rules of inference. Justifications only serve the purpose of making the argument's constructed with the rule of inference defeasible. Another approach is to add the assumptions ¬*Q* (corresponding to the justifications ~*Q*) as defeasible axioms. This approach seems more natural to me, since no new concept of rule of inference is required. In my dissertation, I discuss this possibility as *assumption-type defeat* (Verheij, 1996, p. 159) in the context of Bondarenko *et al.*'s (1993) formalism, one of the versions of which inspired Prakken to his use of assumptions.

[20]   I do not want to say that Prakken has this misunderstanding. However, if he does not, his work suggests otherwise. What he does say, is that *Default modus ponens* is a new rule of inference, where new is meant relative to first-order predicate logic (Definition 6.4.2, p. 154). On the other hand, he seems not to allow all rules of inference of natural deduction (a standard system of rules of inference of first-order predicate logic), such as →-*Introduction*, which involves *withdrawing* premises. This is not in all respects a limitation (since it is well-known that for completeness *Modus ponens* suffices), but could have been explicitly noted.

[21]   Implicitly, we already used this, since we have denoted *Default modus ponens* in another way than Prakken does. See note 19 and section 5 on the logical language.

Modus non excipiens[22]
    Premises:
        There is a rule with antecedens *P* and consequens *Q*.
        P.
        There is no exception to the rule with antecedens *P* and consequens *Q*.
    Conclusion:
        Q.

Modus non excipiens differs from *Modus ponens*, in its third premise 'There is no exception to the rule with antecedens *P* and consequens *Q*'. It says that the consequens of a rule follows from its antecedens, provided that there is no exception to the rule.

As I will attempt to show in the next section, *Modus non excipiens* is as a formal pattern of reasoning even closer related to defeasible rules than *Modus ponens*, and should therefore certainly not be neglected.

6.3 *Formal patterns of reasoning and semantic constraints*

In section 5 on the logical language, semantic constraints distinguishing strict and defeasible rules have been discussed. It is now important to see that the rule-sentences in the formal patterns of reasoning *Modus ponens* and *Modus non excipiens* can be interpreted both for *strict* rules and for *defeasible* rules. The semantic constraints determine which formal patterns of reasoning are truth-preserving, *just as what we are used to in classical logic*, as we will see below.

If the rule-sentences are interpreted as expressing strict rules, the constraint that strict rules are followed determines that the formal pattern of reasoning *Modus ponens* and *Modus non excipiens* are both truth-preserving. However, for strict rules, *Modus non excipiens* does not allow new consequences, since its additional premise ('There is no exception to the rule with antecedens *P* and consequens *Q*') is (semantically) always fulfilled. As a result, Occam's razor suggests that, for strict rules, *Modus ponens* suffices as a formal pattern of reasoning.

If the rules are interpreted as defeasible rules, the constraint that defeasible rules without exceptions are followed, determines that the formal pattern of reasoning *Modus non excipiens* is truth-preserving. *Modus ponens* interpreted for defeasible rules is not truth-preserving since its conclusion does not obtain in all cases (or 'possible worlds') in which its premises obtain: in a case that the rule's antecedens obtains, while there is an exception, the consequens does not always obtain. Below, in section 6.5, we consider the question whether *Modus ponens* interpreted for defeasible rules is nevertheless valid (whatever that may mean!). (The fact that *Modus non excipiens* is truth-preserving with respect to the semantic constraint on defeasible rules, while *Modus ponens* isn't, is why I said at the end of section 6.2 that *Modus non excipiens* is as a formal pattern of reasoning even closer related to defeasible rules than *Modus ponens*.)

Just as in classical logic, the truth-preserving formal patterns of reasoning and the semantic constraints seem to be six of one and half a dozen of the other.[23]

To make this point as clear as possible, I give an example concerning the relation of love and hate, a topic not very fashionable in logic. In a semantics of love and hate, the following semantic constraint could make love and hate mutually excluding:

Hate and love are mutually excluding
    '*P* hates *Q*' and '*P* loves *Q*' are not both true.[24]

(This is not to imply that the constraint holds in our world.) With respect to this semantic constraint (plus a classical interpretation of the connective 'not'), the following formal pattern of reasoning is truth-preserving:

Modus odii
    Premises:
        P hates *Q*.
    Conclusion:
        P does not love *Q*.

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[22]   I thank Bram Roth for suggesting the name of this formal pattern of reasoning.
[23]   There is a review paper on classical logic that in this connection speaks of a 'silly, pedantic exercise', when after the definition of the semantic consequence relation, the proof theory is presented.
[24]   In this case, *P* and *Q* do not express propositions, but refer to persons. See also note 13. It does not follow from the example on love and hate that I consider the relation of love and hate to be a matter of logic proper.

The example concerning love and hate attempts to show that unusual semantic constraints can make unusual formal patterns of reasoning truth-preserving. Similarly, the unusual constraints on defeasible rules make the unusual *Modus non excipiens* truth-preserving.

The discussion above shows that Prakken's distinction between justifying force as a matter of form and as a matter of content (see section 6.2) does not stand: on the one hand, there is semantics and semantic constraints, which I consider to be the logical counterparts of Prakken's 'content',[25] on the other proof theory and formal patterns of reasoning, the logical counterparts of Prakken's 'form'. In the example, we have on the one hand the semantic constraint that hate and love are mutually excluding, and on the other hand the formal pattern of reasoning *Modus odii*. In connection with defeasible rules, we had on the one hand the semantic constraint that defeasible rules without exceptions are followed, and on the other the formal pattern of reasoning *Modus non excipiens*. In each case, content and form are closely related.

One could say that it is a central task of logic to explain the close relation between form and content. Moreover, the justifying force of reasoning is in logic never a matter of form alone, *not even for deduction*;[26] it is always the interpretation (content!) that makes the difference.[27]

The strangest formal pattern of reasoning that I consider here, concerns analogical reasoning.

6.4 *Analogical reasoning*

In contrast with Prakken, to me, the study of analogical reasoning can be done in terms of formal patterns of reasoning (in the sense of rules of inference), *just as any other form of reasoning*.

A formal pattern of reasoning for analogical reasoning might be the following:

Modus analogiae
>   Premises:
>>   There is a rule with antecedens P and consequens Q.
>>   For the rule with antecedens P and consequens Q, P^* is analogical to P.
>> P^*.
>   Conclusion:
>> Q.

Whether this formal pattern of reasoning is truth-preserving is again 'simply' a matter of semantics.[28] A semantic constraint that makes *Modus analogiae* truth-preserving is the following:

Rules are followed if their condition is analogically fulfilled
>   If 'There is a rule with antecedens P and consequens Q', 'For the rule with antecedens P and consequens Q, P^* is analogical to P', and 'P^*' are all three true, then 'Q' is true.

With regards to the semantic constraints for a logic of criminal law (where, at least in the Netherlands, analogical reasoning is taboo) *Modus analogiae* will not be truth-preserving (or are sentences of the form 'For the rule with antecedens P and consequens Q, P^* is analogical to P' never or almost never true).

Prakken is very confident. As said (in section 3.2), he attempts to show that 'the justifying force of an analogy is *entirely* a matter of content, for which reason analogical reasoning should not be regarded as a way of justifying a conclusion, but as a way of suggesting new premises' (p. 27, emphasis added). Prakken remits analogical reasoning to the so-called 'context of discovery' in contrast with the 'context of justification' where, according to him, deduction belongs. The reason why Prakken thinks that the justifying force of analogical reasoning (based on the similarity of cases) is entirely a matter of content, is that 'it is always possible to instead construct from *exactly the same premises* a rule for the opposite

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[25] Apart from semantics, there is another way in which content enters logic: in the assumption of the contingent facts of a case. Obviously, *assuming* that particular sentences express facts (and that therefore require no justification) does not complicate the present discussion of 'justifying force'. See section 6.4 on analogical reasoning for more on the role of contingent facts.

[26] E.g., there is classical and intuitionistic deduction (see, e.g., Troelstra and Van Dalen, 1988). The disagreement is on the formal pattern of reasoning with premise $\neg\neg P$ and conclusion P. *The interpretation of the negation determines whether this pattern is included in the proof theory or not.*

[27] Of course, this is the easy part of posing a problem. The hard part is to find out how form and content are related in the context of defeasible reasoning.

[28] For a semantic view on analogy in terms of the relations between rules and principles, see chapter 3 of my dissertation, or Verheij *et al.* (1998).

conclusion based on the difference between the two cases [i.e., the ordinary and the analogical case]' (p. 28, emphasis added).

That is a matter of course. Moreover, *any* rule (here apparently used by Prakken in the sense of a formal pattern of reasoning) with *arbitrary* premises and conclusion can be *made*. Two questions have to be separated: the question whether a formal pattern of reasoning (e.g., *Modus analogiae*) is valid, and the question whether its premises obtain. The first question is among others[29] determined by the interpretation (the semantics), the second by the contingent facts of the case at hand. Whether some rule for the opposite conclusion can be constructed 'from exactly the same premises' can be *constructed* does certainly not imply that it is valid.

On p. 95, Prakken elaborates somewhat on his view on analogy. He claims that it can be the case that both Q and $\neg Q$ follow analogically from P, for instance, if it is possible to state similarities[30] of the case at hand to cases with contrary outcomes. He states that in such a case still a choice between Q and $\neg Q$ has to be made. (Though the wordings of his views suggest otherwise, I assume that Prakken does not want to say that an inconsistency arises in *all* cases to which analogical reasoning applies.) I agree with him that if analogical reasoning applies an inconsistency *can* occasionally arise, simply because the use of analogies in reasoning (in general) allows more conclusions, and therefore also an inconsistency can arise more easily. However, he seems to imply that this is more of a problem to analogical reasoning than to any other form of reasoning, and that is a point that I do not understand.

Admittedly, some formal patterns of reasoning are less controversial than others. E.g., *Modus ponens* seems 'more obvious' than *Modus analogiae*. Also the truth of some facts of a case seems harder to determine than that of others. E.g., whether 'For the rule with antecedens P and consequens Q, P^* is analogical to P' expresses a fact (given a sensible semantics) will in general be harder to determine than whether 'There is a rule with antecedens P and consequens Q' is. The latter is certainly the case if the rule is interpreted in the *classical* semantics of the material implication: since the material implication is *truth-functional*, its truth only depends on the truth values of its antecedens and consequens. But this is exactly the reason why the material implication is not attractive as the representation of rules. An analogy, such as 'For the rule with antecedens P and consequens Q, P^* is analogical to P' is not truth-functional, so its truth value cannot simply be determined by looking at its parts.

By the way, I agree with Prakken that logic is not about the 'context of discovery'. The only difference is that I recognize that both the upwelling of a rule and of an analogy in the head of a lawyer (or *any* reasoner for that matter) belong to that context, and therefore leave both out of consideration as a logician. I consider it equally remarkable that lawyers (and reasoners in general) are able to find apparently appropriate rules and apparently appropriate analogies.

A final remark about analogical reasoning, concerning Prakken's idea that it is a heuristic for suggesting premises (p. 27) (see section 3.2 above). *Any* formal pattern of reasoning is such a heuristic, even the 'ordinary' ones, like *Modus ponens*. In *Modus ponens*, the premises 'There is a rule with antecedens P and consequens Q' and 'P' are suggested[31], in *Modus analogiae*, also 'For the rule with antecedens P and consequens Q, P^* is analogical to P' is. Moreover, for *any* formal pattern of reasoning the question whether the premises obtain as facts is, in Prakken's terminology, 'entirely a matter of content'. *Modus ponens* is in this connection absolutely not simpler than *Modus analogiae*. This *even* holds if the rule in *Modus ponens* is interpreted as a classical material implication. For the only material implications that do not depend on contingent case information (and therefore *seem* to be less 'a matter of content', but see below), are the tautological ones. But obviously *Modus ponens* on a tautological material implication is pointless. And even the fact that some implications are tautological is, though not based on contingent facts, still based on the semantics, which I also take to be a matter of content (see section 6.3).

As said, all formal patterns of reasoning that we have discussed, are truth-preserving with respect to a suitable semantics. Except for one: Prakken's *Default modus ponens*. The rule that occurs in it is specifically meant to be interpreted as a *defeasible* rule. Therefore the conclusion should not follow *always* (with respect to a suitable semantics), which would be the case if it were truth-preserving. This suggests that not all 'valid' formal patterns of reasoning need to be truth-preserving. What to think of this? What is the relation of formal patterns of reasoning and truth-preservation? What formal patterns of reasoning are valid?

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[29] See section 6.6 on validity.
[30] An obvious restriction is needed: only similarities that are (in some sense) *relevant*, can lead to an analogical conclusion.
[31] Such 'heuristics for suggesting premises' are for instance used in algorithms for automated logical inference that start with the conclusion and attempt to find proofs for it.

In Prakken's work, the (logical) validity of and the choice for the formal pattern of reasoning *Default modus ponens* are presupposed, and do not seem to be supported by a (philosophical) criterion.

In classical logic, the generally accepted sufficient criterion for the validity of formal patterns of reasoning, is truth-preservation:

A formal pattern of reasoning is valid if it is truth-preserving.

In classical logic, the reverse is also true: valid formal patterns of reasoning are always truth-preserving. However, as we have seen, this does not hold for reasoning in a defeasible context. On the one hand, some formal patterns of reasoning are truth-preserving, such as *Modus non excipiens*. Such formal patterns of reasoning can be treated in a relatively 'classical' way, since the difficulties of non-monotonicity do not occur.[32] On the other hand, there are formal patterns of reasoning that are not truth-preserving, but nevertheless considered valid, as many, including Prakken and myself, agree. As examples, we have encountered Prakken's *Default modus ponens*, and *Modus ponens* interpreted for defeasible rules.[33] They are, as we have seen, not truth-preserving since their conclusions do not obtain in all cases (or 'possible worlds') in which their premises obtain. They are considered valid since it apparently makes sense to construct arguments with them; those arguments are however defeasible, in contrast with arguments constructed with truth-preserving formal patterns of reasoning. (In the next section, the topic of the defeat of arguments is addressed.)

For now, a question remains: if truth-preservation is no longer a sufficient criterion for the validity of formal patterns of reasoning, then *what is*? Can we just choose them freely? Prakken pays no attention to this question.

Let me at least suggest another criterion for the validity of formal patterns of reasoning (and it is surely just a suggestion). There is *another* sufficient criterion for the validity of formal patterns of reasoning, different from truth-preservation, and it is available in classical logic. The familiar criterion is the following:

A formal pattern of reasoning is valid if it corresponds to a valid rule.[34]

Here rules must be interpreted as classical material implications; to be explicit:

A formal pattern of reasoning is valid if it corresponds to a valid scheme of material implications.

For instance, classically, the validity of *Modus ponens* follows from the fact that the material implication 'If there is a rule with antecedens P and consequens Q, and P, then Q' is valid, i.e., all its instances are true in all possible worlds.

Above I pointed out that to me there are more formal patterns of reasoning than for Prakken. In fact, I think there are many more. As unusual ones, I only mentioned three: *Modus non excipiens*, *Modus odii*, and *Modus analogiae*. In my opinion, it is however true in general that if a rule with antecedens P and consequens Q is valid, the corresponding formal pattern of reasoning (called *If P, then Q*) is:

If P, then Q.
    Premises:
        P.
    Conclusion:
        Q.

Since there are many valid rules (and certainly not only the tautological material implications of classical logic),[35] there are also many valid formal patterns of reasoning.[36] Which formal patterns of reasoning are valid depends (for me) on contingent information. For instance, the argument

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[32]   See, e.g., chapter 2 of my dissertation, where RBL-deduction is defined. RBL-deduction is a monotonic consequence notion on the basis of defeasible rules, in terms of truth-preservation.

[33]   However, recall my opinion about the formal coincidence of Prakken's *Default modus ponens* and *Modus ponens* (section 6.2)

[34]   Recall that rules are conditionals: they have an antecedens and a consequens. The important difference with formal patterns of reasoning (i.e., rules of inference) is that rules are part of the logical language.

[35]   For many examples of other valid rules, see the work on Reason-Based Logic by Hage (1996, 1997) and Verheij (1996).

The damage was the result of John's violation of a social norm of conduct.
Therefore, John has committed a tort.

can be (validly) constructed because the rule

If the damage was the result of someone's violation of a social norm of conduct, then he has committed a tort.

is valid.

It is beside the point here whether this suggestion for a different criterion for the validity of formal patterns of reasoning, is valuable. I just want to emphasize that it does not suffice to postulate formal patterns of reasoning; they have to be chosen according to a criterion. Since, as we have seen, the criterion of truth-preservation does not suffice in a defeasible context, the choice of formal patterns of reasoning must either be left open (as for instance in Vreeswijk's innovative work (1993, 1997) and my CumulA (Verheij, 1996)), or must be explicitly defended.[37]

Prakken pays hardly any attention to these matters. Moreover, the deviating behavior of *Default modus ponens* with respect to classical *Modus ponens* only becomes apparent in the dialectical proof theory (i.e., in terms of dialogues; cf. the overview of Prakken's book in section 3). The dialogical form of the proof theory is nice; however, Prakken's *interpretation* of defeasible rules and its relation to formal patterns of reasoning remains in the dark.

6.6 *Reasoning about rules*

Prakken claims to have 'given a formal account of legal reasoning as reasoning *about* legal knowledge instead of just mechanically *applying* it' (p. 280, original emphasis), and intends to include reasoning about the validity of rules (cf. p. 176).

In my dissertation, I distinguish two types of reasoning about rules (Verheij, 1996, p. 84ff., p. 103ff.), viz. reasoning on the basis of facts about rules (such as their applicability) and reasoning with (the validity of) rules as conclusion. An example of the latter type of reasoning is the classical derivation of $P \rightarrow R$ from $P \rightarrow Q$ and $Q \rightarrow R$.

In my dissertation, I argue that naming techniques, such as Prakken's, that do not connect names for rules with the rules (as conditionals) themselves, can only satisfyingly represent the first type of reasoning about rules. The reason for this is that in Prakken's formalism, rule names are arbitrary and not related to the antecedens and consequens of the rule. This is in contrast with the technique used in Reason-Based Logic, where rules are terms, encoding the rule's antecedens and consequens.

On p. 176, Prakken mentions two approaches to reasoning with the validity of rules. Prakken suggests to simply include *all rules $P \wedge \sim\neg\text{Valid}(d) \Rightarrow Q$ that are allowed by the language in the set of contingent facts*! This indeed results in 'rule validity by default', but is surely in disagreement with Prakken's formalization criteria of structural resemblance and resemblance to natural language. In no sensible interpretation of rule validity, *all* rules with *any* antecedens and *any* consequens are 'valid by default'. As an alternative, Prakken suggests a second approach: add a condition of the form $\text{Valid}(d)$ to any rule. If one wants to allow that reasoning can have the validity of *any* particular rule as its conclusion, this approach requires that *all* rules of the form $P \wedge \text{Valid}(d) \Rightarrow Q$ are included in the contingent information, again in disagreement with Prakken's formalization criteria. (Note that both approaches, as suggested by Prakken, use the expression of rules (as conditionals) in a disguised form, such as $\text{Valid}(d)$, as discussed in section 5 on the logical language.)

Both approaches to reasoning with the validity of rules, as suggested by Prakken, are unsatisfactory, and would be unnecessary if contingent information about rules could be expressed in its full generality.

7 Attack and defeat

Central to defeasible argumentation are the notion of argument and its relation to defeat. A key idea on which Prakken and I agree, is that not all valid arguments justify their conclusion, since they can be defeated by counterarguments.

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[36] This opinion on the connection between valid rules and formal patterns of reasoning is at the heart of my dissertation (Verheij, 1996). It bridges the gap between its two parts, the first on Reason-Based Logic (dealing with valid rules), the second on CumulA (dealing with formal patterns of reasoning).

[37] A third option is to leave formal patterns of reasoning out of consideration, and focus on the semantics of rules (as conditionals) first.

Prakken bases his work on (among others) the innovative work of Dung (1993, 1995). Dung's important contribution is the formal study of the notion of attack among arguments in an abstract setting. In Dung's terms, an attack relation on arguments is any directed graph on a set of abstract (unstructured) arguments.

In Prakken's system there are two notions of attack, viz. by rebutters and by undercutters (p. 174, extending p. 162).[38] E.g., an argument of the form

$$[P \quad P' \quad P \wedge P' \quad P \wedge P' \Rightarrow Q \quad Q]$$

rebuts an argument of the form

$$[P \quad P \Rightarrow \neg Q \quad \neg Q].$$

The reason is that the arguments have conflicting conclusions Q and $\neg Q$, and the first contains the rule $P \wedge P' \Rightarrow Q$ which is more specific than the rule $P \Rightarrow \neg Q$ of the second (p. 174).[39]

An argument of the form

$$[P \quad P \Rightarrow Q \quad Q]$$

undercuts an argument of the form

$$[P' \quad P' \wedge {\sim}Q \Rightarrow R \quad R].$$

Recall that Prakken's ${\sim}Q$ must be distinguished from the ordinary negation $\neg Q$, and that it stands for 'Assuming that not Q'. Arguments with assumptions are undercut if there is an argument for the opposite of the assumption, as in the example.

It should be noted that Prakken's terminology is confusing: he chooses to use the notion of undercutting in a personal way (viz. in terms of justifications/assumptions), differing from Pollock's standard. In Pollock's sense, undercutters attack the connection between the reason and the conclusion rather than attacking the conclusion itself (Pollock, 1987, p. 484/5).[40]

Prakken is certainly aware of the distinction between Pollock's and his own notion, but also chooses to suggest a relation (e.g., on p. 220, where both forms of undercutting an argument are mentioned as one type of conflict between arguments). The distinction is however fundamental: Pollock's undercutters *cannot* be modeled in Prakken's system. In that system, an argument with conclusion Q undercuts an argument if the latter has an assumption ${\sim}Q$. Let's look at the simplest example: P is a reason for Q. In Prakken's formalism, this would (I think) be represented as the argument:

$$[P \quad P \Rightarrow Q \quad Q]^{41}$$

Suppose now that R is an undercutter in Pollock's sense, i.e., R attacks the connection between the reason P and the conclusion Q. However, in Prakken's system, the argument $[P \quad P \Rightarrow Q \quad Q]$ cannot be undercut, *even though the rule in it is defeasible!*[42] Prakken needs to allow undercutting by adding ${\sim}R$ as a justification:

$$[P \quad P \wedge {\sim}R \Rightarrow Q \quad Q]$$

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[38] Regrettably, Prakken uses attack in a sense different from Dung's. Prakken speaks of an argument attacking another if the arguments have inconsistent conclusions (p. 157). As a result, Prakken's attack is a symmetric notion, while Dung uses attack as a directed notion (in agreement with the directed nature of the common-sense notion of attack). Surprisingly, Prakken continues to define a directed notion that *plays exactly the role of Dung's attacks*, which he chooses to call 'defeat'. Prakken is aware of this as he adds a footnote on p. 156 '*to prevent terminological confusion*' [emphasis added].

[39] On p. 192, Prakken defines a form of rebuttal using priority information, instead of specificity.

[40] I consider this confusing terminology especially regrettable since Pollock's distinction between undercutters and rebutters is among the very few ideas in the young and active field of defeasible reasoning that might deserve the predicate 'generally accepted'.

[41] Or maybe simply as $[P \quad P \Rightarrow Q]$, but that is irrelevant for present purposes.

[42] Though the argument cannot be undercut (in Prakken's sense), it is defeasible since it can be rebutted, e.g., by the argument $[P \quad P' \quad P \wedge P' \quad P \wedge P' \Rightarrow \neg Q \quad \neg Q]$.

This is in disagreement with Prakken's formalization criterion of modularity: we must *change* the argument in order to undercut it.

The reason why I think this problem arises in Prakken's formalism, has already been mentioned in section 6.1 on argument structure: in Prakken's formal arguments, the tree-like reason-conclusion structure of arguments is obscured. It is for instance not clear what Prakken considers to be the reasons and the conclusions in his arguments. For instance, it is unclear what in $[P \quad P \Rightarrow Q \quad Q]$ is the reason for Q: P, or the combination (whatever *that* may be) of P and $P \Rightarrow Q$. Though I take it to be the first, neither interpretation would solve the problem sketched.

The fact that the reason-conclusion structure of arguments is obscured in Prakken's formal arguments, is probably also the cause of Prakken's definition of the relation between defeat by undercutters and by rebutters (p. 174). It seems to be unnecessarily involved.

In my dissertation, I have extensively discussed how types of defeat can be distinguished in terms of the reason-conclusion structure of arguments. For instance, Pollock's original distinction between undercutters and rebutters (which, as noted, is unrelated to Prakken's notion of undercutting) can be interpreted purely in terms of the reason-conclusion structure of arguments (Verheij, 1996, p. 120ff.). In this connection I distinguish two *triggers for defeat*: inconsistency and counterarguments. Inconsistency-triggered defeat starts with the inconsistency of argument conclusions. Counterargument-triggered defeat starts from the attack-relation between arguments (in Dung's sense). The typical example of inconsistency-triggered defeat is defeat by a rebutter, the typical example of counterargument-triggered defeat an undercutter (both in Pollock's standard sense).[43]

In terms of the reason-conclusion structure of arguments, I also distinguished sentence-type, step-type, and composite-type defeat (Verheij, 1996, p. 158ff.). Pollock's and Prakken's use of the notion of undercutters can be distinguished in these terms. Prakken's notion of undercutters (viz. in terms of assumptions, as opposed to Pollock's notion) is naturally described as assumption-type defeat, a special case of sentence-type defeat (see note 19). In contrast, Pollock's notion of undercutters is naturally described as step-type defeat.

In my opinion, the problems of Prakken's definitions of attack and defeat occur for similar reasons as those of his definitions of rules: Prakken focuses too much on the technicalities of formalization, but pays insufficient attention to the interpretation of the formalism.

8 The process of argumentation

Currently, it is widely accepted that argumentation, especially if defeasible arguments are involved, is best considered to be a process. Prakken acknowledges this, e.g., in his four layers of argumentation, where he distinguishes a procedural and a strategic layer.

As said (in section 3), Prakken defines which arguments justify their conclusions in terms of what he calls a dialectical proof theory, developed in cooperation with Sartor (p. 166ff.). In the proof theory, dialogues between a proponent and an opponent play a central role.

Prakken claims that the dialectical proof theory provides a procedure to test whether an argument justifies a conclusion (p. 221). In this respect, he contrasts the *procedural* form of an argumentation formalism, with its *declarative* form, according to Prakken usually in terms of the fixed points of some operator. (According to Prakken, the declarative form of an argumentation formalism can be seen as the argumentation-theoretic counterpart of semantics. Surprisingly, Prakken has not included the argumentation-theoretic semantics of his system in the book, though it has been published elsewhere.) My guess is that Prakken speaks of a procedural form because the dialectical proof theory is based on the notion of dialogues, and each dialogue is easily interpreted as a procedure.

There are two reasons why I do not understand Prakken's claim that the dialectical proof theory provides a procedure to test whether an argument justifies a conclusion. The first is that it is not the dialogues themselves, that determine whether an argument justifies a conclusion, but *dialogue trees* (with certain properties). Those are roughly all possible ways in which a dialogue about a particular argument could proceed. And dialogue trees are not easily interpreted as a procedure to test whether an argument justifies a conclusion. It is unclear to me why Prakken thinks that dialogue trees are closer to such a procedure than a semantic definition in terms of fixed points.

The second reason is that Prakken's procedural description *cannot* provide a procedure to test whether an argument justifies a conclusion. As Prakken himself admits (e.g., p. 253ff.), the computational

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[43] In this respect, it is informative to compare Vreeswijk's (1993, 1997) abstract argumentation systems with my CumulA-model. Vreeswijk's model generalizes rebutters to inconsistency-triggered defeat on any set of arguments with inconsistent conclusions, while CumulA generalizes undercutters to counterargument-triggered defeat between any sets of arguments. Both make defeat relative to the whole reason-conclusion structure of arguments, and not just to one 'reason-conclusion pair' as in defeat by undercutters and rebutters.

complexity of first-order predicate logic and consistency checks (both having central places in Prakken and Sartor's argumentation formalism) prevents finding a procedure to that effect.

It is in this connection interesting what Prakken says about the role of an ordinary model-theoretic semantics, such as that of standard first-order predicate logic (in contrast with an argumentation-theoretic semantics in terms of fixed points). To him, such a model-theoretic semantics can be the semantics of *the underlying logic* of the argumentation formalism (recall Prakken's four layers of argumentation). What Prakken means by this remains in the dark. It cannot mean that all arguments 'that can support conclusions at all' (as are specified at the logical layer!), must be truth-preserving with respect to an ordinary model-theoretic semantics. In that case, a serious problem for Prakken's whole enterprise arises, since the *defeasibility* of a *truth-preserving* argument seems to makes no sense; a truth-preserving argument could merely have false premises. (Cf. also section 6 on arguments.) Clearly, ordinary model-theoretic semantics can play a role in defeasible argumentation, but the meaning of Prakken's logical layer apparently involves something more than just that.

9   Summary of the comments

My comments on Prakken's theory of defeasible argumentation, and its formalization, can be summarized as follows.

- Prakken's formalization criteria of resemblance to natural language, structural resemblance, modularity, implementability, and expressiveness, are not always met with regards to the modeling of legal argument (as he himself partially admits). Two provisoes must be made here. The first is that Prakken only intends the criteria in the context of rules and exceptions. The second is that as a result of the lack of informal examples, the reader's (and the reviewer's) opinion on the estimation of these criteria strongly depends on the reader's own interpretations.
- Prakken has not sufficiently defended his choice of formal patterns of reasoning (in the sense of rules of inference). He does neither sufficiently explain which formal patterns of reasoning to choose from, nor which criteria (such as truth-preservation or validity) can guide the choice. A partial reason for this is Prakken's apparently unwarranted distinction between justifying force as a matter of content and as a matter of form.
- Prakken's claim that he has given a formal account of reasoning about legal information, can only partially be maintained. His naming technique can be used to model reasoning on the basis of facts about rules, but not to model reasoning with (the validity of) rules as conclusion. It is also unclear why Prakken's logical language does not allow the expression of nested defeasible rules, such as $R \Rightarrow (P \Rightarrow Q)$, which is the natural way to express that a rule depends on contingent information.
- Prakken's argument that analogical reasoning is a fundamentally different activity than deductive reasoning is flawed, because both can be regarded as formal patterns of reasoning (in the sense of rules of inference), which are at the same time heuristics for suggesting premises and for suggesting conclusions.
- It is unclear why Prakken claims that the dialectical proof theory of the argumentation system provides a procedure to test whether an argument justifies a conclusion, in contrast with the semantics in terms of fixed points. Also Prakken's opinion on the role of an ordinary model-theoretic semantics for the logic underlying the argumentation formalism seems hard to maintain.

My impression is that Prakken's strong focus on formalism instead of on interpretation is a central cause for these comments. In my opinion, Prakken could have strengthened his central message by giving a more explicit specification of his interpretation of what he considers rules and arguments to be. Formalism alone does not suffice in this respect. Intuitive examples are required to bring formalism to life. Partly as a result of the lack of intuitive background in the book, Prakken sometimes forces himself to focus on minute technical detail, thus sacrificing the main lines leading to his goals.

10 Conclusion

In Part I of the review, I have explained why the second, significantly revised edition of Prakken's dissertation on the formalization of defeasible argumentation in the law is indispensable for researchers interested in the logic of law. Prakken's book leaves no room for the opinion that logic has little to offer to the law. In Part II, I have argued that some comments on Prakken's theory of defeasible argumentation, and its formalization, are in place. I suggested Prakken's strong focus on formalism instead of on interpretation as a common cause for the comments. Notwithstanding the comments, I consider Prakken's book as an important and authoritative contribution to the logic of law, and AI and law in general. However, as my comments attempt to show, the last word on the logic of law has not yet been said.

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